

# Computer algebra independent integration tests

5-Inverse-trig-functions/5.5-Inverse-secant/5.5.1-u-a+b-arcsec-c-x-^n

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## Contents

<b>1</b>	<b>Introduction</b>	<b>11</b>
1.1	Listing of CAS systems tested . . . . .	11
1.2	Results . . . . .	12
1.3	Performance . . . . .	16
1.4	list of integrals that has no closed form antiderivative . . . . .	17
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	18
1.6	list of integrals solved by CAS but failed verification . . . . .	18
1.7	Timing . . . . .	19
1.8	Verification . . . . .	19
1.9	Important notes about some of the results . . . . .	19
1.9.1	Important note about Maxima results . . . . .	19
1.9.2	Important note about FriCAS and Giac/XCAS results . . . . .	20
1.9.3	Important note about finding leaf size of antiderivative . . . . .	20
1.9.4	Important note about Mupad results . . . . .	21
1.10	Design of the test system . . . . .	22
<b>2</b>	<b>detailed summary tables of results</b>	<b>23</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	23
2.1.1	Rubi . . . . .	23
2.1.2	Mathematica . . . . .	23
2.1.3	Maple . . . . .	24
2.1.4	Maxima . . . . .	24

2.1.5	FriCAS . . . . .	24
2.1.6	Sympy . . . . .	25
2.1.7	Giac . . . . .	25
2.1.8	Mupad . . . . .	25
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	27
2.3	Detailed conclusion table specific for Rubi results . . . . .	62
<b>3</b>	<b>Listing of integrals</b>	<b>71</b>
3.1	$\int x^6 (a + b \sec^{-1}(cx)) dx$ . . . . .	71
3.2	$\int x^5 (a + b \sec^{-1}(cx)) dx$ . . . . .	81
3.3	$\int x^4 (a + b \sec^{-1}(cx)) dx$ . . . . .	87
3.4	$\int x^3 (a + b \sec^{-1}(cx)) dx$ . . . . .	94
3.5	$\int x^2 (a + b \sec^{-1}(cx)) dx$ . . . . .	99
3.6	$\int x (a + b \sec^{-1}(cx)) dx$ . . . . .	104
3.7	$\int (a + b \sec^{-1}(cx)) dx$ . . . . .	108
3.8	$\int \frac{a+b \sec^{-1}(cx)}{x} dx$ . . . . .	112
3.9	$\int \frac{a+b \sec^{-1}(cx)}{x^2} dx$ . . . . .	116
3.10	$\int \frac{a+b \sec^{-1}(cx)}{x^3} dx$ . . . . .	119
3.11	$\int \frac{a+b \sec^{-1}(cx)}{x^4} dx$ . . . . .	123
3.12	$\int \frac{a+b \sec^{-1}(cx)}{x^5} dx$ . . . . .	127
3.13	$\int \frac{a+b \sec^{-1}(cx)}{x^6} dx$ . . . . .	131
3.14	$\int \frac{a+b \sec^{-1}(cx)}{x^7} dx$ . . . . .	135
3.15	$\int x^3 (a + b \sec^{-1}(cx))^2 dx$ . . . . .	140
3.16	$\int x^2 (a + b \sec^{-1}(cx))^2 dx$ . . . . .	147
3.17	$\int x (a + b \sec^{-1}(cx))^2 dx$ . . . . .	152
3.18	$\int (a + b \sec^{-1}(cx))^2 dx$ . . . . .	157
3.19	$\int \frac{(a+b \sec^{-1}(cx))^2}{x} dx$ . . . . .	161
3.20	$\int \frac{(a+b \sec^{-1}(cx))^2}{x^2} dx$ . . . . .	166
3.21	$\int \frac{(a+b \sec^{-1}(cx))^2}{x^3} dx$ . . . . .	170
3.22	$\int \frac{(a+b \sec^{-1}(cx))^2}{x^4} dx$ . . . . .	174
3.23	$\int \frac{(a+b \sec^{-1}(cx))^2}{x^5} dx$ . . . . .	178
3.24	$\int x^3 (a + b \sec^{-1}(cx))^3 dx$ . . . . .	182

3.25	$\int x^2 (a + b \sec^{-1}(cx))^3 dx$	187
3.26	$\int x (a + b \sec^{-1}(cx))^3 dx$	193
3.27	$\int (a + b \sec^{-1}(cx))^3 dx$	198
3.28	$\int \frac{(a+b \sec^{-1}(cx))^3}{x} dx$	203
3.29	$\int \frac{(a+b \sec^{-1}(cx))^3}{x^2} dx$	209
3.30	$\int \frac{(a+b \sec^{-1}(cx))^3}{x^3} dx$	213
3.31	$\int \frac{(a+b \sec^{-1}(cx))^3}{x^4} dx$	218
3.32	$\int \frac{(a+b \sec^{-1}(cx))^3}{x^5} dx$	224
3.33	$\int \frac{1}{a+b \sec^{-1}(cx)} dx$	230
3.34	$\int \frac{1}{a+b \sec^{-1}(cx)} dx$	233
3.35	$\int \frac{1}{x(a+b \sec^{-1}(cx))} dx$	236
3.36	$\int \frac{1}{x^2(a+b \sec^{-1}(cx))} dx$	239
3.37	$\int \frac{1}{x^3(a+b \sec^{-1}(cx))} dx$	242
3.38	$\int \frac{1}{x^4(a+b \sec^{-1}(cx))} dx$	246
3.39	$\int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx$	250
3.40	$\int \frac{1}{(a+b \sec^{-1}(cx))^2} dx$	253
3.41	$\int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx$	256
3.42	$\int \frac{1}{x^2(a+b \sec^{-1}(cx))^2} dx$	259
3.43	$\int \frac{1}{x^3(a+b \sec^{-1}(cx))^2} dx$	263
3.44	$\int \frac{1}{x^4(a+b \sec^{-1}(cx))^2} dx$	268
3.45	$\int \frac{x}{(a+b \sec^{-1}(cx))^3} dx$	274
3.46	$\int \frac{1}{(a+b \sec^{-1}(cx))^3} dx$	278
3.47	$\int \frac{1}{x(a+b \sec^{-1}(cx))^3} dx$	282
3.48	$\int \frac{1}{x^2(a+b \sec^{-1}(cx))^3} dx$	286

3.49	$\int \frac{1}{x^3(a+b \sec^{-1}(cx))^3} dx$	291
3.50	$\int \frac{1}{x^4(a+b \sec^{-1}(cx))^3} dx$	297
3.51	$\int (dx)^m (a + b \sec^{-1}(cx))^3 dx$	304
3.52	$\int (dx)^m (a + b \sec^{-1}(cx))^2 dx$	307
3.53	$\int (dx)^m (a + b \sec^{-1}(cx)) dx$	309
3.54	$\int \frac{(dx)^m}{a+b \sec^{-1}(cx)} dx$	312
3.55	$\int \frac{(dx)^m}{(a+b \sec^{-1}(cx))^2} dx$	315
3.56	$\int (d + ex)^3 (a + b \sec^{-1}(cx)) dx$	318
3.57	$\int (d + ex)^2 (a + b \sec^{-1}(cx)) dx$	330
3.58	$\int (d + ex) (a + b \sec^{-1}(cx)) dx$	339
3.59	$\int (a + b \sec^{-1}(cx)) dx$	345
3.60	$\int \frac{a+b \sec^{-1}(cx)}{d+ex} dx$	349
3.61	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex)^2} dx$	353
3.62	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex)^3} dx$	358
3.63	$\int (d + ex)^{3/2} (a + b \sec^{-1}(cx)) dx$	364
3.64	$\int \sqrt{d + ex} (a + b \sec^{-1}(cx)) dx$	372
3.65	$\int \frac{a+b \sec^{-1}(cx)}{\sqrt{d+ex}} dx$	378
3.66	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex)^{3/2}} dx$	384
3.67	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex)^{5/2}} dx$	389
3.68	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex)^{7/2}} dx$	396
3.69	$\int x^4 (d + ex^2) (a + b \sec^{-1}(cx)) dx$	404
3.70	$\int x^2 (d + ex^2) (a + b \sec^{-1}(cx)) dx$	409
3.71	$\int (d + ex^2) (a + b \sec^{-1}(cx)) dx$	419
3.72	$\int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^2} dx$	425
3.73	$\int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^4} dx$	430
3.74	$\int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^6} dx$	434
3.75	$\int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^8} dx$	439
3.76	$\int x^5 (d + ex^2) (a + b \sec^{-1}(cx)) dx$	444
3.77	$\int x^3 (d + ex^2) (a + b \sec^{-1}(cx)) dx$	449

3.78	$\int x (d + ex^2) (a + b \sec^{-1}(cx)) dx$	457
3.79	$\int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x} dx$	463
3.80	$\int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^3} dx$	469
3.81	$\int x^2 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$	475
3.82	$\int (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$	481
3.83	$\int \frac{(d+ex^2)^2(a+b \sec^{-1}(cx))}{x^2} dx$	486
3.84	$\int \frac{(d+ex^2)^2(a+b \sec^{-1}(cx))}{x^4} dx$	494
3.85	$\int \frac{(d+ex^2)^2(a+b \sec^{-1}(cx))}{x^6} dx$	502
3.86	$\int \frac{(d+ex^2)^2(a+b \sec^{-1}(cx))}{x^8} dx$	507
3.87	$\int x^3 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$	513
3.88	$\int x (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$	518
3.89	$\int \frac{(d+ex^2)^2(a+b \sec^{-1}(cx))}{x} dx$	529
3.90	$\int \frac{(d+ex^2)^2(a+b \sec^{-1}(cx))}{x^3} dx$	536
3.91	$\int \frac{x^2(a+b \sec^{-1}(cx))}{d+ex^2} dx$	543
3.92	$\int \frac{x(a+b \sec^{-1}(cx))}{d+ex^2} dx$	550
3.93	$\int \frac{a+b \sec^{-1}(cx)}{d+ex^2} dx$	557
3.94	$\int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)} dx$	563
3.95	$\int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)} dx$	570
3.96	$\int \frac{x^5(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$	577
3.97	$\int \frac{x^3(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$	585
3.98	$\int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$	593
3.99	$\int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^2} dx$	598
3.100	$\int \frac{x^4(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$	605
3.101	$\int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$	614

3.102	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^2} dx$	622
3.103	$\int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)^2} dx$	631
3.104	$\int \frac{x^5(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$	640
3.105	$\int \frac{x^3(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$	649
3.106	$\int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$	655
3.107	$\int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^3} dx$	661
3.108	$\int \frac{x^4(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$	668
3.109	$\int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$	678
3.110	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^3} dx$	687
3.111	$\int x^5 \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx$	697
3.112	$\int x^3 \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx$	704
3.113	$\int x \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx$	711
3.114	$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x} dx$	717
3.115	$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3} dx$	720
3.116	$\int x^2 \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx$	723
3.117	$\int \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx$	726
3.118	$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2} dx$	729
3.119	$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^4} dx$	732
3.120	$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^6} dx$	738
3.121	$\int x^3 (d+ex^2)^{3/2} (a+b \sec^{-1}(cx)) dx$	744
3.122	$\int x (d+ex^2)^{3/2} (a+b \sec^{-1}(cx)) dx$	751
3.123	$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x} dx$	758
3.124	$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3} dx$	761
3.125	$\int x^2 (d+ex^2)^{3/2} (a+b \sec^{-1}(cx)) dx$	764

3.126	$\int (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$	767
3.127	$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2} dx$	770
3.128	$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4} dx$	773
3.129	$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^6} dx$	776
3.130	$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^8} dx$	782
3.131	$\int \frac{x^5 (a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$	788
3.132	$\int \frac{x^3 (a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$	795
3.133	$\int \frac{x (a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$	802
3.134	$\int \frac{a+b \sec^{-1}(cx)}{x \sqrt{d+ex^2}} dx$	807
3.135	$\int \frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$	810
3.136	$\int \frac{x^2 (a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$	813
3.137	$\int \frac{a+b \sec^{-1}(cx)}{\sqrt{d+ex^2}} dx$	816
3.138	$\int \frac{a+b \sec^{-1}(cx)}{x^2 \sqrt{d+ex^2}} dx$	819
3.139	$\int \frac{a+b \sec^{-1}(cx)}{x^4 \sqrt{d+ex^2}} dx$	825
3.140	$\int \frac{a+b \sec^{-1}(cx)}{x^6 \sqrt{d+ex^2}} dx$	831
3.141	$\int \frac{x^5 (a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	838
3.142	$\int \frac{x^3 (a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	845
3.143	$\int \frac{x (a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	852
3.144	$\int \frac{a+b \sec^{-1}(cx)}{x (d+ex^2)^{3/2}} dx$	856
3.145	$\int \frac{a+b \sec^{-1}(cx)}{x^3 (d+ex^2)^{3/2}} dx$	859
3.146	$\int \frac{x^4 (a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	862
3.147	$\int \frac{x^2 (a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	865

3.148	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^{3/2}} dx$	868
3.149	$\int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}} dx$	872
3.150	$\int \frac{a+b \sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}} dx$	878
3.151	$\int \frac{x^5(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	885
3.152	$\int \frac{x^3(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	892
3.153	$\int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	897
3.154	$\int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^{5/2}} dx$	902
3.155	$\int \frac{a+b \sec^{-1}(cx)}{x^3(d+ex^2)^{5/2}} dx$	905
3.156	$\int \frac{x^6(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	908
3.157	$\int \frac{x^4(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	911
3.158	$\int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	914
3.159	$\int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^{5/2}} dx$	920
3.160	$\int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}} dx$	926
3.161	$\int (fx)^m (d+ex^2)^3 (a+b \sec^{-1}(cx)) dx$	934
3.162	$\int (fx)^m (d+ex^2)^2 (a+b \sec^{-1}(cx)) dx$	940
3.163	$\int (fx)^m (d+ex^2) (a+b \sec^{-1}(cx)) dx$	945
3.164	$\int \frac{(fx)^m (a+b \sec^{-1}(cx))}{d+ex^2} dx$	949
3.165	$\int \frac{(fx)^m (a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$	952
3.166	$\int (fx)^m (d+ex^2)^{3/2} (a+b \sec^{-1}(cx)) dx$	955
3.167	$\int (fx)^m \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx$	958
3.168	$\int \frac{(fx)^m (a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$	961
3.169	$\int \frac{(fx)^m (a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	964



3.170	$\int \frac{x^{11}(a+b \sec^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$	967
3.171	$\int \frac{x^7(a+b \sec^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$	973
3.172	$\int \frac{x^3(a+b \sec^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$	979
3.173	$\int \frac{a+b \sec^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx$	984
3.174	$\int \frac{a+b \sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} dx$	987
<b>4</b>	<b>Listing of Grading functions</b>	<b>991</b>
4.0.1	Mathematica and Rubi grading function	991
4.0.2	Maple grading function	993
4.0.3	Sympy grading function	998
4.0.4	SageMath grading function	1001



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 174 ]. This is test number [ 156 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric<sub>2</sub>F<sub>1</sub> functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 174 )	% 0.00 ( 0 )
Mathematica	% 97.13 ( 169 )	% 2.87 ( 5 )
Maple	% 79.89 ( 139 )	% 20.11 ( 35 )
Maxima	% 48.28 ( 84 )	% 51.72 ( 90 )
Fricas	% 62.07 ( 108 )	% 37.93 ( 66 )
Sympy	% 38.51 ( 67 )	% 61.49 ( 107 )
Giac	% 52.87 ( 92 )	% 47.13 ( 82 )
Mupad	% 30.46 ( 53 )	% 69.54 ( 121 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

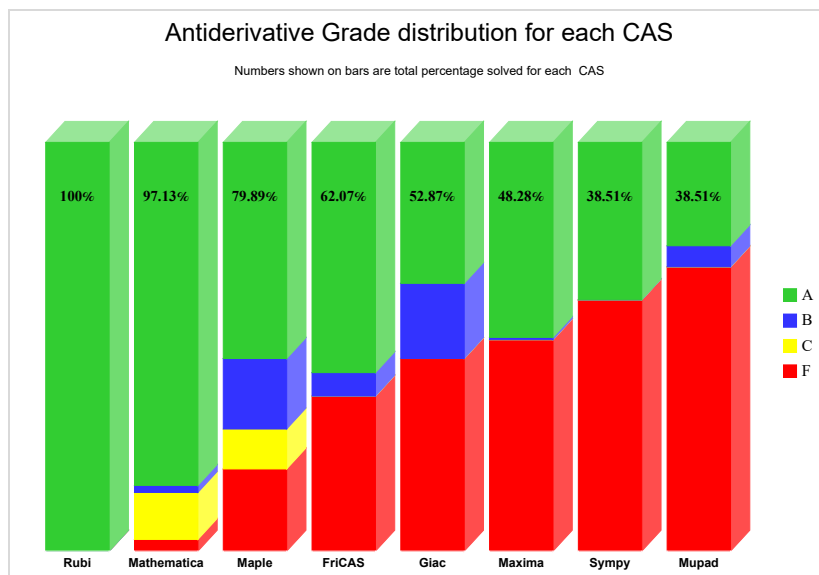
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

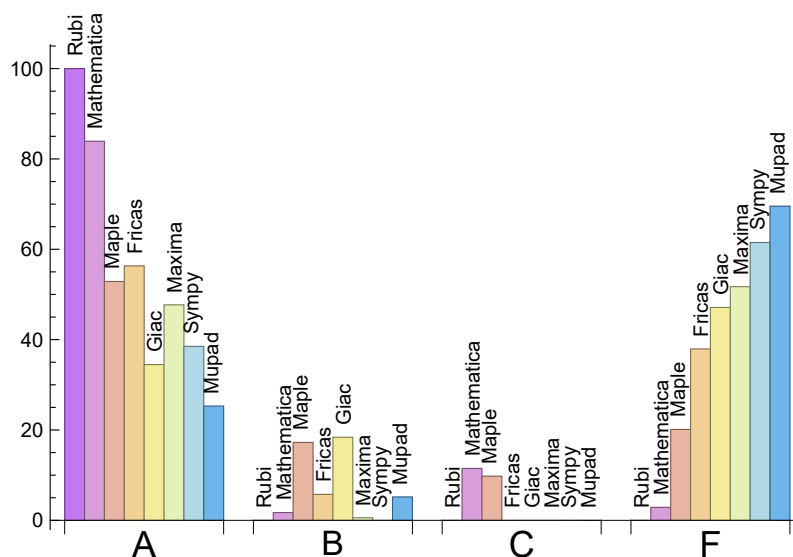
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.91	1.72	11.49	2.87
Maple	52.87	17.24	9.77	20.11
Maxima	47.70	0.57	0.00	51.72
Fricas	56.32	5.75	0.00	37.93
Sympy	38.51	0.00	0.00	61.49
Giac	34.48	18.39	0.00	47.13
Mupad	25.29	5.17	0.00	69.54

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	5	100.00 %	0.00 %	0.00 %
Maple	35	100.00 %	0.00 %	0.00 %
Maxima	90	87.78 %	5.56 %	6.67 %
Fricas	66	89.39 %	6.06 %	4.55 %
Sympy	107	58.88 %	41.12 %	0.00 %
Giac	82	57.32 %	14.63 %	28.05 %
Mupad	121	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

## 1.3 Performance

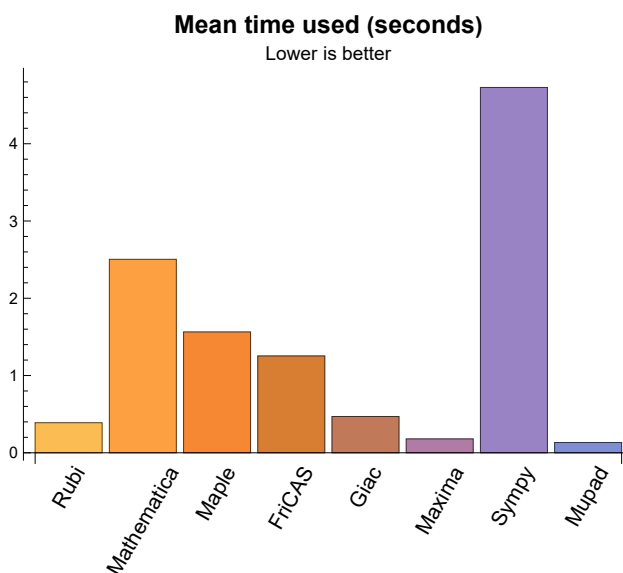
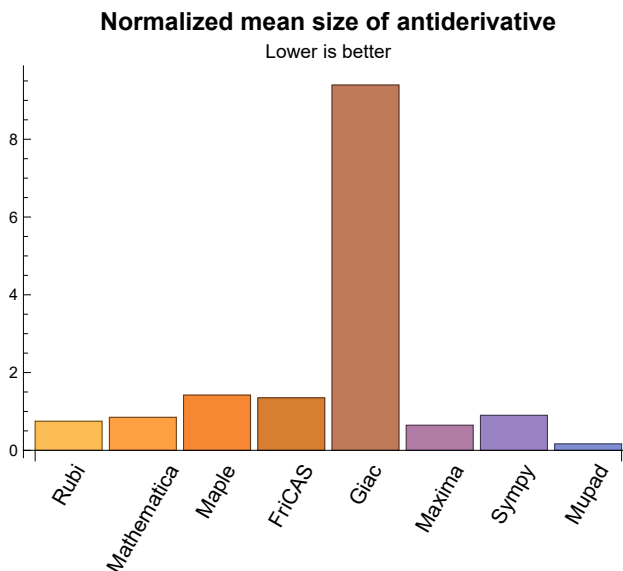
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.39	197.64	0.75	129.50	1.00
Mathematica	2.50	228.67	0.85	124.00	0.89
Maple	1.56	396.19	1.42	145.00	1.15
Maxima	0.18	77.67	0.65	0.00	0.00
Fricas	1.25	243.11	1.35	63.00	0.69
Sympy	4.73	119.03	0.90	32.00	1.00
Giac	0.47	1145.74	9.40	60.50	1.06
Mupad	0.13	10.26	0.17	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {163}

**Mathematica** {8, 16, 19, 24, 25, 26, 27, 28, 60, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 108, 109, 110, 111, 121}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

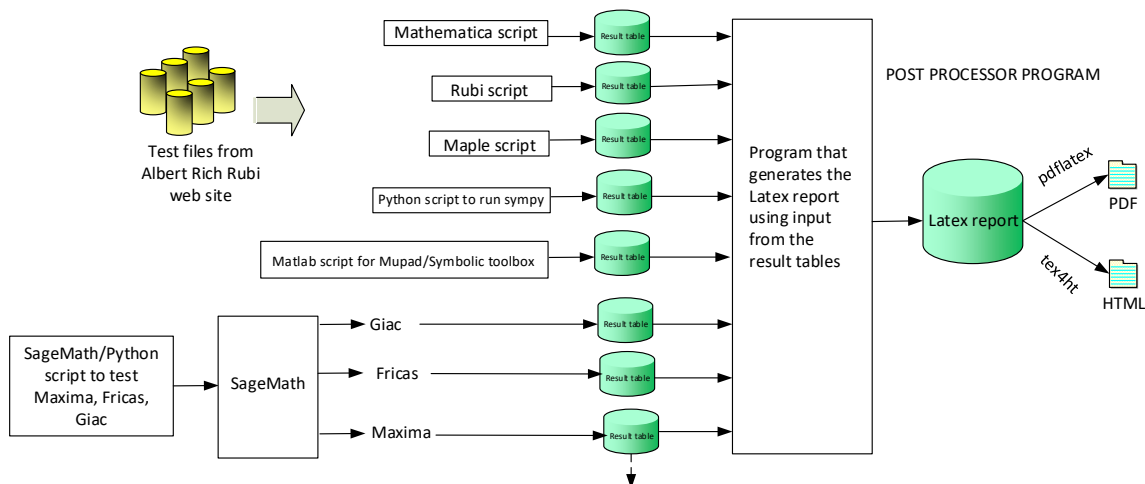
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 122, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 96, 97, 104 }

C grade: { 63, 64, 65, 67, 68, 98, 105, 106, 111, 119, 120, 121, 129, 130, 139, 140, 149, 150, 159, 160 }

F grade: { 99, 107, 161, 162, 163 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 18, 19, 22, 26, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 58, 59, 60, 64, 65, 66, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174 }

B grade: { 5, 10, 12, 15, 16, 17, 20, 21, 23, 24, 25, 28, 29, 30, 31, 32, 56, 57, 61, 62, 63, 67, 68, 70, 71, 81, 82, 98, 105, 106 }

C grade: { 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110 }

F grade: { 27, 53, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 20, 22, 29, 33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 54, 55, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174 }

B grade: { 31 }

C grade: { }

F grade: { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 52, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 56, 57, 58, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 98, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174 }

B grade: { 7, 17, 59, 61, 62, 105, 106, 151, 152, 153 }

C grade: { }



F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 63, 64, 65, 66, 67, 68, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 119, 120, 129, 130, 138, 139, 140, 148, 149, 150, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 114, 115, 116, 117, 118, 123, 124, 126, 127, 128, 134, 135, 136, 147, 164, 167, 168, 173, 174 }

B grade: { }

C grade: { }

F grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 125, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 169, 170, 171, 172 }

## 2.1.7 Giac

A grade: { 9, 10, 11, 12, 13, 14, 21, 22, 23, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 54, 55, 73, 74, 75, 85, 86, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 15, 17, 20, 29, 30, 31, 32, 42, 43, 44, 48, 49, 50, 56, 57, 58, 59, 70, 71, 72, 77, 78, 83, 84, 88 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 45, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 79, 80, 81, 82, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

## 2.1.8 Mupad

A grade: { 33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174 }

B grade: { 6, 7, 9, 10, 20, 29, 58, 59, 72 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	177	162	116	221	8644	-1
normalized size	1	1.00	0.94	1.55	1.42	1.02	1.94	75.82	-0.01
time (sec)	N/A	0.061	0.143	0.075	0.418	0.943	8.461	1.754	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	83	81	63	153	3862	-1
normalized size	1	1.00	0.81	0.93	0.91	0.71	1.72	43.39	-0.01
time (sec)	N/A	0.041	0.091	0.048	0.326	0.713	4.192	0.233	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	97	150	131	107	175	4828	-1
normalized size	1	1.00	1.09	1.69	1.47	1.20	1.97	54.25	-0.01
time (sec)	N/A	0.046	0.076	0.049	0.309	0.686	5.401	1.316	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	74	60	53	107	1926	-1
normalized size	1	1.00	0.97	1.16	0.94	0.83	1.67	30.09	-0.02
time (sec)	N/A	0.027	0.106	0.050	0.308	0.698	2.789	0.197	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	85	123	98	94	107	2101	-1
normalized size	1	1.00	1.33	1.92	1.53	1.47	1.67	32.83	-0.02
time (sec)	N/A	0.035	0.054	0.048	0.314	0.760	3.286	0.824	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	65	37	40	58	634	40
normalized size	1	1.00	1.28	1.67	0.95	1.03	1.49	16.26	1.03
time (sec)	N/A	0.012	0.023	0.046	0.314	0.545	1.992	0.177	0.672

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	34
normalized size	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	1.06
time (sec)	N/A	0.020	0.068	0.043	0.314	0.990	2.326	0.136	0.842

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	59	86	0	0	0	0	-1
normalized size	1	1.00	0.92	1.34	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.017	0.186	0.000	0.576	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	40	62	33	27	36	43	36
normalized size	1	1.00	1.29	2.00	1.06	0.87	1.16	1.39	1.16
time (sec)	N/A	0.020	0.031	0.047	0.313	1.226	1.773	0.147	0.636

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	118	83	39	119	58	50
normalized size	1	1.00	1.29	2.31	1.63	0.76	2.33	1.14	0.98
time (sec)	N/A	0.033	0.035	0.049	0.412	0.581	3.139	0.137	0.729

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	75	58	40	110	65	-1
normalized size	1	1.00	0.98	1.25	0.97	0.67	1.83	1.08	-0.02
time (sec)	N/A	0.039	0.048	0.048	0.318	0.680	2.930	0.148	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	147	125	52	192	83	-1
normalized size	1	1.00	1.03	1.93	1.64	0.68	2.53	1.09	-0.01
time (sec)	N/A	0.045	0.062	0.048	0.422	0.709	5.235	0.151	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	83	76	51	156	87	-1
normalized size	1	1.00	0.84	1.01	0.93	0.62	1.90	1.06	-0.01
time (sec)	N/A	0.049	0.072	0.049	0.335	0.796	6.985	0.133	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	174	165	62	241	104	-1
normalized size	1	1.00	0.87	1.72	1.63	0.61	2.39	1.03	-0.01
time (sec)	N/A	0.059	0.079	0.051	0.426	0.763	8.433	0.138	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	124	208	163	146	0	6625	-1
normalized size	1	1.00	1.16	1.94	1.52	1.36	0.00	61.92	-0.01
time (sec)	N/A	0.107	0.224	0.229	0.688	0.915	0.000	0.808	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	225	343	0	0	0	0	-1
normalized size	1	1.00	1.53	2.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	1.271	0.906	0.000	0.491	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	90	134	87	111	0	2181	-1
normalized size	1	1.00	1.61	2.39	1.55	1.98	0.00	38.95	-0.02
time (sec)	N/A	0.068	0.151	0.257	0.328	0.764	0.000	0.481	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	163	212	0	0	0	0	-1
normalized size	1	1.00	1.77	2.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.182	0.255	0.000	1.022	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	129	215	0	0	0	0	-1
normalized size	1	1.00	1.39	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.130	0.215	0.000	0.413	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	75	117	78	57	0	105	89
normalized size	1	1.00	1.50	2.34	1.56	1.14	0.00	2.10	1.78
time (sec)	N/A	0.059	0.130	0.143	0.328	0.481	0.000	0.168	0.818

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	102	199	0	82	0	147	-1
normalized size	1	1.00	1.09	2.12	0.00	0.87	0.00	1.56	-0.01
time (sec)	N/A	0.079	0.121	0.149	0.000	0.589	0.000	0.164	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	154	164	93	0	168	-1
normalized size	1	1.00	1.06	1.51	1.61	0.91	0.00	1.65	-0.01
time (sec)	N/A	0.094	0.197	0.485	0.704	0.618	0.000	0.156	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	148	265	0	120	0	215	-1
normalized size	1	1.00	1.10	1.98	0.00	0.90	0.00	1.60	-0.01
time (sec)	N/A	0.111	0.180	0.430	0.000	0.708	0.000	0.149	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	288	447	0	0	0	0	-1
normalized size	1	1.00	1.39	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.827	1.019	0.000	0.662	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	403	687	0	0	0	0	-1
normalized size	1	1.00	1.71	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	1.444	1.047	0.000	0.682	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	184	285	0	0	0	0	-1
normalized size	1	1.00	1.46	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.495	0.796	0.000	0.658	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	289	0	0	0	0	0	-1
normalized size	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.246	0.622	0.000	0.631	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	204	390	0	0	0	0	-1
normalized size	1	1.00	1.59	3.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.194	0.263	0.000	0.473	0.000	0.000	0.000



Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	141	198	146	98	0	196	156
normalized size	1	1.00	1.76	2.48	1.82	1.22	0.00	2.45	1.95
time (sec)	N/A	0.083	0.174	0.181	0.335	0.688	0.000	0.185	0.801

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	185	324	0	150	0	278	-1
normalized size	1	1.00	1.35	2.36	0.00	1.09	0.00	2.03	-0.01
time (sec)	N/A	0.105	0.219	0.374	0.000	0.674	0.000	0.177	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	204	299	575	172	0	336	-1
normalized size	1	1.00	1.20	1.76	3.38	1.01	0.00	1.98	-0.01
time (sec)	N/A	0.147	0.305	0.594	0.981	0.706	0.000	0.190	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	283	472	0	225	0	427	-1
normalized size	1	1.00	1.36	2.27	0.00	1.08	0.00	2.05	-0.00
time (sec)	N/A	0.173	0.366	0.641	0.000	0.725	0.000	0.192	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	2.585	2.017	0.000	0.720	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	2.314	0.611	0.000	0.600	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	0.291	0.924	0.000	0.516	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	47	0	0	0	55	-1
normalized size	1	1.00	0.93	1.02	0.00	0.00	0.00	1.20	-0.02
time (sec)	N/A	0.105	0.080	0.140	0.000	0.674	0.000	0.149	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	95	-1
normalized size	1	1.00	0.89	0.92	0.00	0.00	0.00	1.51	-0.02
time (sec)	N/A	0.136	0.077	0.103	0.000	0.535	0.000	0.130	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	102	0	0	0	199	-1
normalized size	1	1.00	0.78	0.87	0.00	0.00	0.00	1.70	-0.01
time (sec)	N/A	0.225	0.179	0.106	0.000	0.426	0.000	0.141	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	10.296	2.241	0.000	0.459	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	22.775	0.593	0.000	0.592	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.024	3.467	1.003	0.000	0.981	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	78	0	0	0	226	-1
normalized size	1	1.00	0.92	1.04	0.00	0.00	0.00	3.01	-0.01
time (sec)	N/A	0.129	0.280	0.150	0.000	1.433	0.000	0.129	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	80	77	0	0	0	357	-1
normalized size	1	1.00	0.95	0.92	0.00	0.00	0.00	4.25	-0.01
time (sec)	N/A	0.149	0.368	0.105	0.000	0.674	0.000	0.155	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	223	153	0	0	0	694	-1
normalized size	1	1.00	1.25	0.86	0.00	0.00	0.00	3.90	-0.01
time (sec)	N/A	0.267	0.476	0.114	0.000	0.705	0.000	0.219	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	3.675	2.550	0.000	0.488	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	12.210	0.689	0.000	0.546	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	1.793	1.183	0.000	0.815	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	88	154	0	0	0	580	-1
normalized size	1	1.00	0.85	1.50	0.00	0.00	0.00	5.63	-0.01
time (sec)	N/A	0.147	0.393	0.156	0.000	0.457	0.000	0.141	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	114	157	0	0	0	929	-1
normalized size	1	1.00	1.02	1.40	0.00	0.00	0.00	8.29	-0.01
time (sec)	N/A	0.180	0.381	0.107	0.000	0.437	0.000	0.144	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	169	307	0	0	0	1640	-1
normalized size	1	1.00	0.74	1.35	0.00	0.00	0.00	7.19	-0.00
time (sec)	N/A	0.313	0.465	0.121	0.000	0.435	0.000	0.160	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	6.791	3.314	0.000	0.467	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	4.514	3.118	0.000	0.473	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	82	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.257	4.371	0.000	0.429	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.832	3.829	0.000	0.401	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.713	2.131	0.000	0.407	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	166	486	272	291	362	9404	-1
normalized size	1	1.00	0.99	2.91	1.63	1.74	2.17	56.31	-0.01
time (sec)	N/A	0.401	0.300	0.056	0.329	0.656	7.876	2.941	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	362	200	209	228	6418	-1
normalized size	1	1.00	1.00	2.92	1.61	1.69	1.84	51.76	-0.01
time (sec)	N/A	0.266	0.190	0.054	0.327	0.576	6.600	4.956	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	114	141	93	130	104	1555	77
normalized size	1	1.00	1.36	1.68	1.11	1.55	1.24	18.51	0.92
time (sec)	N/A	0.167	0.223	0.051	0.322	0.449	4.423	0.677	0.899

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	34
normalized size	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	1.06
time (sec)	N/A	0.022	0.054	0.044	0.322	0.432	2.270	0.150	0.851

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	333	456	0	0	0	0	-1
normalized size	1	1.00	1.35	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.622	0.820	0.000	0.434	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	142	214	0	477	0	0	-1
normalized size	1	1.00	1.37	2.06	0.00	4.59	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.229	0.060	0.000	0.487	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	247	1005	0	1117	0	0	-1
normalized size	1	1.00	1.44	5.84	0.00	6.49	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.495	0.067	0.000	0.994	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	333	810	0	0	0	0	-1
normalized size	1	1.00	0.90	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	1.480	0.204	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	277	388	0	0	0	0	-1
normalized size	1	1.00	0.88	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	6.601	0.069	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	212	254	0	0	0	0	-1
normalized size	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	2.815	0.066	0.000	1.849	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	124	217	0	0	0	0	-1
normalized size	1	1.00	1.04	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.290	0.066	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	326	886	0	0	0	0	-1
normalized size	1	1.00	1.09	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	6.636	0.082	0.000	1.407	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	637	407	1640	0	0	0	0	-1
normalized size	1	1.18	0.75	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	8.914	0.088	0.000	0.000	0.000	0.000	0.000



Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	141	338	296	192	408	0	-1
normalized size	1	1.00	0.68	1.64	1.44	0.93	1.98	0.00	-0.00
time (sec)	N/A	0.128	0.238	0.055	0.330	1.929	11.684	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	123	282	232	170	294	9820	-1
normalized size	1	1.00	0.76	1.75	1.44	1.06	1.83	60.99	-0.01
time (sec)	N/A	0.103	0.173	0.053	0.326	2.032	7.296	3.035	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	150	195	154	141	153	4069	-1
normalized size	1	1.00	1.38	1.79	1.41	1.29	1.40	37.33	-0.01
time (sec)	N/A	0.051	0.283	0.052	0.313	2.086	5.720	2.054	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	104	137	89	123	73	1098	72
normalized size	1	1.00	1.20	1.57	1.02	1.41	0.84	12.62	0.83
time (sec)	N/A	0.063	0.121	0.058	0.319	0.454	5.110	0.619	0.788

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	69	121	94	67	150	116	-1
normalized size	1	1.00	0.66	1.15	0.90	0.64	1.43	1.10	-0.01
time (sec)	N/A	0.075	0.081	0.059	0.320	2.058	4.541	0.163	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	94	140	137	89	279	162	-1
normalized size	1	1.00	0.62	0.92	0.90	0.59	1.84	1.07	-0.01
time (sec)	N/A	0.094	0.130	0.059	0.323	1.066	9.293	0.161	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	110	158	172	110	371	207	-1
normalized size	1	1.00	0.56	0.80	0.87	0.56	1.88	1.05	-0.01
time (sec)	N/A	0.117	0.151	0.062	0.319	1.486	55.096	0.160	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	118	152	185	128	364	0	-1
normalized size	1	1.00	0.60	0.78	0.94	0.65	1.86	0.00	-0.01
time (sec)	N/A	0.147	0.237	0.054	0.333	1.977	7.924	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	134	144	107	272	7840	-1
normalized size	1	1.00	0.64	0.88	0.94	0.70	1.78	51.24	-0.01
time (sec)	N/A	0.122	0.265	0.052	0.313	0.647	5.293	0.310	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	79	115	100	86	177	3360	-1
normalized size	1	1.00	0.57	0.83	0.72	0.62	1.28	24.35	-0.01
time (sec)	N/A	0.093	0.108	0.052	0.317	0.670	3.658	0.228	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	115	142	0	0	0	0	-1
normalized size	1	1.00	0.93	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.090	2.625	0.000	1.981	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	132	145	0	0	0	0	-1
normalized size	1	1.00	0.96	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.097	0.851	0.000	0.579	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	186	494	405	273	542	0	-1
normalized size	1	1.00	0.74	1.96	1.61	1.08	2.15	0.00	-0.00
time (sec)	N/A	0.236	0.374	0.054	0.344	4.040	13.680	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	153	372	296	237	355	0	-1
normalized size	1	1.00	0.80	1.95	1.55	1.24	1.86	0.00	-0.01
time (sec)	N/A	0.115	0.241	0.053	0.344	2.285	9.792	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	136	286	198	230	207	6010	-1
normalized size	1	1.00	0.84	1.77	1.22	1.42	1.28	37.10	-0.01
time (sec)	N/A	0.127	0.205	0.060	0.338	1.106	8.564	2.946	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	127	255	159	221	211	4960	-1
normalized size	1	1.00	0.80	1.61	1.01	1.40	1.34	31.39	-0.01
time (sec)	N/A	0.131	0.228	0.063	0.319	3.064	7.988	14.579	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	127	191	181	127	333	223	-1
normalized size	1	1.00	0.69	1.04	0.99	0.69	1.82	1.22	-0.01
time (sec)	N/A	0.157	0.227	0.063	0.332	1.322	10.529	0.154	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	153	223	241	159	508	294	-1
normalized size	1	1.00	0.63	0.93	1.00	0.66	2.11	1.22	-0.00
time (sec)	N/A	0.193	0.239	0.065	0.330	1.548	57.340	0.158	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	162	214	256	187	493	0	-1
normalized size	1	1.00	0.67	0.88	1.06	0.77	2.04	0.00	-0.00
time (sec)	N/A	0.224	0.292	0.054	0.336	0.983	9.006	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	125	182	192	153	352	11858	-1
normalized size	1	1.00	0.64	0.93	0.98	0.78	1.81	60.81	-0.01
time (sec)	N/A	0.148	0.308	0.052	0.333	1.438	6.163	0.406	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	160	242	0	0	0	0	-1
normalized size	1	1.00	0.86	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.412	0.388	3.521	0.000	0.446	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	187	218	0	0	0	0	-1
normalized size	1	1.00	0.99	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.417	0.664	2.877	0.000	0.498	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	1023	374	0	0	0	0	-1
normalized size	1	1.00	1.87	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.294	1.469	3.800	0.000	0.522	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	891	453	0	0	0	0	-1
normalized size	1	1.00	1.83	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.152	0.458	1.041	0.000	0.605	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	871	272	0	0	0	0	-1
normalized size	1	1.00	1.71	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	0.401	1.893	0.000	0.600	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	402	2933	0	0	0	0	-1
normalized size	1	1.00	0.88	6.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.896	0.926	1.028	0.000	0.502	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	997	331	0	0	0	0	-1
normalized size	1	1.00	1.81	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.096	1.501	3.666	0.000	0.848	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	1255	783	0	0	0	0	-1
normalized size	1	1.00	2.06	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.306	4.251	1.668	0.000	0.594	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	1213	594	0	0	0	0	-1
normalized size	1	1.00	2.13	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.216	1.322	1.369	0.000	0.844	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	286	354	0	384	0	0	-1
normalized size	1	1.00	2.18	2.70	0.00	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.644	0.076	0.000	0.606	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	0	3095	0	0	0	0	-1
normalized size	1	1.00	0.00	5.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.140	37.048	2.679	0.000	0.528	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	784	784	1331	1887	0	0	0	0	-1
normalized size	1	1.00	1.70	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.354	2.537	16.500	0.000	0.646	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	1245	1756	0	0	0	0	-1
normalized size	1	1.00	1.67	2.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.217	1.803	3.780	0.000	1.362	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	1239	1748	0	0	0	0	-1
normalized size	1	1.00	1.68	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.211	2.283	4.343	0.000	0.623	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	785	785	1291	1817	0	0	0	0	-1
normalized size	1	1.00	1.64	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.307	2.247	15.836	0.000	0.771	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	707	707	1805	1626	0	0	0	0	-1
normalized size	1	1.00	2.55	2.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.404	8.014	1.964	0.000	0.710	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	389	1870	0	1015	0	0	-1
normalized size	1	1.00	2.48	11.91	0.00	6.46	0.00	0.00	-0.01
time (sec)	N/A	0.175	1.420	0.082	0.000	1.797	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	386	1840	0	888	0	0	-1
normalized size	1	1.00	2.00	9.53	0.00	4.60	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.977	0.074	0.000	1.888	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	0	5373	0	0	0	0	-1
normalized size	1	1.00	0.00	7.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.291	52.487	6.522	0.000	0.672	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1124	1124	1819	3223	0	0	0	0	-1
normalized size	1	1.00	1.62	2.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.583	6.201	4.414	0.000	0.657	0.000	0.000	0.000



Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1124	1124	1827	2357	0	0	0	0	-1
normalized size	1	1.00	1.63	2.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.022	6.138	6.712	0.000	1.112	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1114	1114	1812	3214	0	0	0	0	-1
normalized size	1	1.00	1.63	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.791	6.062	7.829	0.000	0.651	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	366	0	0	1701	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.22	0.00	0.00	-0.00
time (sec)	N/A	1.275	0.671	4.859	0.000	7.031	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	327	0	0	1383	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	4.70	0.00	0.00	-0.00
time (sec)	N/A	0.398	0.773	4.493	0.000	4.130	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	266	0	0	1100	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	5.64	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.486	3.336	0.000	2.293	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.098	4.472	2.209	0.000	0.956	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.101	4.673	0.947	0.000	0.743	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	9.232	3.904	0.000	1.002	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	4.372	2.306	0.000	0.624	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.086	1.503	1.748	0.000	0.531	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	247	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.651	3.297	0.000	1.427	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	325	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.739	4.681	0.000	0.673	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	339	0	0	1701	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.55	0.00	0.00	-0.00
time (sec)	N/A	0.508	0.627	4.330	0.000	12.021	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	305	0	0	1377	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	5.26	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.768	3.270	0.000	3.166	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.115	4.969	2.196	0.000	0.593	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.119	5.541	0.834	0.000	0.693	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.112	9.593	3.938	0.000	0.813	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	5.534	2.263	0.000	0.775	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.103	8.888	1.678	0.000	1.039	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	11.479	3.175	0.000	0.644	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	303	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	0.678	4.555	0.000	1.221	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	383	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	0.856	5.489	0.000	1.149	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	328	0	0	1385	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	4.31	0.00	0.00	-0.00
time (sec)	N/A	1.012	0.752	5.363	0.000	5.855	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	272	0	0	1111	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	4.94	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.520	4.820	0.000	1.401	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	211	0	0	869	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	6.58	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.238	3.495	0.000	0.880	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.097	1.490	2.375	0.000	1.366	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.108	11.128	1.442	0.000	1.015	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.091	9.689	3.167	0.000	0.550	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	0.849	2.528	0.000	1.979	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	143	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.206	2.226	0.000	0.846	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	249	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	0.665	4.688	0.000	1.380	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1006	1006	329	0	0	0	0	0	-1
normalized size	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.780	0.740	6.309	0.000	0.808	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	303	0	0	1483	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	5.88	0.00	0.00	-0.00
time (sec)	N/A	1.006	0.637	5.162	0.000	1.033	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	221	0	0	1070	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	6.82	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.363	4.655	0.000	1.003	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	95	0	0	283	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	3.54	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.152	3.364	0.000	0.641	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	15.675	2.135	0.000	0.609	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.123	22.172	1.276	0.000	0.887	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	8.194	3.350	0.000	0.582	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.099	3.870	3.146	0.000	1.299	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	113	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.199	2.310	0.000	0.685	0.000	0.000	0.000



Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	212	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.485	2.118	0.000	0.782	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	701	701	292	0	0	0	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.392	0.739	4.457	0.000	1.107	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	312	0	0	2123	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	8.70	0.00	0.00	-0.00
time (sec)	N/A	1.063	0.539	5.329	0.000	1.257	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	172	0	0	664	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	4.07	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.273	4.628	0.000	0.977	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	158	0	0	571	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	4.14	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.210	3.471	0.000	1.537	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.120	26.800	2.136	0.000	0.567	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.138	33.123	1.258	0.000	0.503	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.116	10.322	3.691	0.000	1.490	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	9.674	3.365	0.000	1.650	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	186	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.292	3.151	0.000	0.906	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	248	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.554	2.311	0.000	2.144	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	323	0	0	0	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.396	0.783	2.075	0.000	0.807	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	589	570	0	0	0	0	0	0	-1
normalized size	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.415	0.222	9.227	0.000	1.768	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	355	0	0	0	0	0	0	-1
normalized size	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	0.146	7.521	0.000	0.865	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	204	0	0	0	0	0	0	-1
normalized size	1	1.15	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.110	6.105	0.000	1.527	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	2.040	3.145	0.000	0.561	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	5.387	5.103	0.000	0.555	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.105	0.898	2.835	0.000	0.546	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	0.113	2.798	0.000	1.015	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.096	1.147	3.767	0.000	0.881	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.105	1.417	3.786	0.000	1.647	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	194	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.484	0.251	10.870	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	159	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.018	0.325	7.001	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	118	0	0	0	0	0	-1
normalized size	1	1.07	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.220	4.447	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.360	4.082	0.000	1.767	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	6.548	4.770	0.000	1.772	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [160] had the largest ratio of [.7826]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714
25	A	11	8	1.00	14	0.571
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	A	0	0	0.00	0	0.000
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	4	4	1.00	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	5	5	1.00	14	0.357
43	A	7	7	1.00	14	0.500
44	A	11	6	1.00	14	0.429
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	0	0	0.00	0	0.000
48	A	6	5	1.00	14	0.357
49	A	8	7	1.00	14	0.500
50	A	13	6	1.00	14	0.429
51	A	0	0	0.00	0	0.000
52	A	0	0	0.00	0	0.000
53	A	3	3	1.00	14	0.214
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	11	9	1.00	16	0.562
57	A	10	9	1.00	16	0.562
58	A	9	9	1.00	14	0.643
59	A	5	4	1.00	8	0.500
60	A	4	2	1.00	16	0.125
61	A	7	7	1.00	16	0.438
62	A	8	8	1.00	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	22	13	1.00	18	0.722
64	A	15	11	1.00	18	0.611
65	A	9	9	1.00	18	0.500
66	A	6	6	1.00	18	0.333
67	A	12	11	1.00	18	0.611
68	A	19	14	1.18	18	0.778
69	A	7	7	1.00	19	0.368
70	A	6	7	1.00	19	0.368
71	A	5	5	1.00	16	0.312
72	A	4	5	1.00	19	0.263
73	A	4	5	1.00	19	0.263
74	A	5	6	1.00	19	0.316
75	A	6	6	1.00	19	0.316
76	A	5	5	1.00	19	0.263
77	A	5	5	1.00	19	0.263
78	A	6	5	1.00	17	0.294
79	A	11	11	1.00	19	0.579
80	A	13	13	1.00	19	0.684
81	A	7	8	1.00	21	0.381
82	A	6	7	1.00	18	0.389
83	A	6	7	1.00	21	0.333
84	A	6	7	1.00	21	0.333
85	A	5	6	1.00	21	0.286
86	A	6	7	1.00	21	0.333
87	A	5	6	1.00	21	0.286
88	A	6	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	12	13	1.00	21	0.619
90	A	14	15	1.00	21	0.714
91	A	25	12	1.00	21	0.571
92	A	26	9	1.00	19	0.474
93	A	19	7	1.00	18	0.389
94	A	19	7	1.00	21	0.333
95	A	24	10	1.00	21	0.476
96	A	31	14	1.00	21	0.667
97	A	29	12	1.00	21	0.571
98	A	7	5	1.00	19	0.263
99	A	24	10	1.00	21	0.476
100	A	51	15	1.00	21	0.714
101	A	27	10	1.00	21	0.476
102	A	47	11	1.00	18	0.611
103	A	50	13	1.00	21	0.619
104	A	33	13	1.00	21	0.619
105	A	6	7	1.00	21	0.333
106	A	8	6	1.00	19	0.316
107	A	28	11	1.00	21	0.524
108	A	35	11	1.00	21	0.524
109	A	63	12	1.00	21	0.571
110	A	81	12	1.00	18	0.667
111	A	12	12	1.00	23	0.522
112	A	11	12	1.00	23	0.522
113	A	9	9	1.00	21	0.429
114	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	0	0	0.00	0	0.000
116	A	0	0	0.00	0	0.000
117	A	0	0	0.00	0	0.000
118	A	0	0	0.00	0	0.000
119	A	11	11	1.00	23	0.478
120	A	12	12	1.00	23	0.522
121	A	12	12	1.00	23	0.522
122	A	10	10	1.00	21	0.476
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	0	0	0.00	0	0.000
127	A	0	0	0.00	0	0.000
128	A	0	0	0.00	0	0.000
129	A	12	12	1.00	23	0.522
130	A	13	12	1.00	23	0.522
131	A	11	12	1.00	23	0.522
132	A	10	12	1.00	23	0.522
133	A	8	8	1.00	21	0.381
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000
136	A	0	0	0.00	0	0.000
137	A	0	0	0.00	0	0.000
138	A	11	11	1.00	23	0.478
139	A	11	12	1.00	23	0.522
140	A	32	15	1.00	23	0.652

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
141	A	10	11	1.00	23	0.478
142	A	9	11	1.00	23	0.478
143	A	4	4	1.00	21	0.190
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	0	0	0.00	0	0.000
148	A	5	5	1.00	20	0.250
149	A	10	11	1.00	23	0.478
150	A	25	14	1.00	23	0.609
151	A	10	11	1.00	23	0.478
152	A	7	8	1.00	23	0.348
153	A	5	5	1.00	21	0.238
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000
157	A	0	0	0.00	0	0.000
158	A	10	10	1.00	23	0.435
159	A	10	11	1.00	20	0.550
160	A	26	18	1.00	23	0.783
161	A	6	7	0.97	23	0.304
162	A	6	7	0.95	23	0.304
163	A	5	6	1.15	21	0.286
164	A	0	0	0.00	0	0.000
165	A	0	0	0.00	0	0.000
166	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
167	A	0	0	0.00	0	0.000
168	A	0	0	0.00	0	0.000
169	A	0	0	0.00	0	0.000
170	A	16	11	1.00	26	0.423
171	A	13	11	1.00	26	0.423
172	A	8	9	1.07	26	0.346
173	A	0	0	0.00	0	0.000
174	A	0	0	0.00	0	0.000



# Chapter 3

## Listing of integrals

### 3.1 $\int x^6 \left( a + b \sec^{-1}(cx) \right) dx$

Optimal. Leaf size=114

$$\frac{1}{7}x^7 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^6 \sqrt{1 - \frac{1}{c^2x^2}}}{42c} - \frac{5b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{112c^7} - \frac{5bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{112c^5} - \frac{5bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{168c^3}$$

[Out] 1/7\*x^7\*(a+b\*arcsec(c\*x))-5/112\*b\*arctanh((1-1/c^2/x^2)^(1/2))/c^7-5/112\*b\*x^2\*(1-1/c^2/x^2)^(1/2)/c^5-5/168\*b\*x^4\*(1-1/c^2/x^2)^(1/2)/c^3-1/42\*b\*x^6\*(1-1/c^2/x^2)^(1/2)/c

**Rubi** [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5220, 266, 51, 63, 208}

$$\frac{1}{7}x^7 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^6 \sqrt{1 - \frac{1}{c^2x^2}}}{42c} - \frac{5bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{168c^3} - \frac{5bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{112c^5} - \frac{5b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{112c^7}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*ArcSec[c\*x]),x]

[Out] (-5\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(112\*c^5) - (5\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4)/(168\*c^3) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^6)/(42\*c) + (x^7\*(a + b\*ArcSec[c\*x]))/7 - (5\*b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(112\*c^7)

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int x^6 (a + b \sec^{-1}(cx)) dx &= \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) - \frac{b \int \frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{7c} \\
&= \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) + \frac{b \text{Subst} \left( \int \frac{1}{x^4 \sqrt{1-\frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{14c} \\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}} x^6}{42c} + \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) + \frac{(5b) \text{Subst} \left( \int \frac{1}{x^3 \sqrt{1-\frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{84c^3} \\
&= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^6}{42c} + \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) + \frac{(5b) \text{Subst} \left( \int \frac{1}{x^2 \sqrt{1-\frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{112c^5} \\
&= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^6}{42c} + \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) + \dots \quad (5b) \\
&= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^6}{42c} + \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) - \dots \quad (5b) \\
&= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}} x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^6}{42c} + \frac{1}{7}x^7 (a + b \sec^{-1}(cx)) - \dots \quad 5b \text{ ta}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 107, normalized size = 0.94

$$\frac{ax^7}{7} - \frac{5b \log \left( x \left( \sqrt{\frac{c^2x^2-1}{c^2x^2}} + 1 \right) \right)}{112c^7} + b\sqrt{\frac{c^2x^2-1}{c^2x^2}} \left( -\frac{5x^2}{112c^5} - \frac{5x^4}{168c^3} - \frac{x^6}{42c} \right) + \frac{1}{7}bx^7 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*ArcSec[c\*x]),x]

[Out] (a\*x^7)/7 + b\*sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*((-5\*x^2)/(112\*c^5) - (5\*x^4)/(168\*c^3) - x^6/(42\*c)) + (b\*x^7\*ArcSec[c\*x])/7 - (5\*b\*Log[x\*(1 + Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(112\*c^7)

**fricas [A]** time = 0.94, size = 116, normalized size = 1.02

$$48 ac^7 x^7 + 96 bc^7 \arctan \left( -cx + \sqrt{c^2x^2 - 1} \right) + 48 (bc^7 x^7 - bc^7) \operatorname{arcsec}(cx) + 15b \log \left( -cx + \sqrt{c^2x^2 - 1} \right) - (8b$$





$$\begin{aligned}
& 1/(c*x) + 1)^6) - 525*b*(1/(c^2*x^2) - 1)^3*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) \\
& + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1 \\
& / (c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1 \\
& )^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1) \\
& ^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/( \\
& c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) + 525*b*(1/(c^2*x^2) - 1 \\
& )^3*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^ \\
& 2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c \\
& ^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x \\
& ) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) \\
& - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c* \\
& x) + 1)^6) - 170*b*(1/(c^2*x^2) - 1)^2*\text{sqrt}(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8 \\
& *(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + \\
& 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1 \\
& )^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*( \\
& 1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1) \\
& ^14)*(1/(c*x) + 1)^5) - 1680*a*(1/(c^2*x^2) - 1)^3/((c^8 + 7*c^8*(1/(c^2*x^ \\
& 2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c \\
& ^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x \\
& ) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) \\
& - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c* \\
& x) + 1)^6) + 1680*b*(1/(c^2*x^2) - 1)^4*\text{arccos}(1/(c*x))/((c^8 + 7*c^8*(1/(c \\
& ^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + \\
& 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1 \\
& / (c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2 \\
& *x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*( \\
& 1/(c*x) + 1)^8) - 525*b*(1/(c^2*x^2) - 1)^4*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) \\
& + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1 \\
& / (c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1 \\
& )^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1) \\
& ^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/( \\
& c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) + 525*b*(1/(c^2*x^2) - 1 \\
& )^4*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^ \\
& 2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c \\
& ^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x \\
& ) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) \\
& - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c* \\
& x) + 1)^8) + 1680*a*(1/(c^2*x^2) - 1)^4/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/ \\
& (c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2* \\
& x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + \\
& 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/ \\
& (c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) \\
& - 1008*b*(1/(c^2*x^2) - 1)^5*\text{arccos}(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1) \\
& )/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/ \\
& (c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
\end{aligned}$$



$$\begin{aligned} & ) / (1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4 + 35*c^8*(1/ \\ & (c^2*x^2) - 1)^3 / (1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4 / (1/(c*x) + 1) \\ & ^8 + 21*c^8*(1/(c^2*x^2) - 1)^5 / (1/(c*x) + 1)^{10} + 7*c^8*(1/(c^2*x^2) - 1)^6 / \\ & (1/(c*x) + 1)^{12} + c^8*(1/(c^2*x^2) - 1)^7 / (1/(c*x) + 1)^{14} * (1/(c*x) + 1) \\ & ^{12} - 48*b*(1/(c^2*x^2) - 1)^7 * \arccos(1/(c*x)) / ((c^8 + 7*c^8*(1/(c^2*x^2) \\ & - 1)) / (1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4 + 35*c^8 \\ & *(1/(c^2*x^2) - 1)^3 / (1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4 / (1/(c*x) \\ & + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5 / (1/(c*x) + 1)^{10} + 7*c^8*(1/(c^2*x^2) - \\ & 1)^6 / (1/(c*x) + 1)^{12} + c^8*(1/(c^2*x^2) - 1)^7 / (1/(c*x) + 1)^{14} * (1/(c*x) \\ & + 1)^{14} - 15*b*(1/(c^2*x^2) - 1)^7 * \log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) \\ & + 1)) / ((c^8 + 7*c^8*(1/(c^2*x^2) - 1)) / (1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x \\ & ^2) - 1)^2 / (1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3 / (1/(c*x) + 1)^6 + 3 \\ & 5*c^8*(1/(c^2*x^2) - 1)^4 / (1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5 / (1/( \\ & c*x) + 1)^{10} + 7*c^8*(1/(c^2*x^2) - 1)^6 / (1/(c*x) + 1)^{12} + c^8*(1/(c^2*x^2 \\ & ) - 1)^7 / (1/(c*x) + 1)^{14} * (1/(c*x) + 1)^{14} + 15*b*(1/(c^2*x^2) - 1)^7 * \log \\ & (\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1)) / ((c^8 + 7*c^8*(1/(c^2*x^2) - 1) \\ & / (1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4 + 35*c^8*(1/( \\ & c^2*x^2) - 1)^3 / (1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4 / (1/(c*x) + 1)^ \\ & 8 + 21*c^8*(1/(c^2*x^2) - 1)^5 / (1/(c*x) + 1)^{10} + 7*c^8*(1/(c^2*x^2) - 1)^6 \\ & / (1/(c*x) + 1)^{12} + c^8*(1/(c^2*x^2) - 1)^7 / (1/(c*x) + 1)^{14} * (1/(c*x) + 1) \\ & ^{14} + 66*b*(1/(c^2*x^2) - 1)^6 * \sqrt{-1/(c^2*x^2) + 1} / ((c^8 + 7*c^8*(1/(c^ \\ & 2*x^2) - 1)) / (1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4 + \\ & 35*c^8*(1/(c^2*x^2) - 1)^3 / (1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4 / (1/ \\ & (c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5 / (1/(c*x) + 1)^{10} + 7*c^8*(1/(c^2* \\ & x^2) - 1)^6 / (1/(c*x) + 1)^{12} + c^8*(1/(c^2*x^2) - 1)^7 / (1/(c*x) + 1)^{14} * (1 \\ & / (c*x) + 1)^{13} - 48*a*(1/(c^2*x^2) - 1)^7 / ((c^8 + 7*c^8*(1/(c^2*x^2) - 1) / \\ & (1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4 + 35*c^8*(1/(c \\ & ^2*x^2) - 1)^3 / (1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4 / (1/(c*x) + 1)^8 \\ & + 21*c^8*(1/(c^2*x^2) - 1)^5 / (1/(c*x) + 1)^{10} + 7*c^8*(1/(c^2*x^2) - 1)^6 / \\ & (1/(c*x) + 1)^{12} + c^8*(1/(c^2*x^2) - 1)^7 / (1/(c*x) + 1)^{14} * (1/(c*x) + 1)^{ \\ & 14} ) \end{aligned}$$

**maple [A]** time = 0.08, size = 177, normalized size = 1.55

$$\frac{x^7 a}{7} + \frac{b x^7 \operatorname{arcsec}(c x)}{7} - \frac{b x^6}{42 c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b x^4}{168 c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{5 b x^2}{336 c^5 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{5 b}{112 c^7 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{5 b \sqrt{c^2 x^2 - 1} \ln\left(cx + \sqrt{c^2 x^2 - 1}\right)}{112 c^8 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(a+b\*arcsec(c\*x)),x)

[Out]  $\frac{1}{7} x^7 a + \frac{1}{7} b x^7 \operatorname{arcsec}(c x) - \frac{1}{42} \frac{b}{c} \frac{1}{((c^2 x^2 - 1)/c^2/x^2)^{1/2}} x^6 - \frac{1}{168} \frac{b}{c^3} \frac{1}{((c^2 x^2 - 1)/c^2/x^2)^{1/2}} x^4 - \frac{5}{336} \frac{b}{c^5} \frac{1}{((c^2 x^2 - 1)/c^2/x^2)^{1/2}} x^2 + \frac{5}{112} \frac{b}{c^7} \frac{1}{((c^2 x^2 - 1)/c^2/x^2)^{1/2}} - \frac{5}{112} \frac{b}{c^8} \frac{1}{((c^2 x^2 - 1)/c^2/x^2)^{1/2}} \frac{1}{x} \ln(c x + (c^2 x^2 - 1)^{1/2})$

**maxima** [A] time = 0.42, size = 162, normalized size = 1.42

$$\frac{1}{7}ax^7 + \frac{1}{672} \left( 96x^7 \operatorname{arcsec}(cx) - \frac{2 \left( 15 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 40 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 33 \sqrt{-\frac{1}{c^2x^2} + 1} \right)}{c^6 \left( \frac{1}{c^2x^2} - 1 \right)^3 + 3c^6 \left( \frac{1}{c^2x^2} - 1 \right)^2 + 3c^6 \left( \frac{1}{c^2x^2} - 1 \right) + c^6} + \frac{15 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} + 1 \right)}{c^6} - \frac{15 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} - 1 \right)}{c^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/7\*a\*x^7 + 1/672\*(96\*x^7\*arcsec(c\*x) - (2\*(15\*(-1/(c^2\*x^2) + 1)^(5/2) - 40\*(-1/(c^2\*x^2) + 1)^(3/2) + 33\*sqrt(-1/(c^2\*x^2) + 1))/(c^6\*(1/(c^2\*x^2) - 1)^3 + 3\*c^6\*(1/(c^2\*x^2) - 1)^2 + 3\*c^6\*(1/(c^2\*x^2) - 1) + c^6) + 15\*log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^6 - 15\*log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^6)/c)\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^6\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 8.46, size = 221, normalized size = 1.94

$$\frac{ax^7}{7} + \frac{bx^7}{7} \operatorname{asec}(cx) - \frac{b \left( \begin{array}{l} \left( \frac{cx^7}{6\sqrt{c^2x^2-1}} + \frac{x^5}{24c\sqrt{c^2x^2-1}} + \frac{5x^3}{48c^3\sqrt{c^2x^2-1}} - \frac{5x}{16c^5\sqrt{c^2x^2-1}} + \frac{5 \operatorname{acosh}(cx)}{16c^6} \right) \text{ for } |c^2x^2| > 1 \\ \left( -\frac{icx^7}{6\sqrt{-c^2x^2+1}} - \frac{ix^5}{24c\sqrt{-c^2x^2+1}} - \frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}} + \frac{5ix}{16c^5\sqrt{-c^2x^2+1}} - \frac{5i \operatorname{asin}(cx)}{16c^6} \right) \text{ otherwise} \end{array} \right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(a+b\*asec(c\*x)),x)

[Out] a\*x\*\*7/7 + b\*x\*\*7\*asec(c\*x)/7 - b\*Piecewise((c\*x\*\*7/(6\*sqrt(c\*\*2\*x\*\*2 - 1)) + x\*\*5/(24\*c\*sqrt(c\*\*2\*x\*\*2 - 1)) + 5\*x\*\*3/(48\*c\*\*3\*sqrt(c\*\*2\*x\*\*2 - 1)) -

```

5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2)
> 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 +
1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*
x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)

```



### 3.2 $\int x^5 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=89

$$\frac{1}{6}x^6 (a + b \sec^{-1}(cx)) - \frac{bx^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{30c} - \frac{4bx \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^5} - \frac{2bx^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^3}$$

[Out]  $\frac{1}{6}x^6(a+b*\text{arcsec}(c*x)) - \frac{4}{45}b*x*(1-1/c^2/x^2)^{(1/2)}/c^5 - \frac{2}{45}b*x^3*(1-1/c^2/x^2)^{(1/2)}/c^3 - \frac{1}{30}b*x^5*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5220, 271, 191}

$$\frac{1}{6}x^6 (a + b \sec^{-1}(cx)) - \frac{bx^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{30c} - \frac{2bx^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^3} - \frac{4bx \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^5}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcSec[c\*x]), x]

[Out]  $\frac{(-4*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(45*c^5) - (2*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3)/(45*c^3) - (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*\text{ArcSec}[c*x]))/6}$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[(d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x])/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int x^5 (a + b \sec^{-1}(cx)) dx &= \frac{1}{6}x^6 (a + b \sec^{-1}(cx)) - \frac{b \int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{6c} \\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}} x^5}{30c} + \frac{1}{6}x^6 (a + b \sec^{-1}(cx)) - \frac{(2b) \int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{15c^3} \\
&= -\frac{2b\sqrt{1-\frac{1}{c^2x^2}} x^3}{45c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^5}{30c} + \frac{1}{6}x^6 (a + b \sec^{-1}(cx)) - \frac{(4b) \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{45c^5} \\
&= -\frac{4b\sqrt{1-\frac{1}{c^2x^2}} x}{45c^5} - \frac{2b\sqrt{1-\frac{1}{c^2x^2}} x^3}{45c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^5}{30c} + \frac{1}{6}x^6 (a + b \sec^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 72, normalized size = 0.81

$$\frac{ax^6}{6} + b\sqrt{\frac{c^2x^2-1}{c^2x^2}} \left( -\frac{4x}{45c^5} - \frac{2x^3}{45c^3} - \frac{x^5}{30c} \right) + \frac{1}{6}bx^6 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcSec[c\*x]),x]

[Out] (a\*x^6)/6 + b\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*((-4\*x)/(45\*c^5) - (2\*x^3)/(45\*c^3) - x^5/(30\*c)) + (b\*x^6\*ArcSec[c\*x])/6

**fricas [A]** time = 0.71, size = 63, normalized size = 0.71

$$\frac{15bc^6x^6 \operatorname{arcsec}(cx) + 15ac^6x^6 - (3bc^4x^4 + 4bc^2x^2 + 8b)\sqrt{c^2x^2-1}}{90c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/90\*(15\*b\*c^6\*x^6\*arcsec(c\*x) + 15\*a\*c^6\*x^6 - (3\*b\*c^4\*x^4 + 4\*b\*c^2\*x^2 + 8\*b)\*sqrt(c^2\*x^2 - 1))/c^6

**giac [B]** time = 0.23, size = 3862, normalized size = 43.39

result too large to display



$$\begin{aligned}
& 8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^5 - 300*a*(1/(c^2*x^2) - 1)^3/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^6 + 225*b*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^8) - 156*b*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^7) + 225*a*(1/(c^2*x^2) - 1)^4/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^8) - 90*b*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^{10}) - 70*b*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^9) - 90*a*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^{11}) + 15*a*(1/(c^2*x^2) - 1)^6/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^{12})
\end{aligned}$$

\*(1/(c\*x) + 1)^12))

**maple** [A] time = 0.05, size = 83, normalized size = 0.93

$$\frac{\frac{c^6 x^6 a}{6} + b \left( \frac{c^6 x^6 \operatorname{arcsec}(cx)}{6} - \frac{(c^2 x^2 - 1)(3c^4 x^4 + 4c^2 x^2 + 8)}{90 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} cx} \right)}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arcsec(c\*x)),x)

[Out] 1/c^6\*(1/6\*c^6\*x^6\*a+b\*(1/6\*c^6\*x^6\*arcsec(c\*x)-1/90\*(c^2\*x^2-1)\*(3\*c^4\*x^4+4\*c^2\*x^2+8)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x))

**maxima** [A] time = 0.33, size = 81, normalized size = 0.91

$$\frac{1}{6} ax^6 + \frac{1}{90} \left( 15 x^6 \operatorname{arcsec}(cx) - \frac{3c^4 x^5 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{5}{2}} + 10c^2 x^3 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{3}{2}} + 15x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/6\*a\*x^6 + 1/90\*(15\*x^6\*arcsec(c\*x) - (3\*c^4\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 10\*c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^5)\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^5\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 4.19, size = 153, normalized size = 1.72

$$\frac{ax^6}{6} + \frac{bx^6 \operatorname{asec}(cx)}{6} - \frac{b \left( \begin{cases} \frac{x^4 \sqrt{c^2 x^2 - 1}}{5c} + \frac{4x^2 \sqrt{c^2 x^2 - 1}}{15c^3} + \frac{8\sqrt{c^2 x^2 - 1}}{15c^5} & \text{for } |c^2 x^2| > 1 \\ \frac{ix^4 \sqrt{-c^2 x^2 + 1}}{5c} + \frac{4ix^2 \sqrt{-c^2 x^2 + 1}}{15c^3} + \frac{8i\sqrt{-c^2 x^2 + 1}}{15c^5} & \text{otherwise} \end{cases} \right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asec(c*x)),x)
```

```
[Out] a*x**6/6 + b*x**6*asec(c*x)/6 - b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c)
+ 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5),
Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c*
**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c)
```

### 3.3 $\int x^4 \left( a + b \sec^{-1}(cx) \right) dx$

**Optimal.** Leaf size=89

$$\frac{1}{5}x^5 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{20c} - \frac{3b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{40c^5} - \frac{3bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{40c^3}$$

[Out]  $\frac{1}{5}x^5(a+b*\text{arcsec}(c*x)) - \frac{3}{40}b*\text{arctanh}\left(\left(1-\frac{1}{c^2/x^2}\right)^{(1/2)}\right)/c^5 - \frac{3}{40}b*x^2*(1-\frac{1}{c^2/x^2})^{(1/2)}/c^3 - \frac{1}{20}b*x^4*(1-\frac{1}{c^2/x^2})^{(1/2)}/c$

**Rubi [A]** time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5220, 266, 51, 63, 208}

$$\frac{1}{5}x^5 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{20c} - \frac{3bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{40c^3} - \frac{3b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{40c^5}$$

Antiderivative was successfully verified.

[In] `Int[x^4*(a + b*ArcSec[c*x]),x]`

[Out]  $(-3*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^2)/(40*c^3) - (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^4)/(20*c) + (x^5*(a + b*\text{ArcSec}[c*x]))/5 - (3*b*\text{ArcTanh}[\text{Sqrt}[1 - 1/(c^2*x^2)]])/ (40*c^5)$

#### Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[p[((d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int x^4 (a + b \sec^{-1}(cx)) dx &= \frac{1}{5} x^5 (a + b \sec^{-1}(cx)) - \frac{b \int \frac{x^3}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{5c} \\
 &= \frac{1}{5} x^5 (a + b \sec^{-1}(cx)) + \frac{b \operatorname{Subst} \left( \int \frac{1}{x^3 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{10c} \\
 &= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \sec^{-1}(cx)) + \frac{(3b) \operatorname{Subst} \left( \int \frac{1}{x^2 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{40c^3} \\
 &= -\frac{3b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{40c^3} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \sec^{-1}(cx)) + \frac{(3b) \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{80c^5} \\
 &= -\frac{3b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{40c^3} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \sec^{-1}(cx)) - \frac{(3b) \operatorname{Subst} \left( \int \frac{1}{c^2 - c^2 x^2} dx, x, \frac{1}{x^2} \right)}{40c^3} \\
 &= -\frac{3b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{40c^3} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \sec^{-1}(cx)) - \frac{3b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{40c^5}
 \end{aligned}$$



**Mathematica [A]** time = 0.08, size = 97, normalized size = 1.09

$$\frac{ax^5}{5} - \frac{3b \log\left(x \left(\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 1\right)\right)}{40c^5} + b\sqrt{\frac{c^2x^2-1}{c^2x^2}} \left(-\frac{3x^2}{40c^3} - \frac{x^4}{20c}\right) + \frac{1}{5}bx^5 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcSec[c\*x]),x]

[Out] (a\*x^5)/5 + b\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*((-3\*x^2)/(40\*c^3) - x^4/(20\*c)) + (b\*x^5\*ArcSec[c\*x])/5 - (3\*b\*Log[x\*(1 + Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(40\*c^5)

**fricas [A]** time = 0.69, size = 107, normalized size = 1.20

$$\frac{8ac^5x^5 + 16bc^5 \arctan(-cx + \sqrt{c^2x^2 - 1}) + 8(bc^5x^5 - bc^5) \operatorname{arcsec}(cx) + 3b \log(-cx + \sqrt{c^2x^2 - 1}) - (2bc^3x^3 - 3bc^2x^2 + 2bcx - c)}{40c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/40\*(8\*a\*c^5\*x^5 + 16\*b\*c^5\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + 8\*(b\*c^5\*x^5 - b\*c^5)\*arcsec(c\*x) + 3\*b\*log(-c\*x + sqrt(c^2\*x^2 - 1)) - (2\*b\*c^3\*x^3 + 3\*b\*c^2\*x^2 - 2\*b\*c\*x + c))/c^5

**giac [B]** time = 1.32, size = 4828, normalized size = 54.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] 1/40\*c\*(8\*b\*arccos(1/(c\*x))/(c^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 10\*c^6\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 10\*c^6\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + c^6\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10) - 3\*b\*log(abs(sqrt(-1/(c^2\*x^2) + 1) + 1/(c\*x) + 1))/(c^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 10\*c^6\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 10\*c^6\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + c^6\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10) + 3\*b\*log(abs(sqrt(-1/(c^2\*x^2) + 1) - 1/(c\*x) - 1))/(c^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 10\*c^6\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 10\*c^6\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + c^6\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10) + 8\*a/(c^6 + 5\*c

$$\begin{aligned}
& ^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) \\
& + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - \\
& 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} - 40*b*(1/ \\
& (c^2*x^2) - 1)*\arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1 \\
& )^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1) \\
& ^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^ \\
& 2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^2) - 15*b*(1/(c^2*x^2) - 1)*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^2) + 15*b*(1/(c^2*x^2) - 1)*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^2) - 10*b*\sqrt{-1/(c^2*x^2) + 1}/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)) - 40*a*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})* (1/(c*x) + 1)^2) + 80*b*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^4) - 30*b*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^4) + 30*b*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^4) + 4*b*(-1/(c^2*x^2) + 1)^{(3/2)}/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^3) + 80*a*(1/(c^2*x^2) - 1)^2/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^4) - 80*b*(1/(c^2*x^2) - 1)^3*\arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^4)
\end{aligned}$$



\*sqrt(-1/(c^2\*x^2) + 1)/((c^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 10\*c^6\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 10\*c^6\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + c^6\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10)\*(1/(c\*x) + 1)^9 - 8\*a\*(1/(c^2\*x^2) - 1)^5/((c^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 10\*c^6\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 10\*c^6\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 5\*c^6\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + c^6\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10)\*(1/(c\*x) + 1)^10))

**maple [A]** time = 0.05, size = 150, normalized size = 1.69

$$\frac{ax^5}{5} + \frac{x^5 b \operatorname{arcsec}(cx)}{5} - \frac{bx^4}{20c\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{bx^2}{40c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{3b}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{3b\sqrt{c^2x^2-1} \ln\left(cx + \sqrt{c^2x^2-1}\right)}{40c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arcsec(c\*x)),x)

[Out] 1/5\*a\*x^5+1/5\*x^5\*b\*arcsec(c\*x)-1/20/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4-1/40/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2+3/40/c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)-3/40/c^6\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*ln(c\*x+(c^2\*x^2-1)^(1/2))

**maxima [A]** time = 0.31, size = 131, normalized size = 1.47

$$\frac{1}{5} ax^5 + \frac{1}{80} \left( 16x^5 \operatorname{arcsec}(cx) + \frac{2 \left( 3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 5 \sqrt{-\frac{1}{c^2x^2} + 1} \right)}{c^4 \left( \frac{1}{c^2x^2} - 1 \right)^2 + 2c^4 \left( \frac{1}{c^2x^2} - 1 \right) + c^4} - \frac{3 \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)}{c^4} + \frac{3 \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} - 1\right)}{c^4} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/5\*a\*x^5 + 1/80\*(16\*x^5\*arcsec(c\*x) + (2\*(3\*(-1/(c^2\*x^2) + 1)^(3/2) - 5\*sqrt(-1/(c^2\*x^2) + 1))/(c^4\*(1/(c^2\*x^2) - 1)^2 + 2\*c^4\*(1/(c^2\*x^2) - 1) + c^4) - 3\*log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^4 + 3\*log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^4)/c)\*b

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a + b*acos(1/(c*x))),x)`

[Out] `int(x^4*(a + b*acos(1/(c*x))), x)`

sympy [A] time = 5.40, size = 175, normalized size = 1.97

$$\frac{ax^5}{5} + \frac{bx^5 \operatorname{asec}(cx)}{5} - \frac{b \left( \begin{array}{l} \left( \frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3 \operatorname{acosh}(cx)}{8c^4} \right) \text{ for } |c^2x^2| > 1 \\ \left( -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i \operatorname{asin}(cx)}{8c^4} \right) \text{ otherwise} \end{array} \right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*asec(c*x)),x)`

[Out] `a*x**5/5 + b*x**5*asec(c*x)/5 - b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)`

### 3.4 $\int x^3 \left( a + b \sec^{-1}(cx) \right) dx$

**Optimal.** Leaf size=64

$$\frac{1}{4}x^4 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^3 \sqrt{1 - \frac{1}{c^2x^2}}}{12c} - \frac{bx \sqrt{1 - \frac{1}{c^2x^2}}}{6c^3}$$

[Out]  $\frac{1}{4}x^4(a+b*\text{arcsec}(c*x)) - \frac{1}{6}b*x*(1-1/c^2/x^2)^{(1/2)}/c^3 - \frac{1}{12}b*x^3*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5220, 271, 191}

$$\frac{1}{4}x^4 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^3 \sqrt{1 - \frac{1}{c^2x^2}}}{12c} - \frac{bx \sqrt{1 - \frac{1}{c^2x^2}}}{6c^3}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*ArcSec[c*x]),x]`

[Out]  $-(b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(6*c^3) - (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3)/(12*c) + (x^4*(a + b*\text{ArcSec}[c*x]))/4$

#### Rule 191

`Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

#### Rule 271

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]`

#### Rule 5220

`Int[((a_) + ArcSec[(c_)*(x_)])*(b_)*((d_)*(x_)^(m_)), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m + 1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]`

#### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \sec^{-1}(cx)) dx &= \frac{1}{4}x^4 (a + b \sec^{-1}(cx)) - \frac{b \int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{4c} \\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{1}{4}x^4 (a + b \sec^{-1}(cx)) - \frac{b \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{6c^3} \\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{1}{4}x^4 (a + b \sec^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 62, normalized size = 0.97

$$\frac{ax^4}{4} + b\sqrt{\frac{c^2x^2-1}{c^2x^2}} \left( -\frac{x}{6c^3} - \frac{x^3}{12c} \right) + \frac{1}{4}bx^4 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcSec[c\*x]),x]

[Out] (a\*x^4)/4 + b\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*(-1/6\*x/c^3 - x^3/(12\*c)) + (b\*x^4\*ArcSec[c\*x])/4

**fricas [A]** time = 0.70, size = 53, normalized size = 0.83

$$\frac{3bc^4x^4 \operatorname{arcsec}(cx) + 3ac^4x^4 - (bc^2x^2 + 2b)\sqrt{c^2x^2 - 1}}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/12\*(3\*b\*c^4\*x^4\*arcsec(c\*x) + 3\*a\*c^4\*x^4 - (b\*c^2\*x^2 + 2\*b)\*sqrt(c^2\*x^2 - 1))/c^4

**giac [B]** time = 0.20, size = 1926, normalized size = 30.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out]  $\frac{1}{12}c(3b \arccos(1/(cx)))/(c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) + 3a/(c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) - 12b(1/(c^2x^2) - 1) \arccos(1/(cx))/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^2) - 6b \sqrt{-1/(c^2x^2) + 1}/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)) - 12a(1/(c^2x^2) - 1)/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^2) + 18b(1/(c^2x^2) - 1)^2 \arccos(1/(cx))/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^4) + 10b(-1/(c^2x^2) + 1)^{(3/2)}/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^3) + 18a(1/(c^2x^2) - 1)^2/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^4) - 12b(1/(c^2x^2) - 1)^3 \arccos(1/(cx))/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^6) - 10b(1/(c^2x^2) - 1)^2 \sqrt{-1/(c^2x^2) + 1}/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^5) - 12a(1/(c^2x^2) - 1)^3/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^6) + 3b(1/(c^2x^2) - 1)^4 \arccos(1/(cx))/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^8) - 6b(1/(c^2x^2) - 1)^3 \sqrt{-1/(c^2x^2) + 1}/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^7) + 3a(1/(c^2x^2) - 1)^4/((c^5 + 4c^5(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 6c^5(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 4c^5(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + c^5(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8) * (1/(cx) + 1)^8))$



**maple** [A] time = 0.05, size = 74, normalized size = 1.16

$$\frac{\frac{c^4 x^4 a}{4} + b \left( \frac{c^4 x^4 \operatorname{arcsec}(cx)}{4} - \frac{(c^2 x^2 - 1)(c^2 x^2 + 2)}{12 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} cx} \right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arcsec(c*x)),x)`

[Out] `1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))`

**maxima** [A] time = 0.31, size = 60, normalized size = 0.94

$$\frac{1}{4} ax^4 + \frac{1}{12} \left( 3x^4 \operatorname{arcsec}(cx) - \frac{c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 3x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="maxima")`

[Out] `1/4*a*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b*acos(1/(c*x))),x)`

[Out] `int(x^3*(a + b*acos(1/(c*x))), x)`

**sympy** [A] time = 2.79, size = 107, normalized size = 1.67

$$\frac{ax^4}{4} + \frac{bx^4 \operatorname{asec}(cx)}{4} - \frac{b \left\{ \begin{array}{ll} \frac{x^2 \sqrt{c^2 x^2 - 1}}{3c} + \frac{2\sqrt{c^2 x^2 - 1}}{3c^3} & \text{for } |c^2 x^2| > 1 \\ \frac{ix^2 \sqrt{-c^2 x^2 + 1}}{3c} + \frac{2i\sqrt{-c^2 x^2 + 1}}{3c^3} & \text{otherwise} \end{array} \right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asec(c*x)),x)
```

```
[Out] a*x**4/4 + b*x**4*asec(c*x)/4 - b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c)
+ 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*
x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

### 3.5 $\int x^2 \left( a + b \sec^{-1}(cx) \right) dx$

**Optimal.** Leaf size=64

$$\frac{1}{3}x^3 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{6c} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{6c^3}$$

[Out]  $\frac{1}{3}x^3(a+b\operatorname{arcsec}(cx)) - \frac{1}{6}b\operatorname{arctanh}\left(\left(1-\frac{1}{c^2x^2}\right)^{1/2}\right)/c^3 - \frac{1}{6}b*x^2*(1-1/c^2/x^2)^{1/2}/c$

**Rubi [A]** time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5220, 266, 51, 63, 208}

$$\frac{1}{3}x^3 \left( a + b \sec^{-1}(cx) \right) - \frac{bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{6c} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{6c^3}$$

Antiderivative was successfully verified.

[In] `Int[x^2*(a + b*ArcSec[c*x]),x]`

[Out]  $-(b\sqrt{1 - 1/(c^2*x^2)}*x^2)/(6*c) + (x^3*(a + b*ArcSec[c*x]))/3 - (b*ArcTanh[\sqrt{1 - 1/(c^2*x^2)}])/(6*c^3)$

#### Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[p[((d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int x^2 (a + b \sec^{-1}(cx)) dx &= \frac{1}{3}x^3 (a + b \sec^{-1}(cx)) - \frac{b \int \frac{x}{\sqrt{1 - \frac{1}{c^2x^2}}} dx}{3c} \\
 &= \frac{1}{3}x^3 (a + b \sec^{-1}(cx)) + \frac{b \operatorname{Subst}\left(\int \frac{1}{x^2 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{6c} \\
 &= -\frac{b \sqrt{1 - \frac{1}{c^2x^2}} x^2}{6c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx)) + \frac{b \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{12c^3} \\
 &= -\frac{b \sqrt{1 - \frac{1}{c^2x^2}} x^2}{6c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{c^2 - c^2x^2} dx, x, \sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c} \\
 &= -\frac{b \sqrt{1 - \frac{1}{c^2x^2}} x^2}{6c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx)) - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3}
 \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 85, normalized size = 1.33

$$\frac{ax^3}{3} - \frac{bx^2 \sqrt{\frac{c^2x^2-1}{c^2x^2}}}{6c} - \frac{b \log\left(x \left(\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 1\right)\right)}{6c^3} + \frac{1}{3}bx^3 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcSec[c\*x]),x]

[Out] (a\*x^3)/3 - (b\*x^2\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(6\*c) + (b\*x^3\*ArcSec[c\*x])/3 - (b\*Log[x\*(1 + Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(6\*c^3)

**fricas** [A] time = 0.76, size = 94, normalized size = 1.47

$$\frac{2ac^3x^3 + 4bc^3 \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) - \sqrt{c^2x^2 - 1}bcx + 2(bc^3x^3 - bc^3) \operatorname{arcsec}(cx) + b \log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/6\*(2\*a\*c^3\*x^3 + 4\*b\*c^3\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) - sqrt(c^2\*x^2 - 1)\*b\*c\*x + 2\*(b\*c^3\*x^3 - b\*c^3)\*arcsec(c\*x) + b\*log(-c\*x + sqrt(c^2\*x^2 - 1)))/c^3

**giac** [B] time = 0.82, size = 2101, normalized size = 32.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] 1/6\*c\*(2\*b\*arccos(1/(c\*x))/(c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6) - b\*log(abs(sqrt(-1/(c^2\*x^2) + 1) + 1/(c\*x) + 1))/(c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6) + b\*log(abs(sqrt(-1/(c^2\*x^2) + 1) - 1/(c\*x) - 1))/(c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6) + 2\*a/(c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6) - 6\*b\*(1/(c^2\*x^2) - 1)\*arccos(1/(c\*x))/((c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6)\*(1/(c\*x) + 1)^2) - 3\*b\*(1/(c^2\*x^2) - 1)\*log(abs(sqrt(-1/(c^2\*x^2) + 1) + 1/(c\*x) + 1))/((c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6)\*(1/(c\*x) + 1)^2) + 3\*b\*(1/(c^2\*x^2) - 1)\*log(abs(sqrt(-1/(c^2\*x^2) + 1) - 1/(c\*x) - 1))/((c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6)\*(1/(c\*x) + 1)^2) - 2\*b\*sqrt(-1/(c^2\*x^2) + 1)/((c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6)\*(1/(c\*x) + 1)) - 6\*a\*(1/(c^4 + 3\*c^4\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 3\*c^4\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + c^4\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6)

$2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 6*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4 - 3*b*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 3*b*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*a*(1/(c^2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 2*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - b*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + b*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 2*b*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^5) - 2*a*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6))$

**maple [B]** time = 0.05, size = 123, normalized size = 1.92

$$\frac{x^3 a}{3} + \frac{x^3 b \operatorname{arcsec}(cx)}{3} - \frac{b x^2}{6c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{b}{6c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b \sqrt{c^2 x^2 - 1} \ln\left(cx + \sqrt{c^2 x^2 - 1}\right)}{6c^4 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x)),x)

[Out] 1/3\*x^3\*a+1/3\*x^3\*b\*arcsec(c\*x)-1/6/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2+1/6/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)-1/6/c^4\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*ln(c\*x+(c^2\*x^2-1)^(1/2))

**maxima** [A] time = 0.31, size = 98, normalized size = 1.53

$$\frac{1}{3} ax^3 + \frac{1}{12} \left( 4x^3 \operatorname{arcsec}(cx) - \frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 1/12\*(4\*x^3\*arcsec(c\*x) - (2\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) - 1) + c^2) + log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^2\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 3.29, size = 107, normalized size = 1.67

$$\frac{ax^3}{3} + \frac{bx^3 \operatorname{asec}(cx)}{3} - \frac{b \left( \begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise} \end{cases} \right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*asec(c\*x)),x)

[Out] a\*x\*\*3/3 + b\*x\*\*3\*asec(c\*x)/3 - b\*Piecewise((x\*sqrt(c\*\*2\*x\*\*2 - 1)/(2\*c) + acosh(c\*x)/(2\*c\*\*2), Abs(c\*\*2\*x\*\*2) > 1), (-I\*c\*x\*\*3/(2\*sqrt(-c\*\*2\*x\*\*2 + 1)) + I\*x/(2\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)) - I\*asin(c\*x)/(2\*c\*\*2), True))/(3\*c)

### 3.6 $\int x \left( a + b \sec^{-1}(cx) \right) dx$

**Optimal.** Leaf size=39

$$\frac{1}{2}x^2 \left( a + b \sec^{-1}(cx) \right) - \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{2c}$$

[Out]  $1/2*x^2*(a+b*\text{arcsec}(c*x))-1/2*b*x*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5220, 191}

$$\frac{1}{2}x^2 \left( a + b \sec^{-1}(cx) \right) - \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{2c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-(b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*\text{ArcSec}[c*x]))/2$

#### Rule 191

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^{(p + 1)})/a, x] /;$   $\text{FreeQ}\{a, b, n, p\}, x \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

#### Rule 5220

$\text{Int}[(a_ + \text{ArcSec}[c_*(x_)]*(b_))*((d_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)}*(a + b*\text{ArcSec}[c*x])/(d*(m + 1)), x] - \text{Dist}[(b*d)/(c*(m + 1)), \text{Int}[(d*x)^{(m - 1)}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int x \left( a + b \sec^{-1}(cx) \right) dx &= \frac{1}{2}x^2 \left( a + b \sec^{-1}(cx) \right) - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2x^2}}} dx}{2c} \\ &= -\frac{b\sqrt{1 - \frac{1}{c^2x^2}}}{2c} + \frac{1}{2}x^2 \left( a + b \sec^{-1}(cx) \right) \end{aligned}$$



**Mathematica [A]** time = 0.02, size = 50, normalized size = 1.28

$$\frac{ax^2}{2} - \frac{bx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c} + \frac{1}{2}bx^2 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcSec[c\*x]),x]

[Out] (a\*x^2)/2 - (b\*x\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(2\*c) + (b\*x^2\*ArcSec[c\*x])/2

**fricas [A]** time = 0.55, size = 40, normalized size = 1.03

$$\frac{bc^2x^2 \operatorname{arcsec}(cx) + ac^2x^2 - \sqrt{c^2x^2 - 1}b}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/2\*(b\*c^2\*x^2\*arcsec(c\*x) + a\*c^2\*x^2 - sqrt(c^2\*x^2 - 1)\*b)/c^2

**giac [B]** time = 0.18, size = 634, normalized size = 16.26

$$\frac{1}{2}c \left( \frac{b \arccos\left(\frac{1}{cx}\right)}{c^3 + \frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2} + \frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}} + \frac{a}{c^3 + \frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2} + \frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}} - \frac{2b\left(\frac{1}{c^2x^2}-1\right) \arccos\left(\frac{1}{cx}\right)}{\left(c^3 + \frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2} + \frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}\right)\left(\frac{1}{cx}+1\right)^2} - \frac{1}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] 1/2\*c\*(b\*arccos(1/(c\*x))/(c^3 + 2\*c^3\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4) + a/(c^3 + 2\*c^3\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4) - 2\*b\*(1/(c^2\*x^2) - 1)\*arccos(1/(c\*x))/((c^3 + 2\*c^3\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)\*(1/(c\*x) + 1)^2) - 2\*b\*sqrt(-1/(c^2\*x^2) + 1)/((c^3 + 2\*c^3\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)\*(1/(c\*x) + 1)) - 2\*a\*(1/(c^2\*x^2) - 1)/((c^3 + 2\*c^3\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)\*(1/(c\*x) + 1)^2) + b\*(1/(c^2\*x^2) - 1)^2\*arccos(1/(c\*x))/((c^3 + 2\*c^3\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)\*(1/(c\*x) + 1)^2) + c^3\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)

)<sup>4</sup>\*(1/(c\*x) + 1)<sup>4</sup> + 2\*b\*(-1/(c<sup>2</sup>\*x<sup>2</sup>) + 1)<sup>(3/2)</sup>/((c<sup>3</sup> + 2\*c<sup>3</sup>\*(1/(c<sup>2</sup>\*x<sup>2</sup>) - 1)/(1/(c\*x) + 1)<sup>2</sup> + c<sup>3</sup>\*(1/(c<sup>2</sup>\*x<sup>2</sup>) - 1)<sup>2</sup>/(1/(c\*x) + 1)<sup>4</sup>\*(1/(c\*x) + 1)<sup>3</sup>) + a\*(1/(c<sup>2</sup>\*x<sup>2</sup>) - 1)<sup>2</sup>/((c<sup>3</sup> + 2\*c<sup>3</sup>\*(1/(c<sup>2</sup>\*x<sup>2</sup>) - 1)/(1/(c\*x) + 1)<sup>2</sup> + c<sup>3</sup>\*(1/(c<sup>2</sup>\*x<sup>2</sup>) - 1)<sup>2</sup>/(1/(c\*x) + 1)<sup>4</sup>\*(1/(c\*x) + 1)<sup>4</sup>)

**maple** [A] time = 0.05, size = 65, normalized size = 1.67

$$\frac{\frac{c^2 x^2 a}{2} + b \left( \frac{c^2 x^2 \operatorname{arcsec}(cx)}{2} - \frac{c^2 x^2 - 1}{2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} cx} \right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x)),x)

[Out] 1/c<sup>2</sup>\*(1/2\*c<sup>2</sup>\*x<sup>2</sup>\*a+b\*(1/2\*c<sup>2</sup>\*x<sup>2</sup>\*arcsec(c\*x)-1/2/((c<sup>2</sup>\*x<sup>2</sup>-1)/c<sup>2</sup>/x<sup>2</sup>)<sup>(1/2)</sup>/c/x\*(c<sup>2</sup>\*x<sup>2</sup>-1))

**maxima** [A] time = 0.31, size = 37, normalized size = 0.95

$$\frac{1}{2} ax^2 + \frac{1}{2} \left( x^2 \operatorname{arcsec}(cx) - \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/2\*a\*x<sup>2</sup> + 1/2\*(x<sup>2</sup>\*arcsec(c\*x) - x\*sqrt(-1/(c<sup>2</sup>\*x<sup>2</sup>) + 1)/c)\*b

**mupad** [B] time = 0.67, size = 40, normalized size = 1.03

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{acos}\left(\frac{1}{cx}\right)}{2} - \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*acos(1/(c\*x))),x)

[Out] (a\*x<sup>2</sup>)/2 + (b\*x<sup>2</sup>\*acos(1/(c\*x)))/2 - (b\*x\*(1 - 1/(c<sup>2</sup>\*x<sup>2</sup>))<sup>(1/2)</sup>)/(2\*c)

**sympy** [A] time = 1.99, size = 58, normalized size = 1.49

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{asec}(cx)}{2} - \frac{b \left( \begin{cases} \frac{\sqrt{c^2 x^2 - 1}}{c} & \text{for } |c^2 x^2| > 1 \\ \frac{i\sqrt{-c^2 x^2 + 1}}{c} & \text{otherwise} \end{cases} \right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asec(c*x)),x)
```

```
[Out] a*x**2/2 + b*x**2*asec(c*x)/2 - b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**  
2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

### 3.7 $\int (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=32

$$ax - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \sec^{-1}(cx)$$

[Out] a\*x+b\*x\*arcsec(c\*x)-b\*arctanh((1-1/c^2/x^2)^(1/2))/c

**Rubi [A]** time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5214, 266, 63, 208}

$$ax - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcSec[c\*x], x]

[Out] a\*x + b\*x\*ArcSec[c\*x] - (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 5214

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] := Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

### Rubi steps

$$\begin{aligned}
 \int (a + b \sec^{-1}(cx)) dx &= ax + b \int \sec^{-1}(cx) dx \\
 &= ax + bx \sec^{-1}(cx) - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} dx}}{c} \\
 &= ax + bx \sec^{-1}(cx) + \frac{b \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{2c} \\
 &= ax + bx \sec^{-1}(cx) - (bc) \operatorname{Subst}\left(\int \frac{1}{c^2 - c^2 x^2} dx, x, \sqrt{1 - \frac{1}{c^2 x^2}}\right) \\
 &= ax + bx \sec^{-1}(cx) - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}
 \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 59, normalized size = 1.84

$$ax - \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2 x^2 - 1}}\right)}{\sqrt{c^2 x^2 - 1}} + bx \sec^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSec[c*x], x]
```

```
[Out] a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

**fricas [B]** time = 0.99, size = 63, normalized size = 1.97

$$\frac{acx + 2bc \arctan\left(-cx + \sqrt{c^2 x^2 - 1}\right) + (bcx - bc) \operatorname{arcsec}(cx) + b \log\left(-cx + \sqrt{c^2 x^2 - 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="fricas")
```

[Out]  $(a*c*x + 2*b*c*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + (b*c*x - b*c)*\operatorname{arcsec}(c*x) + b*\log(-c*x + \sqrt{c^2*x^2 - 1}))/c$

**giac** [B] time = 0.14, size = 63, normalized size = 1.97

$$\frac{1}{2}bc \left( \frac{2x \arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)}{c^2} \right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsec(c*x),x, algorithm="giac")`

[Out]  $1/2*b*c*(2*x*\arccos(1/(c*x))/c - (\log(\sqrt{-1/(c^2*x^2) + 1} + 1) - \log(-\sqrt{-1/(c^2*x^2) + 1} + 1))/c^2) + a*x$

**maple** [A] time = 0.04, size = 38, normalized size = 1.19

$$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arcsec(c*x),x)`

[Out]  $a*x + b*x*\operatorname{arcsec}(c*x) - b/c*\ln(c*x + c*x*(1 - 1/c^2/x^2)^{(1/2)})$

**maxima** [A] time = 0.31, size = 53, normalized size = 1.66

$$ax + \frac{\left(2cx \operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsec(c*x),x, algorithm="maxima")`

[Out]  $a*x + 1/2*(2*c*x*\operatorname{arcsec}(c*x) - \log(\sqrt{-1/(c^2*x^2) + 1} + 1) + \log(-\sqrt{-1/(c^2*x^2) + 1} + 1))*b/c$

**mupad** [B] time = 0.84, size = 34, normalized size = 1.06

$$ax + bx \operatorname{acos}\left(\frac{1}{cx}\right) - \frac{b \operatorname{atanh}\left(\frac{1}{\sqrt{1 - \frac{1}{c^2x^2}}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acos(1/(c*x)),x)
```

```
[Out] a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

sympy [A] time = 2.33, size = 32, normalized size = 1.00

$$ax + b \left( x \operatorname{asec}(cx) - \frac{\begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asec(c*x),x)
```

```
[Out] a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c)
```

$$3.8 \quad \int \frac{a+b \sec^{-1}(cx)}{x} dx$$

**Optimal.** Leaf size=64

$$\frac{i(a+b \sec^{-1}(cx))^2}{2b} - \log(1 + e^{2i \sec^{-1}(cx)})(a+b \sec^{-1}(cx)) + \frac{1}{2} i b \text{Li}_2(-e^{2i \sec^{-1}(cx)})$$

[Out]  $1/2 * I * (a + b * \text{arcsec}(c * x))^2 / b - (a + b * \text{arcsec}(c * x)) * \ln(1 + (1/c/x + I * (1 - 1/c^2/x^2))^{1/2}) + 1/2 * I * b * \text{polylog}(2, -(1/c/x + I * (1 - 1/c^2/x^2))^{1/2})^2$

**Rubi [A]** time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5218, 4626, 3719, 2190, 2279, 2391}

$$\frac{1}{2} i b \text{PolyLog}(2, -e^{2i \sec^{-1}(cx)}) + \frac{i(a+b \sec^{-1}(cx))^2}{2b} - \log(1 + e^{2i \sec^{-1}(cx)})(a+b \sec^{-1}(cx))$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x, x]

[Out]  $((I/2) * (a + b * \text{ArcSec}[c * x])^2) / b - (a + b * \text{ArcSec}[c * x]) * \text{Log}[1 + E^{((2 * I) * \text{ArcSec}[c * x])}] + (I/2) * b * \text{PolyLog}[2, -E^{((2 * I) * \text{ArcSec}[c * x])}]$

Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]) / (b\*f\*g\*n \* Log[F]), x] - Dist[(d\*m) / (b\*f\*g\*n \* Log[F]), Int[(c + d\*x)^(m - 1) \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n \* Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3719



```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

### Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] := -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

### Rule 5218

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.))/(x_), x_Symbol] := -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x} \right) \\
&= \text{Subst} \left( \int (a + bx) \tan(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^2}{2b} - 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)}{1 + e^{2ix}} dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^2}{2b} - (a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)}) + b \text{Subst} \left( \int \log(1 + e^{2ix}) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^2}{2b} - (a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)}) - \frac{1}{2}(ib) \text{Subst} \left( \int \frac{\log(1 + e^{2ix})}{x} \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^2}{2b} - (a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)}) + \frac{1}{2}ib \text{Li}_2(-e^{2i \sec^{-1}(cx)})
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 59, normalized size = 0.92

$$a \log(x) + \frac{1}{2}ib \text{Li}_2(-e^{2i \sec^{-1}(cx)}) + \frac{1}{2}ib \sec^{-1}(cx)^2 - b \sec^{-1}(cx) \log(1 + e^{2i \sec^{-1}(cx)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/x,x]

[Out] (I/2)\*b\*ArcSec[c\*x]^2 - b\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] + a\*Log[x] + (I/2)\*b\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]

**fricas** [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/x, x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
by intervals (correct if the argument is real):Check [abs(x)]Undef/Unsigned  
Inf encountered in limitLimit: Max order reached or unable to make series  
expansion Error: Bad Argument Value

**maple** [A] time = 0.19, size = 86, normalized size = 1.34

$$a \ln(cx) + \frac{ib \operatorname{arcsec}(cx)^2}{2} - b \operatorname{arcsec}(cx) \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{ib \operatorname{polylog}\left(2, -\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x,x)

[Out] a\*ln(c\*x)+1/2\*I\*b\*arcsec(c\*x)^2-b\*arcsec(c\*x)\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+1/2\*I\*b\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(c^2 \int \frac{\sqrt{cx+1} \sqrt{cx-1} \log(x)}{c^4x^3 - c^2x} dx - \arctan\left(\sqrt{cx+1} \sqrt{cx-1}\right) \log(x)\right) b + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x,x, algorithm="maxima")

[Out]  $-(c^2 \int \sqrt{cx+1} \sqrt{cx-1} \log(x) / (c^4 x^3 - c^2 x), x) - \arctan(\sqrt{cx+1} \sqrt{cx-1}) \log(x) * b + a \log(x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x,x)

[Out] int((a + b\*acos(1/(c\*x)))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x,x)

[Out] Integral((a + b\*asec(c\*x))/x, x)

$$3.9 \quad \int \frac{a+b \sec^{-1}(cx)}{x^2} dx$$

**Optimal.** Leaf size=31

$$bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b \sec^{-1}(cx)}{x}$$

[Out]  $(-a-b*\text{arcsec}(c*x))/x+b*c*(1-1/c^2/x^2)^{(1/2)}$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5220, 261}

$$bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b \sec^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x^2, x]

[Out] b\*c\*Sqrt[1 - 1/(c^2\*x^2)] - (a + b\*ArcSec[c\*x])/x

**Rule 261**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

**Rule 5220**

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{a+b \sec^{-1}(cx)}{x^2} dx &= -\frac{a+b \sec^{-1}(cx)}{x} + \frac{b \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}} x^3} dx}{c} \\ &= bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b \sec^{-1}(cx)}{x} \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 40, normalized size = 1.29

$$-\frac{a}{x} + bc\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b \sec^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/x^2,x]

[Out] -(a/x) + b\*c\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcSec[c\*x])/x

**fricas** [A] time = 1.23, size = 27, normalized size = 0.87

$$\frac{b \operatorname{arcsec}(cx) - \sqrt{c^2x^2 - 1} b + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2,x, algorithm="fricas")

[Out] -(b\*arcsec(c\*x) - sqrt(c^2\*x^2 - 1)\*b + a)/x

**giac** [A] time = 0.15, size = 43, normalized size = 1.39

$$\left( b\sqrt{-\frac{1}{c^2x^2} + 1} - \frac{b \arccos\left(\frac{1}{cx}\right)}{cx} - \frac{a}{cx} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2,x, algorithm="giac")

[Out] (b\*sqrt(-1/(c^2\*x^2) + 1) - b\*arccos(1/(c\*x))/(c\*x) - a/(c\*x))\*c

**maple** [A] time = 0.05, size = 62, normalized size = 2.00

$$c \left( -\frac{a}{cx} + b \left( -\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}} c^2x^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^2,x)

[Out] c\*(-a/c/x+b\*(-1/c/x\*arcsec(c\*x)+1/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2\*(c^2\*x^2-1)))

**maxima** [A] time = 0.31, size = 33, normalized size = 1.06

$$\left( c\sqrt{-\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsec}(cx)}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2,x, algorithm="maxima")

[Out] (c\*sqrt(-1/(c^2\*x^2) + 1) - arcsec(c\*x)/x)\*b - a/x

**mupad** [B] time = 0.64, size = 36, normalized size = 1.16

$$bc\sqrt{1 - \frac{1}{c^2x^2}} - \frac{a}{x} - \frac{b\operatorname{acos}\left(\frac{1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x^2,x)

[Out] b\*c\*(1 - 1/(c^2\*x^2))^(1/2) - a/x - (b\*acos(1/(c\*x)))/x

**sympy** [A] time = 1.77, size = 36, normalized size = 1.16

$$\begin{cases} -\frac{a}{x} + bc\sqrt{1 - \frac{1}{c^2x^2}} - \frac{b\operatorname{asec}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a+\infty b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*2,x)

[Out] Piecewise((-a/x + b\*c\*sqrt(1 - 1/(c\*\*2\*x\*\*2)) - b\*asec(c\*x)/x, Ne(c, 0)), (-a + zoo\*b)/x, True))

### 3.10 $\int \frac{a+b \sec^{-1}(cx)}{x^3} dx$

**Optimal.** Leaf size=51

$$-\frac{a+b \sec^{-1}(cx)}{2x^2} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2 \csc^{-1}(cx)$$

[Out]  $-1/4*b*c^2*\arccsc(c*x)+1/2*(-a-b*\arcsec(c*x))/x^2+1/4*b*c*(1-1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5220, 335, 321, 216}

$$-\frac{a+b \sec^{-1}(cx)}{2x^2} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x^3, x]

[Out]  $(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) - (b*c^2*\text{ArcCsc}[c*x])/4 - (a + b*\text{ArcSec}[c*x])/(2*x^2)$

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 335

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Subst[Int[(a+b/x^n)^p/x^(m+2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

#### Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] := Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \sec^{-1}(cx)}{x^3} dx &= -\frac{a + b \sec^{-1}(cx)}{2x^2} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{2c} \\ &= -\frac{a + b \sec^{-1}(cx)}{2x^2} - \frac{b \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2c} \\ &= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{4x} - \frac{a + b \sec^{-1}(cx)}{2x^2} - \frac{1}{4}(bc) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\ &= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{4x} - \frac{1}{4}bc^2 \operatorname{csc}^{-1}(cx) - \frac{a + b \sec^{-1}(cx)}{2x^2} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 66, normalized size = 1.29

$$-\frac{a}{2x^2} + \frac{bc\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{4x} - \frac{1}{4}bc^2 \sin^{-1}\left(\frac{1}{cx}\right) - \frac{b \sec^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSec[c*x])/x^3, x]
```

```
[Out] -1/2*a/x^2 + (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcSec[c*x])/
(2*x^2) - (b*c^2*ArcSin[1/(c*x)])/4
```

**fricas** [A] time = 0.58, size = 39, normalized size = 0.76

$$\frac{(bc^2 x^2 - 2b) \operatorname{arcsec}(cx) + \sqrt{c^2 x^2 - 1} b - 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```



[Out]  $1/4*((b*c^2*x^2 - 2*b)*\operatorname{arcsec}(c*x) + \sqrt{c^2*x^2 - 1}*b - 2*a)/x^2$

**giac** [A] time = 0.14, size = 58, normalized size = 1.14

$$\frac{1}{4} \left( bc \arccos\left(\frac{1}{cx}\right) + \frac{b\sqrt{-\frac{1}{c^2x^2} + 1}}{x} - \frac{2b \arccos\left(\frac{1}{cx}\right)}{cx^2} - \frac{2a}{cx^2} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^3,x, algorithm="giac")`

[Out]  $1/4*(b*c*\arccos(1/(c*x)) + b*\sqrt{-1/(c^2*x^2) + 1}/x - 2*b*\arccos(1/(c*x)) / (c*x^2) - 2*a/(c*x^2))*c$

**maple** [B] time = 0.05, size = 118, normalized size = 2.31

$$-\frac{a}{2x^2} - \frac{b \operatorname{arcsec}(cx)}{2x^2} - \frac{cb\sqrt{c^2x^2-1} \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}} x} + \frac{cb}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}} x} - \frac{b}{4c\sqrt{\frac{c^2x^2-1}{c^2x^2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/x^3,x)`

[Out]  $-1/2*a/x^2-1/2*b/x^2*\operatorname{arcsec}(c*x)-1/4*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*\arctan(1/(c^2*x^2-1)^{(1/2)})+1/4*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x-1/4/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^3$

**maxima** [A] time = 0.41, size = 83, normalized size = 1.63

$$-\frac{1}{4} b \left( \frac{\frac{c^4 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^2 x^2 \left(\frac{1}{c^2 x^2} - 1\right) - 1} - c^3 \arctan\left(cx \sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c} + \frac{2 \operatorname{arcsec}(cx)}{x^2} \right) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^3,x, algorithm="maxima")`

[Out]  $-1/4*b*((c^4*x*\sqrt{-1/(c^2*x^2) + 1}/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*\arctan(c*x*\sqrt{-1/(c^2*x^2) + 1}))/c + 2*\operatorname{arcsec}(c*x)/x^2) - 1/2*a/x^2$

mupad [B] time = 0.73, size = 50, normalized size = 0.98

$$\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{bc^2\arccos\left(\frac{1}{cx}\right)\left(\frac{2}{c^2x^2}-1\right)}{4} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x^3,x)

[Out] (b\*c\*(1 - 1/(c^2\*x^2))^(1/2))/(4\*x) - (b\*c^2\*acos(1/(c\*x))\*(2/(c^2\*x^2) - 1))/4 - a/(2\*x^2)

sympy [A] time = 3.14, size = 119, normalized size = 2.33

$$-\frac{a}{2x^2} - \frac{b\operatorname{asec}(cx)}{2x^2} + \frac{b\left(\begin{array}{l} \frac{ic^3\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{ic^2\sqrt{-1+\frac{1}{c^2x^2}}}{2x} \quad \text{for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{c^3\operatorname{asin}\left(\frac{1}{cx}\right)}{2} + \frac{c^2}{2x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{2x^3\sqrt{1-\frac{1}{c^2x^2}}} \quad \text{otherwise} \end{array}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*3,x)

[Out] -a/(2\*x\*\*2) - b\*asec(c\*x)/(2\*x\*\*2) + b\*Piecewise((I\*c\*\*3\*acosh(1/(c\*x))/2 + I\*c\*\*2\*sqrt(-1 + 1/(c\*\*2\*x\*\*2))/(2\*x), 1/Abs(c\*\*2\*x\*\*2) > 1), (-c\*\*3\*asin(1/(c\*x))/2 + c\*\*2/(2\*x\*sqrt(1 - 1/(c\*\*2\*x\*\*2))) - 1/(2\*x\*\*3\*sqrt(1 - 1/(c\*\*2\*x\*\*2))), True))/(2\*c)

$$3.11 \quad \int \frac{a+b \sec^{-1}(cx)}{x^4} dx$$

**Optimal.** Leaf size=60

$$-\frac{a+b \sec^{-1}(cx)}{3x^3} - \frac{1}{9}bc^3 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{3}bc^3 \sqrt{1 - \frac{1}{c^2x^2}}$$

[Out]  $-1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*\text{arcsec}(c*x))/x^3+1/3*b*c^3*(1-1/c^2/x^2)^(1/2)$

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5220, 266, 43}

$$-\frac{a+b \sec^{-1}(cx)}{3x^3} - \frac{1}{9}bc^3 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{3}bc^3 \sqrt{1 - \frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x^4, x]

[Out]  $(b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/3 - (b*c^3*(1 - 1/(c^2*x^2))^(3/2))/9 - (a + b*\text{ArcSec}[c*x])/(3*x^3)$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(((d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^4} dx &= -\frac{a + b \sec^{-1}(cx)}{3x^3} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{3c} \\
&= -\frac{a + b \sec^{-1}(cx)}{3x^3} - \frac{b \operatorname{Subst}\left(\int \frac{x}{\sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{a + b \sec^{-1}(cx)}{3x^3} - \frac{b \operatorname{Subst}\left(\int \left(\frac{c^2}{\sqrt{1 - \frac{x}{c^2}}} - c^2 \sqrt{1 - \frac{x}{c^2}}\right) dx, x, \frac{1}{x^2}\right)}{6c} \\
&= \frac{1}{3}bc^3 \sqrt{1 - \frac{1}{c^2 x^2}} - \frac{1}{9}bc^3 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} - \frac{a + b \sec^{-1}(cx)}{3x^3}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 59, normalized size = 0.98

$$-\frac{a}{3x^3} + b \left( \frac{2c^3}{9} + \frac{c}{9x^2} \right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} - \frac{b \sec^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/x^4,x]

[Out] -1/3\*a/x^3 + b\*((2\*c^3)/9 + c/(9\*x^2))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcSec[c\*x])/(3\*x^3)

**fricas** [A] time = 0.68, size = 40, normalized size = 0.67

$$\frac{3 b \operatorname{arcsec}(cx) - (2 b c^2 x^2 + b) \sqrt{c^2 x^2 - 1} + 3 a}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^4,x, algorithm="fricas")

[Out] -1/9\*(3\*b\*arcsec(c\*x) - (2\*b\*c^2\*x^2 + b)\*sqrt(c^2\*x^2 - 1) + 3\*a)/x^3

**giac** [A] time = 0.15, size = 65, normalized size = 1.08

$$\frac{1}{9} \left( 2 b c^2 \sqrt{-\frac{1}{c^2 x^2} + 1} + \frac{b \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^2} - \frac{3 b \arccos\left(\frac{1}{c x}\right)}{c x^3} - \frac{3 a}{c x^3} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^4,x, algorithm="giac")

[Out]  $\frac{1}{9}*(2*b*c^2*\sqrt{-1/(c^2*x^2) + 1}) + b*\sqrt{-1/(c^2*x^2) + 1}/x^2 - 3*b*\arccos(1/(c*x))/(c*x^3) - 3*a/(c*x^3))*c$

**maple** [A] time = 0.05, size = 75, normalized size = 1.25

$$c^3 \left( -\frac{a}{3c^3x^3} + b \left( -\frac{\operatorname{arcsec}(cx)}{3c^3x^3} + \frac{(c^2x^2 - 1)(2c^2x^2 + 1)}{9\sqrt{\frac{c^2x^2 - 1}{c^2x^2}} c^4x^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^4,x)

[Out]  $c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*\operatorname{arcsec}(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))$

**maxima** [A] time = 0.32, size = 58, normalized size = 0.97

$$-\frac{1}{9}b \left( \frac{c^4 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} + \frac{3 \operatorname{arcsec}(cx)}{x^3} \right) - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^4,x, algorithm="maxima")

[Out]  $-1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*\sqrt{-1/(c^2*x^2) + 1}))/c + 3*\operatorname{arcsec}(c*x)/x^3) - 1/3*a/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x^4,x)

[Out] int((a + b\*acos(1/(c\*x)))/x^4, x)

sympy [A] time = 2.93, size = 110, normalized size = 1.83

$$-\frac{a}{3x^3} - \frac{b \operatorname{asec}(cx)}{3x^3} + \frac{b \left( \begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases} \right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*4,x)

[Out] -a/(3\*x\*\*3) - b\*asec(c\*x)/(3\*x\*\*3) + b\*Piecewise((2\*c\*\*3\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*x) + c\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*x\*\*3), Abs(c\*\*2\*x\*\*2) > 1), (2\*I\*c\*\*3\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*x) + I\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*x\*\*3), True))/(3\*c)

$$3.12 \quad \int \frac{a+b \sec^{-1}(cx)}{x^5} dx$$

**Optimal.** Leaf size=76

$$-\frac{a+b \sec^{-1}(cx)}{4x^4} - \frac{3}{32}bc^4 \csc^{-1}(cx) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}$$

[Out]  $-3/32*b*c^4*\text{arccsc}(c*x)+1/4*(-a-b*\text{arcsec}(c*x))/x^4+1/16*b*c*(1-1/c^2/x^2)^{(1/2)}/x^3+3/32*b*c^3*(1-1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5220, 335, 321, 216}

$$-\frac{a+b \sec^{-1}(cx)}{4x^4} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3}{32}bc^4 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x^5, x]

[Out]  $(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(16*x^3) + (3*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(32*x) - (3*b*c^4*\text{ArcCsc}[c*x])/32 - (a + b*\text{ArcSec}[c*x])/(4*x^4)$

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 335

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Subst[Int[(a+b/x^n)^p/x^(m+2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

### Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] := Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^5} dx &= -\frac{a + b \sec^{-1}(cx)}{4x^4} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{4c} \\
&= -\frac{a + b \sec^{-1}(cx)}{4x^4} - \frac{b \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{4c} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{16x^3} - \frac{a + b \sec^{-1}(cx)}{4x^4} - \frac{1}{16} (3bc) \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{16x^3} + \frac{3bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{32x} - \frac{a + b \sec^{-1}(cx)}{4x^4} - \frac{1}{32} (3bc^3) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{16x^3} + \frac{3bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{32x} - \frac{3}{32} bc^4 \csc^{-1}(cx) - \frac{a + b \sec^{-1}(cx)}{4x^4}
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 78, normalized size = 1.03

$$-\frac{a}{4x^4} - \frac{3}{32} bc^4 \sin^{-1}\left(\frac{1}{cx}\right) + b \left(\frac{3c^3}{32x} + \frac{c}{16x^3}\right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} - \frac{b \sec^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/x^5,x]

[Out] -1/4\*a/x^4 + b\*(c/(16\*x^3) + (3\*c^3)/(32\*x))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcSec[c\*x])/(4\*x^4) - (3\*b\*c^4\*ArcSin[1/(c\*x)])/32

**fricas** [A] time = 0.71, size = 52, normalized size = 0.68

$$\frac{(3bc^4x^4 - 8b) \operatorname{arcsec}(cx) + (3bc^2x^2 + 2b)\sqrt{c^2x^2 - 1} - 8a}{32x^4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^5,x, algorithm="fricas")

[Out]  $\frac{1}{32} * ((3*b*c^4*x^4 - 8*b)*arcsec(c*x) + (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 - 1) - 8*a)/x^4$

**giac** [A] time = 0.15, size = 83, normalized size = 1.09

$$\frac{1}{32} \left( 3bc^3 \arccos\left(\frac{1}{cx}\right) + \frac{3bc^2 \sqrt{-\frac{1}{c^2x^2} + 1}}{x} + \frac{2b \sqrt{-\frac{1}{c^2x^2} + 1}}{x^3} - \frac{8b \arccos\left(\frac{1}{cx}\right)}{cx^4} - \frac{8a}{cx^4} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^5,x, algorithm="giac")

[Out]  $\frac{1}{32} * (3*b*c^3*arccos(1/(c*x)) + 3*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 2*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 8*b*arccos(1/(c*x))/(c*x^4) - 8*a/(c*x^4)) * c$

**maple** [B] time = 0.05, size = 147, normalized size = 1.93

$$\frac{a}{4x^4} - \frac{b \operatorname{arcsec}(cx)}{4x^4} - \frac{3c^3b\sqrt{c^2x^2-1} \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}} x} + \frac{3c^3b}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}} x} - \frac{cb}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}} x^3} - \frac{b}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^5,x)

[Out]  $-1/4*a/x^4 - 1/4*b/x^4*arcsec(c*x) - 3/32*c^3*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*arctan(1/(c^2*x^2-1)^{(1/2)}) + 3/32*c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x - 1/32*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^3 - 1/16*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^5$

**maxima** [A] time = 0.42, size = 125, normalized size = 1.64

$$\frac{1}{32} b \left( \frac{3c^5 \arctan\left(cx \sqrt{-\frac{1}{c^2x^2} + 1}\right) + \frac{3c^8x^3\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 5c^6x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^4x^4\left(\frac{1}{c^2x^2} - 1\right)^2 - 2c^2x^2\left(\frac{1}{c^2x^2} - 1\right) + 1}}{c} - \frac{8 \operatorname{arcsec}(cx)}{x^4} \right) - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^5,x, algorithm="maxima")

[Out]  $\frac{1}{32}b\left(\left(3c^5\arctan\left(\frac{cx\sqrt{-1/(c^2x^2)+1}}{1}\right)\right) + \left(3c^8x^3\left(-1/(c^2x^2)+1\right)^{3/2}\right) + 5c^6x\sqrt{-1/(c^2x^2)+1}\right)/(c^4x^4\left(1/(c^2x^2)-1\right)^2 - 2c^2x^2\left(1/(c^2x^2)-1\right)+1)/c - 8\operatorname{arcsec}(cx)/x^4 - 1/4a/x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x^5,x)

[Out] int((a + b\*acos(1/(c\*x)))/x^5, x)

sympy [A] time = 5.23, size = 192, normalized size = 2.53

$$\frac{a}{4x^4} - \frac{b \operatorname{asec}(cx)}{4x^4} + \frac{b \left( \begin{array}{l} \left( \frac{3ic^5 \operatorname{acosh}\left(\frac{1}{cx}\right)}{8} - \frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} \right) \text{ for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{3c^5 \operatorname{asin}\left(\frac{1}{cx}\right)}{8} + \frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} \text{ otherwise} \end{array} \right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*5,x)

[Out]  $-a/(4x^{**4}) - b\operatorname{asec}(cx)/(4x^{**4}) + b\operatorname{Piecewise}\left(\left(\frac{3Ic^{**5}\operatorname{acosh}(1/(cx))}{8} - \frac{3Ic^{**4}}{8x\sqrt{-1+1/(c^{**2}x^{**2})}}\right) + \frac{Ic^{**2}}{8x^{**3}\sqrt{-1+1/(c^{**2}x^{**2})}} + \frac{I}{4x^{**5}\sqrt{-1+1/(c^{**2}x^{**2})}}\right), \frac{1}{\operatorname{Abs}(c^{**2}x^{**2})} > 1), \left(-\frac{3c^{**5}\operatorname{asin}(1/(cx))}{8} + \frac{3c^{**4}}{8x\sqrt{1-1/(c^{**2}x^{**2})}}\right) - \frac{c^{**2}}{8x^{**3}\sqrt{1-1/(c^{**2}x^{**2})}} - \frac{1}{4x^{**5}\sqrt{1-1/(c^{**2}x^{**2})}}\right), \operatorname{True})/(4c)$

### 3.13 $\int \frac{a+b \sec^{-1}(cx)}{x^6} dx$

**Optimal.** Leaf size=82

$$-\frac{a+b \sec^{-1}(cx)}{5x^5} + \frac{1}{25}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{5/2} - \frac{2}{15}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{5}bc^5 \sqrt{1 - \frac{1}{c^2x^2}}$$

[Out]  $-2/15*b*c^5*(1-1/c^2/x^2)^{(3/2)}+1/25*b*c^5*(1-1/c^2/x^2)^{(5/2)}+1/5*(-a-b*\arccsc(c*x))/x^5+1/5*b*c^5*(1-1/c^2/x^2)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5220, 266, 43}

$$-\frac{a+b \sec^{-1}(cx)}{5x^5} + \frac{1}{25}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{5/2} - \frac{2}{15}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{5}bc^5 \sqrt{1 - \frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x^6,x]

[Out]  $(b*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)])/5 - (2*b*c^5*(1 - 1/(c^2*x^2))^{(3/2)})/15 + (b*c^5*(1 - 1/(c^2*x^2))^{(5/2)})/25 - (a + b*\text{ArcSec}[c*x])/(5*x^5)$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 5220

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(((d\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(d\*(m + 1)), x] - Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^6} dx &= -\frac{a + b \sec^{-1}(cx)}{5x^5} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{5c} \\
&= -\frac{a + b \sec^{-1}(cx)}{5x^5} - \frac{b \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{10c} \\
&= -\frac{a + b \sec^{-1}(cx)}{5x^5} - \frac{b \operatorname{Subst}\left(\int \left(\frac{c^4}{\sqrt{1 - \frac{x}{c^2}}} - 2c^4 \sqrt{1 - \frac{x}{c^2}} + c^4 \left(1 - \frac{x}{c^2}\right)^{3/2}\right) dx, x, \frac{1}{x^2}\right)}{10c} \\
&= \frac{1}{5} bc^5 \sqrt{1 - \frac{1}{c^2 x^2}} - \frac{2}{15} bc^5 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} + \frac{1}{25} bc^5 \left(1 - \frac{1}{c^2 x^2}\right)^{5/2} - \frac{a + b \sec^{-1}(cx)}{5x^5}
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 69, normalized size = 0.84

$$-\frac{a}{5x^5} + b \left( \frac{8c^5}{75} + \frac{4c^3}{75x^2} + \frac{c}{25x^4} \right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} - \frac{b \sec^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/x^6, x]

[Out] -1/5\*a/x^5 + b\*((8\*c^5)/75 + c/(25\*x^4) + (4\*c^3)/(75\*x^2))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcSec[c\*x])/(5\*x^5)

**fricas** [A] time = 0.80, size = 51, normalized size = 0.62

$$-\frac{15 b \operatorname{arcsec}(cx) - (8 bc^4 x^4 + 4 bc^2 x^2 + 3 b) \sqrt{c^2 x^2 - 1} + 15 a}{75 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^6,x, algorithm="fricas")

[Out] -1/75\*(15\*b\*arcsec(c\*x) - (8\*b\*c^4\*x^4 + 4\*b\*c^2\*x^2 + 3\*b)\*sqrt(c^2\*x^2 - 1) + 15\*a)/x^5

**giac** [A] time = 0.13, size = 87, normalized size = 1.06

$$\frac{1}{75} \left( 8 bc^4 \sqrt{-\frac{1}{c^2 x^2} + 1} + \frac{4 bc^2 \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^2} + \frac{3 b \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^4} - \frac{15 b \arccos\left(\frac{1}{cx}\right)}{cx^5} - \frac{15 a}{cx^5} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^6,x, algorithm="giac")

[Out]  $\frac{1}{75}*(8*b*c^4*\sqrt{-1/(c^2*x^2) + 1} + 4*b*c^2*\sqrt{-1/(c^2*x^2) + 1}/x^2 + 3*b*\sqrt{-1/(c^2*x^2) + 1}/x^4 - 15*b*\arccos(1/(c*x))/(c*x^5) - 15*a/(c*x^5))*c$

**maple** [A] time = 0.05, size = 83, normalized size = 1.01

$$c^5 \left( -\frac{a}{5c^5x^5} + b \left( -\frac{\arcsin(cx)}{5c^5x^5} + \frac{(c^2x^2 - 1)(8c^4x^4 + 4c^2x^2 + 3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}} c^6x^6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^6,x)

[Out]  $c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*\arcsin(c*x)+1/75*(c^2*x^2-1)*(8*c^4*x^4+4*c^2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6)$

**maxima** [A] time = 0.34, size = 76, normalized size = 0.93

$$\frac{1}{75} b \left( \frac{3c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 10c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} - \frac{15 \arcsin(cx)}{x^5} \right) - \frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^6,x, algorithm="maxima")

[Out]  $\frac{1}{75}*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*\sqrt{-1/(c^2*x^2) + 1})/c - 15*\arcsin(c*x)/x^5) - 1/5*a/x^5$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x^6,x)

[Out] int((a + b\*acos(1/(c\*x)))/x^6, x)

sympy [A] time = 6.99, size = 156, normalized size = 1.90

$$-\frac{a}{5x^5} - \frac{b \operatorname{asec}(cx)}{5x^5} + \frac{b \left( \begin{cases} \frac{8c^5 \sqrt{c^2x^2-1}}{15x} + \frac{4c^3 \sqrt{c^2x^2-1}}{15x^3} + \frac{c \sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5 \sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3 \sqrt{-c^2x^2+1}}{15x^3} + \frac{ic \sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases} \right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*6,x)

[Out] -a/(5\*x\*\*5) - b\*asec(c\*x)/(5\*x\*\*5) + b\*Piecewise((8\*c\*\*5\*sqrt(c\*\*2\*x\*\*2 - 1)/(15\*x) + 4\*c\*\*3\*sqrt(c\*\*2\*x\*\*2 - 1)/(15\*x\*\*3) + c\*sqrt(c\*\*2\*x\*\*2 - 1)/(5\*x\*\*5), Abs(c\*\*2\*x\*\*2) > 1), (8\*I\*c\*\*5\*sqrt(-c\*\*2\*x\*\*2 + 1)/(15\*x) + 4\*I\*c\*\*3\*sqrt(-c\*\*2\*x\*\*2 + 1)/(15\*x\*\*3) + I\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)/(5\*x\*\*5), True))/(5\*c)

$$3.14 \quad \int \frac{a+b \sec^{-1}(cx)}{x^7} dx$$

**Optimal.** Leaf size=101

$$-\frac{a+b \sec^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6 \csc^{-1}(cx) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} + \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} + \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}$$

[Out]  $-5/96*b*c^6*\arccsc(c*x)+1/6*(-a-b*\arcsec(c*x))/x^6+1/36*b*c*(1-1/c^2/x^2)^{(1/2)}/x^5+5/144*b*c^3*(1-1/c^2/x^2)^{(1/2)}/x^3+5/96*b*c^5*(1-1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5220, 335, 321, 216}

$$-\frac{a+b \sec^{-1}(cx)}{6x^6} + \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} + \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5}{96}bc^6 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/x^7, x]

[Out]  $(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(36*x^5) + (5*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(144*x^3) + (5*b*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)])/(96*x) - (5*b*c^6*\text{ArcCsc}[c*x])/96 - (a + b*\text{ArcSec}[c*x])/(6*x^6)$

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 335

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Subst[Int[(a+b/x^n)^p/x^(m+2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

## Rule 5220

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))*((d_.)*(x_.))^(m_.), x_Symbol] := Sim
p[((d*x)^(m + 1)*(a + b*ArcSec[c*x]))/(d*(m + 1)), x] - Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^7} dx &= -\frac{a + b \sec^{-1}(cx)}{6x^6} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^8} dx}{6c} \\
&= -\frac{a + b \sec^{-1}(cx)}{6x^6} - \frac{b \operatorname{Subst}\left(\int \frac{x^6}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{6c} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} - \frac{a + b \sec^{-1}(cx)}{6x^6} - \frac{1}{36} (5bc) \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} + \frac{5bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{144x^3} - \frac{a + b \sec^{-1}(cx)}{6x^6} - \frac{1}{48} (5bc^3) \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} + \frac{5bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{144x^3} + \frac{5bc^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{96x} - \frac{a + b \sec^{-1}(cx)}{6x^6} - \frac{1}{96} (5bc^5) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} + \frac{5bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{144x^3} + \frac{5bc^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{96x} - \frac{5}{96} bc^6 \csc^{-1}(cx) - \frac{a + b \sec^{-1}(cx)}{6x^6}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 88, normalized size = 0.87

$$-\frac{a}{6x^6} - \frac{5}{96} bc^6 \sin^{-1}\left(\frac{1}{cx}\right) + b \left( \frac{5c^5}{96x} + \frac{5c^3}{144x^3} + \frac{c}{36x^5} \right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} - \frac{b \sec^{-1}(cx)}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/x^7, x]

[Out] -1/6\*a/x^6 + b\*(c/(36\*x^5) + (5\*c^3)/(144\*x^3) + (5\*c^5)/(96\*x))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcSec[c\*x])/(6\*x^6) - (5\*b\*c^6\*ArcSin[1/(c\*x)])/

96



**fricas** [A] time = 0.76, size = 62, normalized size = 0.61

$$\frac{3(5bc^6x^6 - 16b) \operatorname{arcsec}(cx) + (15bc^4x^4 + 10bc^2x^2 + 8b)\sqrt{c^2x^2 - 1} - 48a}{288x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^7,x, algorithm="fricas")

[Out] 1/288\*(3\*(5\*b\*c^6\*x^6 - 16\*b)\*arcsec(c\*x) + (15\*b\*c^4\*x^4 + 10\*b\*c^2\*x^2 + 8\*b)\*sqrt(c^2\*x^2 - 1) - 48\*a)/x^6

**giac** [A] time = 0.14, size = 104, normalized size = 1.03

$$\frac{1}{288} \left( 15bc^5 \arccos\left(\frac{1}{cx}\right) + \frac{15bc^4 \sqrt{-\frac{1}{c^2x^2} + 1}}{x} + \frac{10bc^2 \sqrt{-\frac{1}{c^2x^2} + 1}}{x^3} + \frac{8b \sqrt{-\frac{1}{c^2x^2} + 1}}{x^5} - \frac{48b \arccos\left(\frac{1}{cx}\right)}{cx^6} - \frac{48a}{cx^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^7,x, algorithm="giac")

[Out] 1/288\*(15\*b\*c^5\*arccos(1/(c\*x)) + 15\*b\*c^4\*sqrt(-1/(c^2\*x^2) + 1)/x + 10\*b\*c^2\*sqrt(-1/(c^2\*x^2) + 1)/x^3 + 8\*b\*sqrt(-1/(c^2\*x^2) + 1)/x^5 - 48\*b\*arccos(1/(c\*x))/(c\*x^6) - 48\*a/(c\*x^6))\*c

**maple** [A] time = 0.05, size = 174, normalized size = 1.72

$$\frac{a}{6x^6} - \frac{b \operatorname{arcsec}(cx)}{6x^6} - \frac{5c^5b\sqrt{c^2x^2-1} \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}} x} + \frac{5c^5b}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}} x} - \frac{5c^3b}{288\sqrt{\frac{c^2x^2-1}{c^2x^2}} x^3} - \frac{cb}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}} x^5} - \frac{cb}{36c\sqrt{\frac{c^2x^2-1}{c^2x^2}} x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^7,x)

[Out] -1/6\*a/x^6-1/6\*b/x^6\*arcsec(c\*x)-5/96\*c^5\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2))+5/96\*c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x-5/288\*c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3-1/144\*c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^5-1/36/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^7

**maxima** [A] time = 0.43, size = 165, normalized size = 1.63

$$\frac{1}{288} b \left( \frac{15 c^7 \arctan \left( c x \sqrt{-\frac{1}{c^2 x^2} + 1} \right) - \frac{15 c^{12} x^5 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} + 40 c^{10} x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 33 c^8 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^6 x^6 \left( \frac{1}{c^2 x^2} - 1 \right)^3 - 3 c^4 x^4 \left( \frac{1}{c^2 x^2} - 1 \right)^2 + 3 c^2 x^2 \left( \frac{1}{c^2 x^2} - 1 \right) - 1}}{c} - \frac{48 \operatorname{arcsec}(c x)}{x^6} \right) - \frac{a}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^7,x, algorithm="maxima")

[Out] 1/288\*b\*((15\*c^7\*arctan(c\*x\*sqrt(-1/(c^2\*x^2) + 1)) - (15\*c^12\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 40\*c^10\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 33\*c^8\*x\*sqrt(-1/(c^2\*x^2) + 1))/(c^6\*x^6\*(1/(c^2\*x^2) - 1)^3 - 3\*c^4\*x^4\*(1/(c^2\*x^2) - 1)^2 + 3\*c^2\*x^2\*(1/(c^2\*x^2) - 1) - 1))/c - 48\*arcsec(c\*x)/x^6) - 1/6\*a/x^6

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acos} \left( \frac{1}{c x} \right)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/x^7,x)

[Out] int((a + b\*acos(1/(c\*x)))/x^7, x)

**sympy** [A] time = 8.43, size = 241, normalized size = 2.39

$$-\frac{a}{6x^6} - \frac{b \operatorname{asec}(cx)}{6x^6} + \frac{b \left( \begin{array}{l} \frac{5ic^7 \operatorname{acosh} \left( \frac{1}{cx} \right)}{16} - \frac{5ic^6}{16x \sqrt{-1 + \frac{1}{c^2 x^2}}} + \frac{5ic^4}{48x^3 \sqrt{-1 + \frac{1}{c^2 x^2}}} + \frac{ic^2}{24x^5 \sqrt{-1 + \frac{1}{c^2 x^2}}} + \frac{i}{6x^7 \sqrt{-1 + \frac{1}{c^2 x^2}}} \quad \text{for } \frac{1}{|c^2 x^2|} > 1 \\ -\frac{5c^7 \operatorname{asin} \left( \frac{1}{cx} \right)}{16} + \frac{5c^6}{16x \sqrt{1 - \frac{1}{c^2 x^2}}} - \frac{5c^4}{48x^3 \sqrt{1 - \frac{1}{c^2 x^2}}} - \frac{c^2}{24x^5 \sqrt{1 - \frac{1}{c^2 x^2}}} - \frac{1}{6x^7 \sqrt{1 - \frac{1}{c^2 x^2}}} \quad \text{otherwise} \end{array} \right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*7,x)

```
[Out] -a/(6*x**6) - b*asec(c*x)/(6*x**6) + b*Piecewise((5*I*c**7*acosh(1/(c*x))/16 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/16 + 5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c**2*x**2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1/(c**2*x**2))), True))/(6*c)
```

### 3.15 $\int x^3 \left( a + b \sec^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=107

$$-\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{6c}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^3}+\frac{1}{4}x^4(a+b\sec^{-1}(cx))^2+\frac{b^2\log(x)}{3c^4}+\frac{b^2x^2}{12c^2}$$

[Out]  $1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arcsec(c*x))^2+1/3*b^2*\ln(x)/c^4-1/3*b*x*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/c^3-1/6*b*x^3*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/c$

**Rubi [A]** time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5222, 4409, 4185, 4184, 3475}

$$-\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{6c}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^3}+\frac{1}{4}x^4(a+b\sec^{-1}(cx))^2+\frac{b^2x^2}{12c^2}+\frac{b^2\log(x)}{3c^4}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*ArcSec[c*x])^2, x]`

[Out]  $(b^2*x^2)/(12*c^2) - (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(a + b*\text{ArcSec}[c*x]))/(3*c^3) - (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3*(a + b*\text{ArcSec}[c*x]))/(6*c) + (x^4*(a + b*\text{ArcSec}[c*x])^2)/4 + (b^2*\text{Log}[x])/(3*c^4)$

#### Rule 3475

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

#### Rule 4184

`Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

#### Rule 4185

`Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] := -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]`

Rule 4409

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*Sec[(a_.) + (b_.)*(x_.)]^(n_.)*Tan[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned} \int x^3 (a + b \sec^{-1}(cx))^2 dx &= \frac{\text{Subst}\left(\int (a + bx)^2 \sec^4(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^4} \\ &= \frac{1}{4}x^4 (a + b \sec^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \sec^4(x) dx, x, \sec^{-1}(cx)\right)}{2c^4} \\ &= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3 (a + b \sec^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \sec^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \sec^4(x) dx, x, \sec^{-1}(cx)\right)}{2c^4} \\ &= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \sec^{-1}(cx))}{3c^3} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3 (a + b \sec^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \sec^{-1}(cx))^2 \\ &= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \sec^{-1}(cx))}{3c^3} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3 (a + b \sec^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \sec^{-1}(cx))^2 \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 124, normalized size = 1.16

$$\frac{cx \left( 3a^2c^3x^3 - 2ab\sqrt{1 - \frac{1}{c^2x^2}} (c^2x^2 + 2) + b^2cx \right) - 2bcx \sec^{-1}(cx) \left( b\sqrt{1 - \frac{1}{c^2x^2}} (c^2x^2 + 2) - 3ac^3x^3 \right) + 3b^2c^4x^4 \sec^{-1}(cx)}{12c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcSec[c\*x])^2,x]

[Out] (c\*x\*(b^2\*c\*x + 3\*a^2\*c^3\*x^3 - 2\*a\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*(2 + c^2\*x^2)) - 2\*b\*c\*x\*(-3\*a\*c^3\*x^3 + b\*Sqrt[1 - 1/(c^2\*x^2)]\*(2 + c^2\*x^2))\*ArcSec[c\*x] + 3\*b^2\*c^4\*x^4\*ArcSec[c\*x]^2 + 4\*b^2\*Log[x])/(12\*c^4)

**fricas** [A] time = 0.92, size = 146, normalized size = 1.36

$$\frac{3b^2c^4x^4 \operatorname{arcsec}(cx)^2 + 3a^2c^4x^4 + 12abc^4 \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + b^2c^2x^2 + 4b^2 \log(x) + 6(abc^4x^4 - abc^4)a}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] 1/12\*(3\*b^2\*c^4\*x^4\*arcsec(c\*x)^2 + 3\*a^2\*c^4\*x^4 + 12\*a\*b\*c^4\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + b^2\*c^2\*x^2 + 4\*b^2\*log(x) + 6\*(a\*b\*c^4\*x^4 - a\*b\*c^4)\*arcsec(c\*x) - 2\*(a\*b\*c^2\*x^2 + 2\*a\*b + (b^2\*c^2\*x^2 + 2\*b^2)\*arcsec(c\*x))\*sqrt(c^2\*x^2 - 1))/c^4

**giac** [B] time = 0.81, size = 6625, normalized size = 61.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] 1/12\*(3\*b^2\*arccos(1/(c\*x))^2/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) + 6\*a\*b\*arccos(1/(c\*x))/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) - 12\*b^2\*(1/(c^2\*x^2) - 1)\*arccos(1/(c\*x))^2/((c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8)\*(1/(c\*x) + 1)^2 - 4\*b^2\*log(2)/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) + 4\*b^2\*log(2/(c\*x) + 2)/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) - 4\*b^2\*log(abs(sqrt(-1/(c^2\*x^2) + 1) + 1/(c\*x) + 1))/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) - 12\*b^2\*sqrt(-1/(c^2\*x^2) + 1)\*arccos(1/(c\*x))/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1))^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8)







$(c*x + 1)^8 * (1/(c*x) + 1)^5 - 12*b^2*(1/(c^2*x^2) - 1)^3*\sqrt{-1/(c^2*x^2) + 1}*\arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7 - 12*a^2*(1/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6 + 6*a*b*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 - 4*b^2*(1/(c^2*x^2) - 1)^4*\log(2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 + 4*b^2*(1/(c^2*x^2) - 1)^4*\log(2/(c*x) + 2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 - 4*b^2*(1/(c^2*x^2) - 1)^4*\log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 - 4*b^2*(1/(c^2*x^2) - 1)^4*\log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 - 12*a*b*(1/(c^2*x^2) - 1)^3*\sqrt{-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7 + 3*a^2*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 + b^2*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8))*c$

**maple [B]** time = 0.23, size = 208, normalized size = 1.94

$$\frac{a^2x^4}{4} + \frac{b^2\operatorname{arcsec}(cx)^2x^4}{4} - \frac{b^2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}x^3}{6c} + \frac{b^2x^2}{12c^2} - \frac{b^2\operatorname{arcsec}(cx)x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^3} - \frac{b^2\ln\left(\frac{1}{cx}\right)}{3c^4} + \frac{abx^4\operatorname{arcsec}(cx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arcsec(c\*x))^2,x)

[Out] 1/4\*a^2\*x^4+1/4\*b^2\*arcsec(c\*x)^2\*x^4-1/6/c\*b^2\*arcsec(c\*x)\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^3+1/12\*b^2\*x^2/c^2-1/3/c^3\*b^2\*arcsec(c\*x)\*x\*((c^2\*x^2-1)/c^2/x^2)^(1/2)

$\frac{2}{x^2} \sqrt{x^2-1} - \frac{1}{3} c^4 b^2 \ln(1/c/x) + \frac{1}{2} a b x^4 \operatorname{arcsec}(c x) - \frac{1}{6} c a b \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{1/2} x^3 - \frac{1}{6} c^3 a b \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{1/2} x + \frac{1}{3} c^5 a b \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{1/2} / x$

**maxima** [A] time = 0.69, size = 163, normalized size = 1.52

$$\frac{1}{4} b^2 x^4 \operatorname{arcsec}(c x)^2 + \frac{1}{4} a^2 x^4 + \frac{1}{6} \left( 3 x^4 \operatorname{arcsec}(c x) - \frac{c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{3/2} + 3 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^3} \right) a b + \frac{\left( (c^2 x^2 + 2 \log(x^2)) \sqrt{\dots} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} b^2 x^4 \operatorname{arcsec}(c x)^2 + \frac{1}{4} a^2 x^4 + \frac{1}{6} (3 x^4 \operatorname{arcsec}(c x) - (c^2 x^3 * (-1/(c^2 x^2) + 1)^{3/2} + 3 x * \sqrt{-1/(c^2 x^2) + 1}) / c^3) a b + \frac{1}{12} ((c^2 x^2 + 2 * \log(x^2)) * \sqrt{c x + 1} * \sqrt{c x - 1} - 2 * (c^4 x^4 + c^2 x^2 - 2) * \arctan(\sqrt{c x + 1} * \sqrt{c x - 1})) * b^2 / (\sqrt{c x + 1} * \sqrt{c x - 1} * c^4)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b\*acos(1/(c\*x)))^2,x)

[Out] int(x^3\*(a + b\*acos(1/(c\*x)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{asec}(c x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*asec(c\*x))\*\*2,x)

[Out] Integral(x\*\*3\*(a + b\*asec(c\*x))\*\*2, x)

### 3.16 $\int x^2 (a + b \sec^{-1}(cx))^2 dx$

**Optimal.** Leaf size=147

$$\frac{2ib \tan^{-1}(e^{i \sec^{-1}(cx)}) (a + b \sec^{-1}(cx))}{3c^3} - \frac{bx^2 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))}{3c} + \frac{1}{3} x^3 (a + b \sec^{-1}(cx))^2 - \frac{ib^2 \text{Li}_2(-ie^{i \sec^{-1}(cx)})}{3c^3}$$

[Out]  $\frac{1}{3} b^2 x / c^2 + \frac{1}{3} x^3 (a + b \operatorname{arcsec}(c x))^2 + \frac{2}{3} I b (a + b \operatorname{arcsec}(c x)) \arctan\left(\frac{1/c/x + I(1 - 1/c^2/x^2)^{1/2}}{c^3 - 1/3 I b^2 \operatorname{polylog}(2, -I(1/c/x + I(1 - 1/c^2/x^2)^{1/2}))}\right) / c^3 + \frac{1}{3} I b^2 \operatorname{polylog}(2, I(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) / c^3 - \frac{1}{3} b^2 x^2 (a + b \operatorname{arcsec}(c x)) (1 - 1/c^2/x^2)^{1/2} / c$

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4409, 4185, 4181, 2279, 2391}

$$-\frac{ib^2 \operatorname{PolyLog}(2, -ie^{i \sec^{-1}(cx)})}{3c^3} + \frac{ib^2 \operatorname{PolyLog}(2, ie^{i \sec^{-1}(cx)})}{3c^3} - \frac{bx^2 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))}{3c} + \frac{2ib \tan^{-1}(e^{i \sec^{-1}(cx)})}{3c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcSec[c\*x])^2,x]

[Out]  $\frac{b^2 x}{3c^2} - \frac{b \sqrt{1 - 1/(c^2 x^2)} x^2 (a + b \operatorname{ArcSec}[c x])}{3c} + \frac{x^3 (a + b \operatorname{ArcSec}[c x])^2}{3} + \frac{((2I)/3) b (a + b \operatorname{ArcSec}[c x]) \operatorname{ArcTan}[E^{I \operatorname{ArcSec}[c x]}]}{c^3} - \frac{((I/3) b^2 \operatorname{PolyLog}[2, (-I) E^{I \operatorname{ArcSec}[c x]}]}{c^3} + \frac{((I/3) b^2 \operatorname{PolyLog}[2, I E^{I \operatorname{ArcSec}[c x]}]}{c^3}$

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m \* ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x],

$x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + E^{(I*k*Pi)} * E^{(I*(e + f*x))}], x], x) /;$  FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4185

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_)]*(b_.)^{(n_)}*((c_.) + (d_.)*(x_)), x\_Symbol] :>$   
 $-\text{Simp}[(b^2*(c + d*x)*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x]$   
 $+ (\text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(c + d*x)*(b*\text{Csc}[e + f*x])^{(n-2)}, x]$   
 $, x] - \text{Simp}[(b^2*d*(b*\text{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /;$   
 FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 4409

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_)} * \text{Sec}[(a_.) + (b_.)*(x_)]^{(n_)} * \text{Tan}[(a_.) + (b_.)*(x_)]^{(p_)}, x\_Symbol] :>$   $\text{Simp}[(c + d*x)^m * \text{Sec}[a + b*x]^n / (b*n), x] -$   
 $\text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m-1)} * \text{Sec}[a + b*x]^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 5222

$\text{Int}[(a_.) + \text{ArcSec}[(c_.)*(x_)]*(b_.)^{(n_)}*(x_)]^{(m_)}, x\_Symbol] :>$   $\text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n * \text{Sec}[x]^{(m+1)} * \text{Tan}[x], x], x, \text{ArcSec}[c*x]$   
 $], x] /;$  FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \sec^{-1}(cx))^2 dx &= \frac{\text{Subst}\left(\int (a + bx)^2 \sec^3(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^3} \\
&= \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^2 - \frac{(2b) \text{Subst}\left(\int (a + bx) \sec^3(x) dx, x, \sec^{-1}(cx)\right)}{3c^3} \\
&= \frac{b^2x}{3c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \sec^3(x) dx, x, \sec^{-1}(cx)\right)}{3c^3} \\
&= \frac{b^2x}{3c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^2 + \frac{2ib(a + b \sec^{-1}(cx))}{3c} \\
&= \frac{b^2x}{3c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^2 + \frac{2ib(a + b \sec^{-1}(cx))}{3c} \\
&= \frac{b^2x}{3c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^2 + \frac{2ib(a + b \sec^{-1}(cx))}{3c}
\end{aligned}$$

**Mathematica [A]** time = 1.27, size = 225, normalized size = 1.53

$$\frac{1}{3} \left( a^2x^3 + \frac{ab \left( 2x^4 \sec^{-1}(cx) - \frac{c^3x^3 + \sqrt{c^2x^2 - 1} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right) - cx}{c^4 \sqrt{1 - \frac{1}{c^2x^2}}} \right)}{x} + \frac{b^2 \left( c^3x^3 \sec^{-1}(cx)^2 - c^2x^2 \sqrt{1 - \frac{1}{c^2x^2}} \sec^{-1}(cx) - \dots \right)}{3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcSec[c\*x])^2,x]

[Out] (a^2\*x^3 + (a\*b\*(2\*x^4\*ArcSec[c\*x] - ((-c\*x) + c^3\*x^3 + Sqrt[-1 + c^2\*x^2]\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(c^4\*Sqrt[1 - 1/(c^2\*x^2)])))/x + (b^2\*(c\*x - c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2\*ArcSec[c\*x] + c^3\*x^3\*ArcSec[c\*x]^2 - ArcSec[c\*x]\*Log[1 - I\*E^(I\*ArcSec[c\*x])] + ArcSec[c\*x]\*Log[1 + I\*E^(I\*ArcSec[c\*x])] - I\*PolyLog[2, (-I)\*E^(I\*ArcSec[c\*x])] + I\*PolyLog[2, I\*E^(I\*ArcSec[c\*x])]))/c^3)/3

**fricas [F]** time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2 \operatorname{arcsec}(cx)^2 + 2abx^2 \operatorname{arcsec}(cx) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*arcsec(c\*x)^2 + 2\*a\*b\*x^2\*arcsec(c\*x) + a^2\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^2\*x^2, x)

**maple** [B] time = 0.91, size = 343, normalized size = 2.33

$$\frac{x^3 a^2}{3} + \frac{b^2 x^3 \operatorname{arcsec}(cx)^2}{3} - \frac{b^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} \operatorname{arcsec}(cx) x^2}{3c} + \frac{b^2 x}{3c^2} + \frac{b^2 \operatorname{arcsec}(cx) \ln\left(1 + i\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)}{3c^3} - \frac{b^2 \operatorname{arcsec}(cx)}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x))^2,x)

[Out]  $\frac{1}{3}x^3a^2 + \frac{1}{3}b^2x^3\operatorname{arcsec}(cx)^2 - \frac{1}{3}cb^2\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{(1/2)} \operatorname{arcsec}(cx)x^2 + \frac{1}{3}b^2x/c^2 + \frac{1}{3}c^3b^2\operatorname{arcsec}(cx)\ln\left(1 + I\left(\frac{1}{c/x} + I\left(1 - \frac{1}{c^2/x^2}\right)^{(1/2)}\right)\right) - \frac{1}{3}c^3b^2\operatorname{arcsec}(cx)\ln\left(1 - I\left(\frac{1}{c/x} + I\left(1 - \frac{1}{c^2/x^2}\right)^{(1/2)}\right)\right) - \frac{1}{3}I/c^3b^2\operatorname{dilog}\left(1 + I\left(\frac{1}{c/x} + I\left(1 - \frac{1}{c^2/x^2}\right)^{(1/2)}\right)\right) + \frac{1}{3}I/c^3b^2\operatorname{dilog}\left(1 - I\left(\frac{1}{c/x} + I\left(1 - \frac{1}{c^2/x^2}\right)^{(1/2)}\right)\right) + \frac{2}{3}x^3ab\operatorname{arcsec}(cx) - \frac{1}{3}c^4ab\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{(1/2)}x^2 + \frac{1}{3}c^3ab\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{(1/2)} - \frac{1}{3}c^4ab\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{(1/2)}/\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{(1/2)}/x\ln(cx + \left(\frac{c^2x^2-1}{c^2/x^2}\right)^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3 + \frac{1}{6}\left(4x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)ab + \frac{1}{12}\left(4x^3\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1))/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integrate(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) - 24*integrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acos(1/(c*x)))^2,x)
```

```
[Out] int(x^2*(a + b*acos(1/(c*x)))^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asec}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*asec(c*x))**2, x)
```

### 3.17 $\int x \left( a + b \sec^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=56

$$-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{c} + \frac{1}{2}x^2(a+b\sec^{-1}(cx))^2 + \frac{b^2\log(x)}{c^2}$$

[Out]  $1/2*x^2*(a+b*\text{arcsec}(c*x))^2+b^2*\ln(x)/c^2-b*x*(a+b*\text{arcsec}(c*x))*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5222, 4409, 4184, 3475}

$$-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{c} + \frac{1}{2}x^2(a+b\sec^{-1}(cx))^2 + \frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcSec[c\*x])^2,x]

[Out]  $-((b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(a + b*\text{ArcSec}[c*x]))/c) + (x^2*(a + b*\text{ArcSec}[c*x])^2)/2 + (b^2*\text{Log}[x])/c^2$

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 4409

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sec[a + b\*x]^n)/(b\*n), x] - Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sec[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 5222



```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned} \int x(a + b \sec^{-1}(cx))^2 dx &= \frac{\text{Subst}\left(\int (a + bx)^2 \sec^2(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^2} \\ &= \frac{1}{2}x^2(a + b \sec^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \sec^2(x) dx, x, \sec^{-1}(cx)\right)}{c^2} \\ &= -\frac{b\sqrt{1 - \frac{1}{c^2x^2}}x(a + b \sec^{-1}(cx))}{c} + \frac{1}{2}x^2(a + b \sec^{-1}(cx))^2 + \frac{b^2 \text{Subst}\left(\int \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^2} \\ &= -\frac{b\sqrt{1 - \frac{1}{c^2x^2}}x(a + b \sec^{-1}(cx))}{c} + \frac{1}{2}x^2(a + b \sec^{-1}(cx))^2 + \frac{b^2 \log(x)}{c^2} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 90, normalized size = 1.61

$$\frac{acx \left( acx - 2b\sqrt{1 - \frac{1}{c^2x^2}} \right) + 2bcx \sec^{-1}(cx) \left( acx - b\sqrt{1 - \frac{1}{c^2x^2}} \right) + b^2c^2x^2 \sec^{-1}(cx)^2 + 2b^2 \log(cx)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcSec[c\*x])^2,x]

[Out] (a\*c\*x\*(-2\*b\*Sqrt[1 - 1/(c^2\*x^2)] + a\*c\*x) + 2\*b\*c\*x\*(-(b\*Sqrt[1 - 1/(c^2\*x^2)])) + a\*c\*x)\*ArcSec[c\*x] + b^2\*c^2\*x^2\*ArcSec[c\*x]^2 + 2\*b^2\*Log[c\*x])/(2\*c^2)

**fricas [B]** time = 0.76, size = 111, normalized size = 1.98

$$\frac{b^2c^2x^2 \operatorname{arcsec}(cx)^2 + a^2c^2x^2 + 4abc^2 \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + 2b^2 \log(x) + 2(abc^2x^2 - abc^2) \operatorname{arcsec}(cx) - 2c^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] 1/2\*(b^2\*c^2\*x^2\*arcsec(c\*x)^2 + a^2\*c^2\*x^2 + 4\*a\*b\*c^2\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + 2\*b^2\*log(x) + 2\*(a\*b\*c^2\*x^2 - a\*b\*c^2)\*arcsec(c\*x) - 2\*sqrt(c^2\*x^2 - 1)\*(b^2\*arcsec(c\*x) + a\*b))/c^2

**giac [B]** time = 0.48, size = 2181, normalized size = 38.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out]  $\frac{1}{2}*(b^2*\arccos(1/(c*x))^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*b*\arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)*\arccos(1/(c*x))^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b^2*\log(2)/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b^2*\log(2/(c*x) + 2)/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*b^2*\sqrt{-1/(c^2*x^2) + 1}*\arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + a^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*a*b*(1/(c^2*x^2) - 1)*\arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + b^2*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 4*b^2*(1/(c^2*x^2) - 1)*\log(2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 4*b^2*(1/(c^2*x^2) - 1)*\log(2/(c*x) + 2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 4*b^2*(1/(c^2*x^2) - 1)*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 4*b^2*(1/(c^2*x^2) - 1)*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 4*a*b*\sqrt{-1/(c^2*x^2) + 1}/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + 4*b^2*(-1/(c^2*x^2) + 1)^(3/2)*\arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) - 2*a^2*(1/(c^2*x^2) - 1)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 2*a*b*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)^2*\log(2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)$

$$x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b^2*(1/(c^2*x^2) - 1)^2*\log(2/(c*x) + 2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 4*a*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) + a^2*(1/(c^2*x^2) - 1)^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4))*c$$

**maple [B]** time = 0.26, size = 134, normalized size = 2.39

$$\frac{a^2x^2}{2} + \frac{x^2b^2\text{arcsec}(cx)^2}{2} - \frac{b^2\text{arcsec}(cx)x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{c} - \frac{b^2\ln\left(\frac{1}{cx}\right)}{c^2} + abx^2\text{arcsec}(cx) - \frac{abx}{c\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{ab}{c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x))^2,x)

[Out] 1/2\*a^2\*x^2+1/2\*x^2\*b^2\*arcsec(c\*x)^2-1/c\*b^2\*arcsec(c\*x)\*x\*((c^2\*x^2-1)/c^2/x^2)^(1/2)-1/c^2\*b^2\*ln(1/c/x)+a\*b\*x^2\*arcsec(c\*x)-1/c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x+1/c^3\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x

**maxima [A]** time = 0.33, size = 87, normalized size = 1.55

$$\frac{1}{2}b^2x^2\text{arcsec}(cx)^2 + \frac{1}{2}a^2x^2 + \left(x^2\text{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c}\right)ab - \left(\frac{x\sqrt{-\frac{1}{c^2x^2} + 1}\text{arcsec}(cx)}{c} - \frac{\log(x)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2\*arcsec(c\*x)^2 + 1/2\*a^2\*x^2 + (x^2\*arcsec(c\*x) - x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*a\*b - (x\*sqrt(-1/(c^2\*x^2) + 1)\*arcsec(c\*x)/c - log(x)/c^2)\*b^2

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x \left( a + b \arccos\left(\frac{1}{cx}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acos(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*acos(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{asec}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(x*(a + b*asec(c*x))**2, x)
```

### 3.18 $\int (a + b \sec^{-1}(cx))^2 dx$

**Optimal.** Leaf size=92

$$x(a + b \sec^{-1}(cx))^2 + \frac{4ib \tan^{-1}(e^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c} - \frac{2ib^2 \text{Li}_2(-ie^{i \sec^{-1}(cx)})}{c} + \frac{2ib^2 \text{Li}_2(ie^{i \sec^{-1}(cx)})}{c}$$

[Out]  $x*(a+b*\text{arcsec}(c*x))^2+4*I*b*(a+b*\text{arcsec}(c*x))*\arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))/c-2*I*b^2*\text{polylog}(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c+2*I*b^2*\text{polylog}(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c$

**Rubi [A]** time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5216, 4409, 4181, 2279, 2391}

$$-\frac{2ib^2 \text{PolyLog}(2, -ie^{i \sec^{-1}(cx)})}{c} + \frac{2ib^2 \text{PolyLog}(2, ie^{i \sec^{-1}(cx)})}{c} + x(a + b \sec^{-1}(cx))^2 + \frac{4ib \tan^{-1}(e^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^2, x]

[Out]  $x*(a + b*\text{ArcSec}[c*x])^2 + ((4*I)*b*(a + b*\text{ArcSec}[c*x])* \text{ArcTan}[E^{(I*\text{ArcSec}[c*x])}])/c - ((2*I)*b^2*\text{PolyLog}[2, (-I)*E^{(I*\text{ArcSec}[c*x])}])/c + ((2*I)*b^2*\text{PolyLog}[2, I*E^{(I*\text{ArcSec}[c*x])}])/c$

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))]^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5216

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[1/c, Subst[Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int (a + b \sec^{-1}(cx))^2 dx &= \frac{\text{Subst}\left(\int (a + bx)^2 \sec(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c} \\ &= x(a + b \sec^{-1}(cx))^2 - \frac{(2b) \text{Subst}\left(\int (a + bx) \sec(x) dx, x, \sec^{-1}(cx)\right)}{c} \\ &= x(a + b \sec^{-1}(cx))^2 + \frac{4ib(a + b \sec^{-1}(cx)) \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} + \frac{(2b^2) \text{Subst}\left(\int \log(1 - i \sec^{-1}(cx)) dx, x, \sec^{-1}(cx)\right)}{c} \\ &= x(a + b \sec^{-1}(cx))^2 + \frac{4ib(a + b \sec^{-1}(cx)) \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{(2ib^2) \text{Subst}\left(\int \frac{\log(1 - i \sec^{-1}(cx))}{x} dx, x, \sec^{-1}(cx)\right)}{c} \\ &= x(a + b \sec^{-1}(cx))^2 + \frac{4ib(a + b \sec^{-1}(cx)) \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{2ib^2 \text{Li}_2\left(-ie^{i \sec^{-1}(cx)}\right)}{c} \end{aligned}$$

**Mathematica** [A] time = 0.18, size = 163, normalized size = 1.77

$$\frac{a^2 cx + 2ab \left( cx \sec^{-1}(cx) + \log\left(\cos\left(\frac{1}{2} \sec^{-1}(cx)\right) - \sin\left(\frac{1}{2} \sec^{-1}(cx)\right)\right) - \log\left(\cos\left(\frac{1}{2} \sec^{-1}(cx)\right) + \sin\left(\frac{1}{2} \sec^{-1}(cx)\right)\right) \right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSec[c*x])^2, x]
```

```
[Out] (a^2*c*x + 2*a*b*(c*x*ArcSec[c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*x]/2]) - Log[Cos[ArcSec[c*x]/2] + Sin[ArcSec[c*x]/2])) + b^2*(ArcSec[c*x]*(c*x*ArcSec[c*x] - 2*Log[1 - I*E^(I*ArcSec[c*x])]) + 2*Log[1 + I*E^(I*ArcSec[c*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (2*I)*PolyLog[2, I*E^(I*ArcSec[c*x])])/c
```

**fricas** [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}(b^2 \operatorname{arcsec}(cx)^2 + 2ab \operatorname{arcsec}(cx) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^2, x)

**maple** [A] time = 0.26, size = 212, normalized size = 2.30

$$x b^2 \operatorname{arcsec}(cx)^2 + 2xab \operatorname{arcsec}(cx) - \frac{2b^2 \operatorname{arcsec}(cx) \ln\left(1 - i\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)\right)}{c} + \frac{2b^2 \operatorname{arcsec}(cx) \ln\left(1 + i\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^2,x)

[Out]  $x*b^2*\operatorname{arcsec}(c*x)^2+2*x*a*b*\operatorname{arcsec}(c*x)-2/c*b^2*\operatorname{arcsec}(c*x)*\ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2/c*b^2*\operatorname{arcsec}(c*x)*\ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-2*I/c*\operatorname{dilog}(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))*b^2+2*I/c*\operatorname{dilog}(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))*b^2+a^2*x-2/c*\ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))*a*b$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(2c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\log(c)^2 - 4c^2 \int \frac{x^2 \log(c^2x^2)}{c^2x^2-1} dx \log(c) + 8c^2 \int \frac{x^2 \log(x)}{c^2x^2-1} dx \log(c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out]  $-1/4*(2*c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3)*\log(c)^2 - 4*c^2*\int(x^2*\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) + 8*c^2*\int(x^2$

```
*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*integrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(log(x)/(c^2*x^2 - 1), x)*log(c) + 8*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x) + 4*integrate(log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x) + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a*b/c
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^2,x)

[Out] int((a + b\*acos(1/(c\*x)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asec}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*2,x)

[Out] Integral((a + b\*asec(c\*x))\*\*2, x)



$$3.19 \quad \int \frac{(a+b \sec^{-1}(cx))^2}{x} dx$$

Optimal. Leaf size=93

$$ibLi_2(-e^{2i \sec^{-1}(cx)})(a+b \sec^{-1}(cx)) + \frac{i(a+b \sec^{-1}(cx))^3}{3b} - \log(1+e^{2i \sec^{-1}(cx)})(a+b \sec^{-1}(cx))^2 - \frac{1}{2}b^2Li_3(-e^{2i \sec^{-1}(cx)})$$

[Out] 1/3\*I\*(a+b\*arcsec(c\*x))^3/b - (a+b\*arcsec(c\*x))^2\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+I\*b\*(a+b\*arcsec(c\*x))\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)-1/2\*b^2\*polylog(3,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 3719, 2190, 2531, 2282, 6589}

$$ibPolyLog(2,-e^{2i \sec^{-1}(cx)})(a+b \sec^{-1}(cx)) - \frac{1}{2}b^2PolyLog(3,-e^{2i \sec^{-1}(cx)}) + \frac{i(a+b \sec^{-1}(cx))^3}{3b} - \log(1+e^{2i \sec^{-1}(cx)})$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^2/x, x]

[Out] ((I/3)\*(a + b\*ArcSec[c\*x])^3)/b - (a + b\*ArcSec[c\*x])^2\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] + I\*b\*(a + b\*ArcSec[c\*x])\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])] - (b^2\*PolyLog[3, -E^((2\*I)\*ArcSec[c\*x])])/2

Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3719

```
Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
LtQ[m, -1])
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec^{-1}(cx))^2}{x} dx &= \text{Subst} \left( \int (a + bx)^2 \tan(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^3}{3b} - 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)^2}{1 + e^{2ix}} dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^3}{3b} - (a + b \sec^{-1}(cx))^2 \log(1 + e^{2i \sec^{-1}(cx)}) + (2b) \text{Subst} \left( \int (a + bx) \tan(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^3}{3b} - (a + b \sec^{-1}(cx))^2 \log(1 + e^{2i \sec^{-1}(cx)}) + ib(a + b \sec^{-1}(cx)) \text{Li}_2(-e^{2i \sec^{-1}(cx)}) \\
&= \frac{i(a + b \sec^{-1}(cx))^3}{3b} - (a + b \sec^{-1}(cx))^2 \log(1 + e^{2i \sec^{-1}(cx)}) + ib(a + b \sec^{-1}(cx)) \text{Li}_2(-e^{2i \sec^{-1}(cx)}) \\
&= \frac{i(a + b \sec^{-1}(cx))^3}{3b} - (a + b \sec^{-1}(cx))^2 \log(1 + e^{2i \sec^{-1}(cx)}) + ib(a + b \sec^{-1}(cx)) \text{Li}_2(-e^{2i \sec^{-1}(cx)})
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 129, normalized size = 1.39

$$a^2 \log(cx) + ib \text{Li}_2(-e^{2i \sec^{-1}(cx)}) (a + b \sec^{-1}(cx)) + iab \sec^{-1}(cx)^2 - 2ab \sec^{-1}(cx) \log(1 + e^{2i \sec^{-1}(cx)}) - \frac{1}{2} b^2 \text{Li}_3(-e^{2i \sec^{-1}(cx)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])^2/x, x]

[Out] I\*a\*b\*ArcSec[c\*x]^2 + (I/3)\*b^2\*ArcSec[c\*x]^3 - 2\*a\*b\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - b^2\*ArcSec[c\*x]^2\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] + a^2\*Log[c\*x] + I\*b\*(a + b\*ArcSec[c\*x])\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])] - (b^2\*PolyLog[3, -E^((2\*I)\*ArcSec[c\*x])])/2

**fricas [F]** time = 0.41, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \text{arcsec}(cx)^2 + 2ab \text{arcsec}(cx) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x, x, algorithm="fricas")

[Out] integral((b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2)/x, x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
by intervals (correct if the argument is real):Check [abs(x)]Evaluation time:  
0.52ln of unsigned or minus infinity Error: Bad Argument Value

**maple** [A] time = 0.22, size = 215, normalized size = 2.31

$$a^2 \ln(cx) + \frac{ib^2 \operatorname{arcsec}(cx)^3}{3} - b^2 \operatorname{arcsec}(cx)^2 \ln \left( 1 + \left( \frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2 x^2}} \right)^2 \right) + ib^2 \operatorname{arcsec}(cx) \operatorname{polylog} \left( 2, - \left( \frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2 x^2}} \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^2/x,x)

[Out] a^2\*ln(c\*x)+1/3\*I\*b^2\*arcsec(c\*x)^3-b^2\*arcsec(c\*x)^2\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+I\*b^2\*arcsec(c\*x)\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)-1/2\*b^2\*polylog(3,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+I\*a\*b\*arcsec(c\*x)^2-2\*a\*b\*arcsec(c\*x)\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+I\*a\*b\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} b^2 c^2 \left( \frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2} \right) \log(c)^2 + b^2 c^2 \int \frac{x^2 \log(c^2 x^2)}{c^2 x^3 - x} dx \log(c) - 2 b^2 c^2 \int \frac{x^2 \log(x)}{c^2 x^3 - x} dx \log(c) + 2 b^2 c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x,x, algorithm="maxima")

[Out] -1/2\*b^2\*c^2\*(log(c\*x + 1)/c^2 + log(c\*x - 1)/c^2)\*log(c)^2 + b^2\*c^2\*integrate(x^2\*log(c^2\*x^2)/(c^2\*x^3 - x), x)\*log(c) - 2\*b^2\*c^2\*integrate(x^2\*log(x)/(c^2\*x^3 - x), x)\*log(c) + 2\*b^2\*c^2\*integrate(x^2\*log(c^2\*x^2)\*log(x)/(c^2\*x^3 - x), x) - b^2\*c^2\*integrate(x^2\*log(x)^2/(c^2\*x^3 - x), x) + 2\*a\*b\*c^2\*integrate(x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(c^2\*x^3 - x), x) + 1/2\*b^2\*(log(c\*x + 1) + log(c\*x - 1) - 2\*log(x))\*log(c)^2 + b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2\*log(x) - 1/4\*b^2\*log(c^2\*x^2)^2\*log(x) - b^2\*integrate(log(c^2\*x^2)/(c^2\*x^3 - x), x)\*log(c) + 2\*b^2\*integrate(log(x)/(c^2\*x^3 - x), x)\*log(c) - 2\*b^2\*integrate(sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*log(x)/(c^2\*x^3 - x), x) - 2\*b^2\*integrate(log(c^2\*x^2)\*log(x)/(c^2\*x^3 - x), x) + b^2\*integrate(log(x)^2/(c^2\*x^3 - x),

$x) - 2*a*b*\text{integrate}(\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})/(c^2*x^3 - x), x) + a^2*\log(x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^2/x, x)

[Out] int((a + b\*acos(1/(c\*x)))^2/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*2/x, x)

[Out] Integral((a + b\*asec(c\*x))\*\*2/x, x)

$$3.20 \quad \int \frac{(a+b \sec^{-1}(cx))^2}{x^2} dx$$

Optimal. Leaf size=50

$$2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx)) - \frac{(a+b \sec^{-1}(cx))^2}{x} + \frac{2b^2}{x}$$

[Out]  $2*b^2/x-(a+b*\operatorname{arcsec}(c*x))^2/x+2*b*c*(a+b*\operatorname{arcsec}(c*x))*(1-1/c^2/x^2)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5222, 3296, 2638}

$$2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx)) - \frac{(a+b \sec^{-1}(cx))^2}{x} + \frac{2b^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^2/x^2, x]

[Out]  $(2*b^2)/x + 2*b*c*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*(a + b*\operatorname{ArcSec}[c*x]) - (a + b*\operatorname{ArcSec}[c*x])^2/x$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m+1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m+1)\*Tan[x], x], x, ArcSec[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec^{-1}(cx))^2}{x^2} dx &= c \operatorname{Subst} \left( \int (a + bx)^2 \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{(a + b \sec^{-1}(cx))^2}{x} + (2bc) \operatorname{Subst} \left( \int (a + bx) \cos(x) dx, x, \sec^{-1}(cx) \right) \\
&= 2bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx)) - \frac{(a + b \sec^{-1}(cx))^2}{x} - (2b^2 c) \operatorname{Subst} \left( \int \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{2b^2}{x} + 2bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx)) - \frac{(a + b \sec^{-1}(cx))^2}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 75, normalized size = 1.50

$$\frac{-a^2 + 2abcx \sqrt{1 - \frac{1}{c^2 x^2}} + 2b \sec^{-1}(cx) \left( bcx \sqrt{1 - \frac{1}{c^2 x^2}} - a \right) - b^2 \sec^{-1}(cx)^2 + 2b^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^2/x^2,x]

[Out] (-a^2 + 2\*b^2 + 2\*a\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + 2\*b\*(-a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)\*ArcSec[c\*x] - b^2\*ArcSec[c\*x]^2)/x

**fricas [A]** time = 0.48, size = 57, normalized size = 1.14

$$\frac{b^2 \operatorname{arcsec}(cx)^2 + 2ab \operatorname{arcsec}(cx) + a^2 - 2b^2 - 2\sqrt{c^2 x^2 - 1} (b^2 \operatorname{arcsec}(cx) + ab)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^2,x, algorithm="fricas")

[Out] -(b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2 - 2\*b^2 - 2\*sqrt(c^2\*x^2 - 1)\*(b^2\*arcsec(c\*x) + a\*b))/x

**giac [B]** time = 0.17, size = 105, normalized size = 2.10

$$\left( 2b^2 \sqrt{-\frac{1}{c^2 x^2} + 1} \arccos\left(\frac{1}{cx}\right) + 2ab \sqrt{-\frac{1}{c^2 x^2} + 1} - \frac{b^2 \arccos\left(\frac{1}{cx}\right)^2}{cx} - \frac{2ab \arccos\left(\frac{1}{cx}\right)}{cx} - \frac{a^2}{cx} + \frac{2b^2}{cx} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^2,x, algorithm="giac")

[Out] (2\*b^2\*sqrt(-1/(c^2\*x^2) + 1)\*arccos(1/(c\*x)) + 2\*a\*b\*sqrt(-1/(c^2\*x^2) + 1) - b^2\*arccos(1/(c\*x))^2/(c\*x) - 2\*a\*b\*arccos(1/(c\*x))/(c\*x) - a^2/(c\*x) + 2\*b^2/(c\*x))\*c

**maple [B]** time = 0.14, size = 117, normalized size = 2.34

$$c \left( -\frac{a^2}{cx} + b^2 \left( -\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2 \operatorname{arcsec}(cx) \sqrt{\frac{c^2x^2-1}{c^2x^2}} \right) + 2ab \left( -\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{c^2x^2-1} c^2x^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^2/x^2,x)

[Out] c\*(-a^2/c/x+b^2\*(-1/c/x\*arcsec(c\*x)^2+2/c/x+2\*arcsec(c\*x)\*((c^2\*x^2-1)/c^2/x^2)^(1/2))+2\*a\*b\*(-1/c/x\*arcsec(c\*x)+1/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2\*(c^2\*x^2-1)))

**maxima [A]** time = 0.33, size = 78, normalized size = 1.56

$$2 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsec}(cx)}{x} \right) ab + 2 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} \operatorname{arcsec}(cx) + \frac{1}{x} \right) b^2 - \frac{b^2 \operatorname{arcsec}(cx)^2}{x} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^2,x, algorithm="maxima")

[Out] 2\*(c\*sqrt(-1/(c^2\*x^2) + 1) - arcsec(c\*x)/x)\*a\*b + 2\*(c\*sqrt(-1/(c^2\*x^2) + 1)\*arcsec(c\*x) + 1/x)\*b^2 - b^2\*arcsec(c\*x)^2/x - a^2/x

**mupad [B]** time = 0.82, size = 89, normalized size = 1.78

$$2b^2c \operatorname{acos}\left(\frac{1}{cx}\right) \sqrt{1 - \frac{1}{c^2x^2}} - \frac{b^2 \left( \operatorname{acos}\left(\frac{1}{cx}\right)^2 - 2 \right)}{x} - \frac{a^2}{x} + 2abc \left( \sqrt{1 - \frac{1}{c^2x^2}} - \frac{\operatorname{acos}\left(\frac{1}{cx}\right)}{cx} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^2/x^2,x)

[Out] 2\*b^2\*c\*acos(1/(c\*x))\*(1 - 1/(c^2\*x^2))^(1/2) - (b^2\*(acos(1/(c\*x))^2 - 2))/x - a^2/x + 2\*a\*b\*c\*((1 - 1/(c^2\*x^2))^(1/2) - acos(1/(c\*x))/(c\*x))



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*2/x\*\*2,x)

[Out] Integral((a + b\*asec(c\*x))\*\*2/x\*\*2, x)

$$3.21 \quad \int \frac{(a+b \sec^{-1}(cx))^2}{x^3} dx$$

**Optimal.** Leaf size=94

$$\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{2x} + \frac{1}{2}\left(c^2 - \frac{1}{x^2}\right)(a+b \sec^{-1}(cx))^2 - \frac{1}{2}abc^2 \sec^{-1}(cx) - \frac{1}{4}b^2c^2 \sec^{-1}(cx)^2 + \frac{b^2}{4x^2}$$

[Out]  $1/4*b^2/x^2-1/2*a*b*c^2*\text{arcsec}(c*x)-1/4*b^2*c^2*\text{arcsec}(c*x)^2+1/2*(c^2-1/x^2)*(a+b*\text{arcsec}(c*x))^2+1/2*b*c*(a+b*\text{arcsec}(c*x))*(1-1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5222, 4404, 3310}

$$\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{2x} + \frac{1}{2}\left(c^2 - \frac{1}{x^2}\right)(a+b \sec^{-1}(cx))^2 - \frac{1}{2}abc^2 \sec^{-1}(cx) - \frac{1}{4}b^2c^2 \sec^{-1}(cx)^2 + \frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^2/x^3, x]

[Out]  $b^2/(4*x^2) - (a*b*c^2*ArcSec[c*x])/2 - (b^2*c^2*ArcSec[c*x]^2)/4 + (b*c*sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(2*x) + ((c^2 - x^{-2})*(a + b*ArcSec[c*x])^2)/2$

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sine[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sine[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)^(n_)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
```

], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec^{-1}(cx))^2}{x^3} dx &= c^2 \text{Subst} \left( \int (a + bx)^2 \cos(x) \sin(x) dx, x, \sec^{-1}(cx) \right) \\ &= \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^2 - (bc^2) \text{Subst} \left( \int (a + bx) \sin^2(x) dx, x, \sec^{-1}(cx) \right) \\ &= \frac{b^2}{4x^2} + \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))}{2x} + \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^2 - \frac{1}{2} (bc^2) \text{Subst} \left( \int (a + bx) \sin^2(x) dx, x, \sec^{-1}(cx) \right) \\ &= \frac{b^2}{4x^2} - \frac{1}{2} abc^2 \sec^{-1}(cx) - \frac{1}{4} b^2 c^2 \sec^{-1}(cx)^2 + \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))}{2x} + \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^2 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 102, normalized size = 1.09

$$\frac{-2a^2 + 2abcx \sqrt{1 - \frac{1}{c^2 x^2}} - 2abc^2 x^2 \sin^{-1} \left( \frac{1}{cx} \right) + 2b \sec^{-1}(cx) \left( bcx \sqrt{1 - \frac{1}{c^2 x^2}} - 2a \right) + b^2 (c^2 x^2 - 2) \sec^{-1}(cx)^2 + b^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^2/x^3,x]

[Out]  $\frac{(-2a^2 + b^2 + 2a*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x + 2*b*(-2*a + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)*\text{ArcSec}[c*x] + b^2*(-2 + c^2*x^2)*\text{ArcSec}[c*x]^2 - 2*a*b*c^2*x^2*\text{ArcSin}[1/(c*x)])}{4*x^2}$

**fricas [A]** time = 0.59, size = 82, normalized size = 0.87

$$\frac{(b^2 c^2 x^2 - 2 b^2) \text{arcsec}(cx)^2 - 2 a^2 + b^2 + 2 (abc^2 x^2 - 2 ab) \text{arcsec}(cx) + 2 \sqrt{c^2 x^2 - 1} (b^2 \text{arcsec}(cx) + ab)}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^3,x, algorithm="fricas")

[Out]  $\frac{1}{4} * ((b^2 * c^2 * x^2 - 2 * b^2) * \text{arcsec}(c * x)^2 - 2 * a^2 + b^2 + 2 * (a * b * c^2 * x^2 - 2 * a * b) * \text{arcsec}(c * x) + 2 * \text{sqrt}(c^2 * x^2 - 1) * (b^2 * \text{arcsec}(c * x) + a * b)) / x^2$

**giac** [A] time = 0.16, size = 147, normalized size = 1.56

$$\frac{1}{8} \left( 2b^2c \arccos\left(\frac{1}{cx}\right)^2 + 4abc \arccos\left(\frac{1}{cx}\right) - b^2c + \frac{4b^2\sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right)}{x} + \frac{4ab\sqrt{-\frac{1}{c^2x^2} + 1}}{x} - \frac{4b^2 \arccos\left(\frac{1}{cx}\right)}{cx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^3,x, algorithm="giac")

[Out] 1/8\*(2\*b^2\*c\*arccos(1/(c\*x))^2 + 4\*a\*b\*c\*arccos(1/(c\*x)) - b^2\*c + 4\*b^2\*sqrt(-1/(c^2\*x^2) + 1)\*arccos(1/(c\*x))/x + 4\*a\*b\*sqrt(-1/(c^2\*x^2) + 1)/x - 4\*b^2\*arccos(1/(c\*x))^2/(c\*x^2) - 8\*a\*b\*arccos(1/(c\*x))/(c\*x^2) - 4\*a^2/(c\*x^2) + 2\*b^2/(c\*x^2))\*c

**maple** [B] time = 0.15, size = 199, normalized size = 2.12

$$-\frac{a^2}{2x^2} - \frac{b^2 \operatorname{arcsec}(cx)^2}{2x^2} + \frac{b^2 c^2 \operatorname{arcsec}(cx)^2}{4} + \frac{c b^2 \operatorname{arcsec}(cx) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{2x} - \frac{b^2 c^2}{4} + \frac{b^2}{4x^2} - \frac{ab \operatorname{arcsec}(cx)}{x^2} - \frac{cab \sqrt{c^2 x^2 - 1} \operatorname{arcsec}(cx)}{2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^2/x^3,x)

[Out] -1/2\*a^2/x^2-1/2\*b^2/x^2\*arcsec(c\*x)^2+1/4\*b^2\*c^2\*arcsec(c\*x)^2+1/2\*c\*b^2\*arcsec(c\*x)/x\*((c^2\*x^2-1)/c^2/x^2)^(1/2)-1/4\*b^2\*c^2+1/4\*b^2/x^2-a\*b/x^2\*arcsec(c\*x)-1/2\*c\*a\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2))+1/2\*c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x-1/2\*c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^3,x, algorithm="maxima")

[Out] -1/2\*a\*b\*((c^4\*x\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*x^2\*(1/(c^2\*x^2) - 1) - 1) - c^3\*arctan(c\*x\*sqrt(-1/(c^2\*x^2) + 1)))/c + 2\*arcsec(c\*x)/x^2) - 1/8\*(4\*(c^2\*(log(c\*x + 1) + log(c\*x - 1) - 2\*log(x))\*log(c)^2 - 4\*c^2\*integrate(1/2\*x^2\*log(c^2\*x^2)/(c^2\*x^5 - x^3), x)\*log(c) + 8\*c^2\*integrate(1/2\*x^2\*log(x)/(c^2\*x^5 - x^3), x)\*log(c) - 4\*c^2\*integrate(1/2\*x^2\*log(c^2\*x^2)\*log(x)/(c^2\*x^5 - x^3), x) + 4\*c^2\*integrate(1/2\*x^2\*log(x)^2/(c^2\*x^5 - x^3), x) +

```

2*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1)
) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2*log(c)^2 + 4*integrate(1/2*log
(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2*x^5 - x^
3), x)*log(c) - 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c^2*x^2)*log
(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x^3), x) - 2*
integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*x^2 + 4*arctan(sqrt(c*x + 1)
)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2*b^2/x^2 - 1/2*a^2/x^2

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^2/x^3, x)

[Out] int((a + b\*acos(1/(c\*x)))^2/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*2/x\*\*3, x)

[Out] Integral((a + b\*asec(c\*x))\*\*2/x\*\*3, x)

$$3.22 \quad \int \frac{(a+b \sec^{-1}(cx))^2}{x^4} dx$$

**Optimal.** Leaf size=102

$$\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{9x^2} + \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx)) - \frac{(a+b \sec^{-1}(cx))^2}{3x^3} + \frac{4b^2c^2}{9x} + \frac{2b^2}{27x^3}$$

[Out]  $2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*\text{arcsec}(c*x))^2/x^3+4/9*b*c^3*(a+b*\text{arcsec}(c*x))*(1-1/c^2/x^2)^{(1/2)}+2/9*b*c*(a+b*\text{arcsec}(c*x))*(1-1/c^2/x^2)^{(1/2)}/x^2$

**Rubi [A]** time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5222, 4405, 3310, 3296, 2638}

$$\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx)) + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{9x^2} - \frac{(a+b \sec^{-1}(cx))^2}{3x^3} + \frac{4b^2c^2}{9x} + \frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^2/x^4, x]

[Out]  $(2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) + (4*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*(a + b*\text{ArcSec}[c*x]))/9 + (2*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*(a + b*\text{ArcSec}[c*x]))/(9*x^2) - (a + b*\text{ArcSec}[c*x])^2/(3*x^3)$

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n-1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n-2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n-1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

]

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec^{-1}(cx))^2}{x^4} dx &= c^3 \operatorname{Subst} \left( \int (a + bx)^2 \cos^2(x) \sin(x) dx, x, \sec^{-1}(cx) \right) \\
 &= -\frac{(a + b \sec^{-1}(cx))^2}{3x^3} + \frac{1}{3} (2bc^3) \operatorname{Subst} \left( \int (a + bx) \cos^3(x) dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{2b^2}{27x^3} + \frac{2bc \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{9x^2} - \frac{(a + b \sec^{-1}(cx))^2}{3x^3} + \frac{1}{9} (4bc^3) \operatorname{Subst} \left( \int (a + bx) \cos(x) dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{2b^2}{27x^3} + \frac{4}{9} bc^3 \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx)) + \frac{2bc \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{9x^2} - \frac{(a + b \sec^{-1}(cx))^2}{3x^3} \\
 &= \frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} + \frac{4}{9} bc^3 \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx)) + \frac{2bc \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{9x^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 108, normalized size = 1.06

$$\frac{-9a^2 + 6abcx \sqrt{1 - \frac{1}{c^2x^2}} (2c^2x^2 + 1) + 6b \sec^{-1}(cx) \left( bcx \sqrt{1 - \frac{1}{c^2x^2}} (2c^2x^2 + 1) - 3a \right) + 2b^2 (6c^2x^2 + 1) - 9b^2 \sec^{-1}(cx)^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^2/x^4, x]

[Out]  $(-9a^2 + 6ab\sqrt{1 - 1/(c^2x^2)})x^2(1 + 2c^2x^2) + 2b^2(1 + 6c^2x^2) + 6b(-3a + b\sqrt{1 - 1/(c^2x^2)})x^2(1 + 2c^2x^2) + \text{ArcSec}[cx] - 9b^2\text{ArcSec}[cx]^2)/(27x^3)$

**fricas** [A] time = 0.62, size = 93, normalized size = 0.91

$$\frac{12b^2c^2x^2 - 9b^2 \text{arcsec}(cx)^2 - 18ab \text{arcsec}(cx) - 9a^2 + 2b^2 + 6(2abc^2x^2 + ab + (2b^2c^2x^2 + b^2) \text{arcsec}(cx))\sqrt{c^2x^2 - 1}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="fricas")`

[Out]  $1/27*(12b^2c^2x^2 - 9b^2\text{arcsec}(c*x)^2 - 18a*b*\text{arcsec}(c*x) - 9a^2 + 2b^2 + 6*(2a*b*c^2x^2 + a*b + (2b^2*c^2x^2 + b^2)*\text{arcsec}(c*x))*\text{sqrt}(c^2x^2 - 1))/x^3$

**giac** [A] time = 0.16, size = 168, normalized size = 1.65

$$\frac{1}{27} \left( 12b^2c^2 \sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right) + 12abc^2 \sqrt{-\frac{1}{c^2x^2} + 1} + \frac{12b^2c}{x} + \frac{6b^2 \sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right)}{x^2} + \frac{6ab \sqrt{-\frac{1}{c^2x^2} + 1}}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="giac")`

[Out]  $1/27*(12b^2c^2*\text{sqrt}(-1/(c^2*x^2) + 1)*\arccos(1/(c*x)) + 12a*b*c^2*\text{sqrt}(-1/(c^2*x^2) + 1) + 12b^2*c/x + 6*b^2*\text{sqrt}(-1/(c^2*x^2) + 1)*\arccos(1/(c*x)))/x^2 + 6*a*b*\text{sqrt}(-1/(c^2*x^2) + 1)/x^2 - 9*b^2*\arccos(1/(c*x))^2/(c*x^3) - 18*a*b*\arccos(1/(c*x))/(c*x^3) - 9*a^2/(c*x^3) + 2*b^2/(c*x^3))*c$

**maple** [A] time = 0.48, size = 154, normalized size = 1.51

$$c^3 \left( -\frac{a^2}{3c^3x^3} + b^2 \left( -\frac{\text{arcsec}(cx)^2}{3c^3x^3} + \frac{2 \text{arcsec}(cx) (2c^2x^2 + 1) \sqrt{\frac{c^2x^2 - 1}{c^2x^2}}}{9c^2x^2} + \frac{2}{27c^3x^3} + \frac{4}{9cx} \right) \right) + 2ab \left( -\frac{\text{arcsec}(cx)}{3c^3x^3} + \frac{c^2}{9cx} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))^2/x^4,x)`

[Out]  $c^3*(-1/3*a^2/c^3/x^3 + b^2*(-1/3/c^3/x^3*\text{arcsec}(c*x)^2 + 2/9*\text{arcsec}(c*x)*(2*c^2*x^2 + 1)/c^2/x^2*((c^2*x^2 - 1)/c^2/x^2)^{(1/2)} + 2/27/c^3/x^3 + 4/9/c/x) + 2*a*b*(-1/3/c^3/x^3*\text{arcsec}(c*x) + 1/9*(c^2*x^2 - 1)*(2*c^2*x^2 + 1)/((c^2*x^2 - 1)/c^2/x^2)^{(1/2)})/c^4/x^4)$



**maxima** [A] time = 0.70, size = 164, normalized size = 1.61

$$-\frac{2}{9}ab \left( \frac{c^4 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} + \frac{3 \operatorname{arcsec}(cx)}{x^3} \right) - \frac{b^2 \operatorname{arcsec}(cx)^2}{3x^3} - \frac{a^2}{3x^3} + \frac{2 \left( (6c^3 x^2 + c) \sqrt{cx+1} \sqrt{cx-1} \right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^4,x, algorithm="maxima")

[Out]  $-2/9*a*b*((c^4*(-1/(c^2*x^2) + 1))^{3/2} - 3*c^4*\sqrt{-1/(c^2*x^2) + 1})/c + 3*\operatorname{arcsec}(c*x)/x^3 - 1/3*b^2*\operatorname{arcsec}(c*x)^2/x^3 - 1/3*a^2/x^3 + 2/27*((6*c^3*x^2 + c)*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 3*(2*c^5*x^4 - c^3*x^2 - c)*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*b^2/(\sqrt{c*x + 1}*\sqrt{c*x - 1}*c*x^3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^2/x^4,x)

[Out] int((a + b\*acos(1/(c\*x)))^2/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*2/x\*\*4,x)

[Out] Integral((a + b\*asec(c\*x))\*\*2/x\*\*4, x)

$$3.23 \quad \int \frac{(a+b \sec^{-1}(cx))^2}{x^5} dx$$

**Optimal.** Leaf size=134

$$\frac{3}{16}abc^4 \sec^{-1}(cx) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{8x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{16x} - \frac{(a+b \sec^{-1}(cx))^2}{4x^4} + \frac{3}{32}b^2c^4 \sec^{-1}(cx)$$

[Out] 1/32\*b^2/x^4+3/32\*b^2\*c^2/x^2+3/16\*a\*b\*c^4\*arcsec(c\*x)+3/32\*b^2\*c^4\*arcsec(c\*x)^2-1/4\*(a+b\*arcsec(c\*x))^2/x^4+1/8\*b\*c\*(a+b\*arcsec(c\*x))\*(1-1/c^2/x^2)^(1/2)/x^3+3/16\*b\*c^3\*(a+b\*arcsec(c\*x))\*(1-1/c^2/x^2)^(1/2)/x

**Rubi [A]** time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5222, 4405, 3310}

$$\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{16x} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{8x^3} + \frac{3}{16}abc^4 \sec^{-1}(cx) - \frac{(a+b \sec^{-1}(cx))^2}{4x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{32}b^2c^4 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^2/x^5,x]

[Out] b^2/(32\*x^4) + (3\*b^2\*c^2)/(32\*x^2) + (3\*a\*b\*c^4\*ArcSec[c\*x])/16 + (3\*b^2\*c^4\*ArcSec[c\*x]^2)/32 + (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcSec[c\*x]))/(8\*x^3) + (3\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcSec[c\*x]))/(16\*x) - (a + b\*ArcSec[c\*x])^2/(4\*x^4)

#### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
Simp[((d*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Ssin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

#### Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*cos[a + b*x]^(n + 1))/(b*(n + 1)), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

#### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^(n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec^{-1}(cx))^2}{x^5} dx &= c^4 \text{Subst} \left( \int (a + bx)^2 \cos^3(x) \sin(x) dx, x, \sec^{-1}(cx) \right) \\ &= -\frac{(a + b \sec^{-1}(cx))^2}{4x^4} + \frac{1}{2} (bc^4) \text{Subst} \left( \int (a + bx) \cos^4(x) dx, x, \sec^{-1}(cx) \right) \\ &= \frac{b^2}{32x^4} + \frac{bc\sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{8x^3} - \frac{(a + b \sec^{-1}(cx))^2}{4x^4} + \frac{1}{8} (3bc^4) \text{Subst} \left( \int (a + bx) \cos^2(x) dx, x, \sec^{-1}(cx) \right) \\ &= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{bc\sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{8x^3} + \frac{3bc^3\sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{16x} \\ &= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16} abc^4 \sec^{-1}(cx) + \frac{3}{32} b^2c^4 \sec^{-1}(cx)^2 + \frac{bc\sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{8x^3} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 148, normalized size = 1.10

$$\frac{-8a^2 - 6abc^4x^4 \sin^{-1}\left(\frac{1}{cx}\right) + 4abcx\sqrt{1 - \frac{1}{c^2x^2}} + 2b \sec^{-1}(cx) \left( bcx\sqrt{1 - \frac{1}{c^2x^2}} (3c^2x^2 + 2) - 8a \right) + 6abc^3x^3\sqrt{1 - \frac{1}{c^2x^2}}}{32x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^2/x^5, x]

[Out]  $(-8a^2 + b^2 + 4a*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 + 6*a*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3 + 2*b*(-8*a + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*\text{ArcSec}[c*x] + b^2*(-8 + 3*c^4*x^4)*\text{ArcSec}[c*x]^2 - 6*a*b*c^4*x^4*\text{ArcSin}[1/(c*x)])/(32*x^4)$

**fricas [A]** time = 0.71, size = 120, normalized size = 0.90

$$\frac{3b^2c^2x^2 + (3b^2c^4x^4 - 8b^2) \text{arcsec}(cx)^2 - 8a^2 + b^2 + 2(3abc^4x^4 - 8ab) \text{arcsec}(cx) + 2(3abc^2x^2 + 2ab + (3bc^3x^3\sqrt{1 - \frac{1}{c^2x^2}}))}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^5,x, algorithm="fricas")

[Out]  $\frac{1}{32}*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arcsec(c*x)^2 - 8*a^2 + b^2 + 2*(3*a*b*c^4*x^4 - 8*a*b)*arcsec(c*x) + 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^2*c^2*x^2 + 2*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^4$

**giac** [A] time = 0.15, size = 215, normalized size = 1.60

$$\frac{1}{256} \left( 24 b^2 c^3 \arccos\left(\frac{1}{cx}\right)^2 + 48 abc^3 \arccos\left(\frac{1}{cx}\right) - 15 b^2 c^3 + \frac{48 b^2 c^2 \sqrt{-\frac{1}{c^2 x^2} + 1} \arccos\left(\frac{1}{cx}\right)}{x} + \frac{48 abc^2 \sqrt{-\frac{1}{c^2 x^2} + 1}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^5,x, algorithm="giac")

[Out]  $\frac{1}{256}*(24*b^2*c^3*\arccos(1/(c*x))^2 + 48*a*b*c^3*\arccos(1/(c*x)) - 15*b^2*c^3 + 48*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*\arccos(1/(c*x))/x + 48*a*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 24*b^2*c/x^2 + 32*b^2*sqrt(-1/(c^2*x^2) + 1)*\arccos(1/(c*x))/x^3 + 32*a*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 64*b^2*\arccos(1/(c*x))^2/(c*x^4) - 128*a*b*\arccos(1/(c*x))/(c*x^4) - 64*a^2/(c*x^4) + 8*b^2/(c*x^4))*c$

**maple** [B] time = 0.43, size = 265, normalized size = 1.98

$$-\frac{a^2}{4x^4} - \frac{b^2 \arccos(cx)^2}{4x^4} + \frac{3b^2 c^4 \arccos(cx)^2}{32} + \frac{3c^3 b^2 \arccos(cx) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{16x} + \frac{c b^2 \arccos(cx) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{8x^3} + \frac{b^2}{32x^4} + \frac{3b^2 c^2}{32x^2} - \frac{ab}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^2/x^5,x)

[Out]  $-1/4*a^2/x^4 - 1/4*b^2/x^4*arcsec(c*x)^2 + 3/32*b^2*c^4*arcsec(c*x)^2 + 3/16*c^3*b^2*arcsec(c*x)/x*((c^2*x^2-1)/c^2/x^2)^{(1/2)} + 1/8*c*b^2*arcsec(c*x)/x^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)} + 1/32*b^2/x^4 + 3/32*b^2*c^2/x^2 - 1/2*a*b/x^4*arcsec(c*x) - 3/16*c^3*a*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*\arctan(1/(c^2*x^2-1)^{(1/2)}) + 3/16*c^3*a*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x - 1/16*c*a*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^3 - 1/8*c*a*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^5$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^2/x^5,x, algorithm="maxima")

[Out]  $\frac{1}{16}ab\left(3c^5\arctan\left(\frac{c^2x^2+1}{c^2x^2}\right) + (3c^8x^3(-1/(c^2x^2)+1)^{3/2} + 5c^6x\sqrt{-1/(c^2x^2)+1})/(c^4x^4(1/(c^2x^2)-1)^2 - 2c^2x^2(1/(c^2x^2)-1)+1)/c - 8\operatorname{arcsec}(cx)/x^4 - \frac{1}{16}(4(2(c^2\log(cx+1) + c^2\log(cx-1) - 2c^2\log(x) + 1/x^2)c^2\log(c)^2 - 16c^2\int \frac{1/4x^2\log(c^2x^2)}{c^2x^7-x^5} dx)\log(c) + 32c^2\int \frac{1/4x^2\log(x)}{c^2x^7-x^5} dx)\log(c) - 16c^2\int \frac{1/4x^2\log(c^2x^2)\log(x)}{c^2x^7-x^5} dx + 16c^2\int \frac{1/4x^2\log(x)^2}{c^2x^7-x^5} dx + 4c^2\int \frac{1/4x^2\log(c^2x^2)}{c^2x^7-x^5} dx - (2c^4\log(cx+1) + 2c^4\log(cx-1) - 4c^4\log(x) + (2c^2x^2+1)/x^4)\log(c)^2 + 16\int \frac{1/4\log(c^2x^2)}{c^2x^7-x^5} dx)\log(c) - 32\int \frac{1/4\log(x)}{c^2x^7-x^5} dx)\log(c) - 8\int \frac{1/4\sqrt{cx+1}\sqrt{cx-1}\arctan(\sqrt{cx+1}\sqrt{cx-1})}{c^2x^7-x^5} dx + 16\int \frac{1/4\log(c^2x^2)\log(x)}{c^2x^7-x^5} dx - 16\int \frac{1/4\log(x)^2}{c^2x^7-x^5} dx - 4\int \frac{1/4\log(c^2x^2)}{c^2x^7-x^5} dx)\log(c)^2 + 4\arctan(\sqrt{cx+1}\sqrt{cx-1})^2 - \log(c^2x^2)^2\right)b^2/x^4 - \frac{1}{4}a^2/x^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^2/x^5,x)

[Out] int((a + b\*acos(1/(c\*x)))^2/x^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*2/x\*\*5,x)

[Out] Integral((a + b\*asec(c\*x))\*\*2/x\*\*5, x)

### 3.24 $\int x^3 \left(a + b \sec^{-1}(cx)\right)^3 dx$

**Optimal.** Leaf size=207

$$\frac{b^2 \log\left(1 + e^{2i \sec^{-1}(cx)}\right) \left(a + b \sec^{-1}(cx)\right)}{c^4} + \frac{b^2 x^2 \left(a + b \sec^{-1}(cx)\right)}{4c^2} + \frac{ib \left(a + b \sec^{-1}(cx)\right)^2}{2c^4} - \frac{bx^3 \sqrt{1 - \frac{1}{c^2 x^2}} \left(a + b \sec^{-1}(cx)\right)}{4c}$$

[Out]  $\frac{1}{4} b^2 x^2 (a + b \operatorname{arcsec}(cx)) / c^2 + \frac{1}{2} i b (a + b \operatorname{arcsec}(cx))^2 / c^4 + \frac{1}{4} x^4 (a + b \operatorname{arcsec}(cx))^3 - b^2 (a + b \operatorname{arcsec}(cx)) \ln\left(1 + \frac{1}{c/x + i \sqrt{1 - 1/c^2 x^2}}\right) / c^4 + \frac{1}{2} i b^3 \operatorname{polylog}\left(2, -\frac{1}{c/x + i \sqrt{1 - 1/c^2 x^2}}\right) / c^4 - \frac{1}{4} b^3 x (1 - 1/c^2 x^2)^{1/2} / c^3 - \frac{1}{2} b x (a + b \operatorname{arcsec}(cx))^2 (1 - 1/c^2 x^2)^{1/2} / c^3 - \frac{1}{4} b x^3 (a + b \operatorname{arcsec}(cx))^2 (1 - 1/c^2 x^2)^{1/2} / c$

**Rubi [A]** time = 0.21, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5222, 4409, 4186, 3767, 8, 4184, 3719, 2190, 2279, 2391}

$$\frac{ib^3 \operatorname{PolyLog}\left(2, -e^{2i \sec^{-1}(cx)}\right)}{2c^4} + \frac{b^2 x^2 \left(a + b \sec^{-1}(cx)\right)}{4c^2} - \frac{b^2 \log\left(1 + e^{2i \sec^{-1}(cx)}\right) \left(a + b \sec^{-1}(cx)\right)}{c^4} - \frac{bx^3 \sqrt{1 - \frac{1}{c^2 x^2}} \left(a + b \sec^{-1}(cx)\right)}{4c}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*ArcSec[c*x])^3, x]`

[Out]  $-\frac{b^3 \sqrt{1 - 1/(c^2 x^2)} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{ArcSec}[c x])}{4c^2} + \left(\frac{i}{2}\right) b (a + b \operatorname{ArcSec}[c x])^2 / c^4 - \frac{b \sqrt{1 - 1/(c^2 x^2)} x (a + b \operatorname{ArcSec}[c x])^2}{2c^3} - \frac{b \sqrt{1 - 1/(c^2 x^2)} x^3 (a + b \operatorname{ArcSec}[c x])^2}{4c} + \frac{x^4 (a + b \operatorname{ArcSec}[c x])^3}{4} - \frac{b^2 (a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 + E^{(2i) \operatorname{ArcSec}[c x]}\right]}{c^4} + \left(\frac{i}{2}\right) b^3 \operatorname{PolyLog}\left[2, -E^{(2i) \operatorname{ArcSec}[c x]}\right] / c^4$

#### Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))
]^(n)], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \sec^{-1}(cx))^3 dx &= \frac{\text{Subst}\left(\int (a + bx)^3 \sec^4(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4}x^4 (a + b \sec^{-1}(cx))^3 - \frac{(3b) \text{Subst}\left(\int (a + bx)^2 \sec^4(x) dx, x, \sec^{-1}(cx)\right)}{4c^4} \\
&= \frac{b^2 x^2 (a + b \sec^{-1}(cx))}{4c^2} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^3 (a + b \sec^{-1}(cx))^2}{4c} + \frac{1}{4}x^4 (a + b \sec^{-1}(cx))^3 \\
&= \frac{b^2 x^2 (a + b \sec^{-1}(cx))}{4c^2} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \sec^{-1}(cx))^2}{2c^3} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^3 (a + b \sec^{-1}(cx))^3}{4c} \\
&= -\frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \sec^{-1}(cx))}{4c^2} + \frac{ib (a + b \sec^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \sec^{-1}(cx))^3}{2c} \\
&= -\frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \sec^{-1}(cx))}{4c^2} + \frac{ib (a + b \sec^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \sec^{-1}(cx))^3}{2c} \\
&= -\frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \sec^{-1}(cx))}{4c^2} + \frac{ib (a + b \sec^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \sec^{-1}(cx))^3}{2c} \\
&= -\frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \sec^{-1}(cx))}{4c^2} + \frac{ib (a + b \sec^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \sec^{-1}(cx))^3}{2c}
\end{aligned}$$

**Mathematica [A]** time = 0.83, size = 288, normalized size = 1.39

$$a^3 c^4 x^4 + b \sec^{-1}(cx) \left( cx \left( 3a^2 c^3 x^3 - 2ab \sqrt{1 - \frac{1}{c^2 x^2}} (c^2 x^2 + 2) + b^2 cx \right) - 4b^2 \log(1 + e^{2i \sec^{-1}(cx)}) \right) - 2a^2 bcx \sqrt{1 - \frac{1}{c^2 x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (-2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^2
*c^2*x^2 - a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 - b^2*(-3*a*c^
```



$4*x^4 + b*(-2*I + 2*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x + c^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3)*\text{ArcSec}[c*x]^2 + b^3*c^4*x^4*\text{ArcSec}[c*x]^3 + b*\text{ArcSec}[c*x]*(c*x*(b^2*c*x + 3*a^2*c^3*x^3 - 2*a*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2*\text{Log}[1 + E^((2*I)*\text{ArcSec}[c*x])]) - 4*a*b^2*\text{Log}[1/(c*x)] + (2*I)*b^3*\text{PolyLog}[2, -E^((2*I)*\text{ArcSec}[c*x])])/(4*c^4)$

**fricas** [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}(b^3x^3 \text{arcsec}(cx)^3 + 3ab^2x^3 \text{arcsec}(cx)^2 + 3a^2bx^3 \text{arcsec}(cx) + a^3x^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*x^3\*arcsec(c\*x)^2 + 3\*a^2\*b\*x^3\*arcsec(c\*x) + a^3\*x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \text{arcsec}(cx) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^3\*x^3, x)

**maple** [B] time = 1.02, size = 447, normalized size = 2.16

$$\frac{x^4 a^3}{4} + \frac{b^3 \text{arcsec}(cx)^3 x^4}{4} - \frac{b^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} \text{arcsec}(cx)^2 x^3}{4c} - \frac{b^3 \text{arcsec}(cx)^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x}{2c^3} + \frac{ib^3 \text{arcsec}(cx)^2}{2c^4} + \frac{b^3 \text{arcsec}(cx)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arcsec(c\*x))^3,x)

[Out]  $\frac{1}{4}x^4a^3 + \frac{1}{4}b^3\text{arcsec}(c*x)^3x^4 - \frac{1}{4}c*b^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*\text{arcsec}(c*x)^2*x^3 - \frac{1}{2}c^3*b^3*\text{arcsec}(c*x)^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x + \frac{1}{2}I/c^4*b^3*\text{arcsec}(c*x)^2 + \frac{1}{4}c^2*b^3*\text{arcsec}(c*x)*x^2 - \frac{1}{4}c^3*b^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x - \frac{1}{4}I/c^4*b^3 - \frac{1}{c^4}b^3*\text{arcsec}(c*x)*\ln(1+(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2) + \frac{1}{2}I*b^3*\text{polylog}(2, -(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2) / c^4 + \frac{3}{4}a^2*b*x^4*\text{arcsec}(c*x) - \frac{1}{4}c*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^3 - \frac{1}{4}c^3*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x + \frac{1}{2}c^5*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x + \frac{3}{4}a*b^2*\text{arcsec}(c*x)^2*x^4 - \frac{1}{2}c*a*b^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}$

$(1/2)*\operatorname{arcsec}(c*x)*x^3+1/4/c^2*x^2*a*b^2-1/c^3*a*b^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)*\operatorname{arcsec}(c*x)*x-1/c^4*a*b^2*\ln(1/c/x)}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}ab^2x^4\operatorname{arcsec}(cx)^2+\frac{1}{4}a^3x^4+\frac{1}{4}\left(3x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)a^2b+\frac{1}{16}\left(4x^4\arctan\left(\sqrt{c^2x^2-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $\frac{3}{4}a^3b^2x^4\operatorname{arcsec}(c*x)^2 + \frac{1}{4}a^3x^4 + \frac{1}{4}(3x^4\operatorname{arcsec}(c*x) - (c^2x^3(-1/(c^2x^2) + 1)^{(3/2)} + 3x*\sqrt{-1/(c^2x^2) + 1})/c^3)*a^2b + \frac{1}{16}(4x^4\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^3 - 3x^4\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c^2*x^2)^2 - 16*\int(3/16*((4*x^3*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - x^3*\log(c^2*x^2)^2)*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 4*(4*c^2*x^5*\log(c)^2 - 4*x^3*\log(c)^2 + 4*(c^2*x^5 - x^3)*\log(x)^2 - ((4*c^2*\log(c) + c^2)*x^5 - x^3*(4*\log(c) + 1) + 4*(c^2*x^5 - x^3)*\log(x))*\log(c^2*x^2) + 8*(c^2*x^5*\log(c) - x^3*\log(c))*\log(x))*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))/((c^2*x^2 - 1), x)*b^3 + \frac{1}{4}*((c^2*x^2 + 2*\log(x^2))*\sqrt{c*x + 1})*\sqrt{c*x - 1} - 2*(c^4*x^4 + c^2*x^2 - 2)*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*a*b^2/(\sqrt{c*x + 1}*\sqrt{c*x - 1})*c^4)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left( a + b \operatorname{acos} \left( \frac{1}{c*x} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b\*acos(1/(c\*x)))^3,x)

[Out] int(x^3\*(a + b\*acos(1/(c\*x)))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{asec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*asec(c\*x))\*\*3,x)

[Out] Integral(x\*\*3\*(a + b\*asec(c\*x))\*\*3, x)

### 3.25 $\int x^2 (a + b \sec^{-1}(cx))^3 dx$

**Optimal.** Leaf size=236

$$\frac{ib^2 \text{Li}_2(-ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c^3} + \frac{ib^2 \text{Li}_2(ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c^3} + \frac{b^2 x (a + b \sec^{-1}(cx))}{c^2} + \frac{ib \tan^{-1}(a + b \sec^{-1}(cx))}{c}$$

[Out]  $b^2 x (a + b \text{arcsec}(cx)) / c^2 + 1/3 x^3 (a + b \text{arcsec}(cx))^3 + I b (a + b \text{arcsec}(cx))^2 \arctan(1/c/x + I(1 - 1/c^2/x^2)^{1/2}) / c^3 - b^3 \text{arctanh}((1 - 1/c^2/x^2)^{1/2}) / c^3 - I b^2 (a + b \text{arcsec}(cx)) \text{polylog}(2, -I(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) / c^3 + I b^2 (a + b \text{arcsec}(cx)) \text{polylog}(2, I(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) / c^3 + b^3 \text{polylog}(3, -I(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) / c^3 - b^3 \text{polylog}(3, I(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) / c^3 - 1/2 b x^2 (a + b \text{arcsec}(cx))^2 (1 - 1/c^2/x^2)^{1/2} / c$

**Rubi [A]** time = 0.19, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5222, 4409, 4186, 3770, 4181, 2531, 2282, 6589}

$$\frac{ib^2 \text{PolyLog}(2, -ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c^3} + \frac{ib^2 \text{PolyLog}(2, ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c^3} + \frac{b^3 \text{PolyLog}(3, -I(1/c/x + I(1 - 1/c^2/x^2)^{1/2}))}{c^3} - \frac{b^3 \text{PolyLog}(3, I(1/c/x + I(1 - 1/c^2/x^2)^{1/2}))}{c^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2(a + b \text{ArcSec}[c*x])^3, x]$

[Out]  $(b^2 x (a + b \text{ArcSec}[c*x])) / c^2 - (b \sqrt{1 - 1/(c^2 x^2)}) x^2 (a + b \text{ArcSec}[c*x])^2 / (2c) + (x^3 (a + b \text{ArcSec}[c*x])^3) / 3 + (I b (a + b \text{ArcSec}[c*x])^2 \text{ArcTan}[E^{I \text{ArcSec}[c*x]}]) / c^3 - (b^3 \text{ArcTanh}[\sqrt{1 - 1/(c^2 x^2)}]) / c^3 - (I b^2 (a + b \text{ArcSec}[c*x]) \text{PolyLog}[2, (-I) E^{I \text{ArcSec}[c*x]}]) / c^3 + (I b^2 (a + b \text{ArcSec}[c*x]) \text{PolyLog}[2, I E^{I \text{ArcSec}[c*x]}]) / c^3 + (b^3 \text{PolyLog}[3, (-I) E^{I \text{ArcSec}[c*x]}]) / c^3 - (b^3 \text{PolyLog}[3, I E^{I \text{ArcSec}[c*x]}]) / c^3$

#### Rule 2282

$\text{Int}[u, x\_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$   $\text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w\_)((a\_)(v\_)^{(n\_))^{(m\_)} /;$   $\text{FreeQ}[\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& \text{!MatchQ}[u, E^{((c\_)((a\_)(b\_)*x))} (F\_)] [v_] /;$   $\text{FreeQ}[\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
LtQ[m, -1])
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \sec^{-1}(cx))^3 dx &= \frac{\text{Subst}\left(\int (a + bx)^3 \sec^3(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^3} \\
&= \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^3 - \frac{b \text{Subst}\left(\int (a + bx)^2 \sec^3(x) dx, x, \sec^{-1}(cx)\right)}{c^3} \\
&= \frac{b^2x (a + b \sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^3 \\
&= \frac{b^2x (a + b \sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^3 \\
&= \frac{b^2x (a + b \sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^3 \\
&= \frac{b^2x (a + b \sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^3 \\
&= \frac{b^2x (a + b \sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1 - \frac{1}{c^2x^2}} x^2 (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \sec^{-1}(cx))^3
\end{aligned}$$

**Mathematica [A]** time = 1.44, size = 403, normalized size = 1.71

$$2a^3c^3x^3 + 6a^2bc^3x^3 \sec^{-1}(cx) - 3a^2bc^2x^2\sqrt{1 - \frac{1}{c^2x^2}} - 3a^2b \log\left(x\left(\sqrt{1 - \frac{1}{c^2x^2}} + 1\right)\right) + 6ab^2c^3x^3 \sec^{-1}(cx)^2 - 6ab$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcSec[c\*x])^3,x]

[Out] (6\*a\*b^2\*c\*x - 3\*a^2\*b\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2 + 2\*a^3\*c^3\*x^3 + 6\*b^3\*c\*x\*ArcSec[c\*x] - 6\*a\*b^2\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2\*ArcSec[c\*x] + 6\*a^2\*b\*c^3\*x^3\*ArcSec[c\*x] - 3\*b^3\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2\*ArcSec[c\*x]^2 + 6\*a\*b^2\*c^3\*x^3\*ArcSec[c\*x]^2 + 2\*b^3\*c^3\*x^3\*ArcSec[c\*x]^3 + (6\*I)\*b^3\*ArcSec[c\*x]^2\*ArcTan[E^(I\*ArcSec[c\*x])] - 6\*b^3\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]] - 6\*a\*b^2\*ArcSec[c\*x]\*Log[1 - I\*E^(I\*ArcSec[c\*x])] + 6\*a\*b^2\*ArcSec[c\*

$x \cdot \text{Log}[1 + I \cdot E^{(I \cdot \text{ArcSec}[c \cdot x])}] - 3 \cdot a^2 \cdot b \cdot \text{Log}[(1 + \text{Sqrt}[1 - 1/(c^2 \cdot x^2)]) \cdot x] - (6 \cdot I) \cdot b^2 \cdot (a + b \cdot \text{ArcSec}[c \cdot x]) \cdot \text{PolyLog}[2, (-I) \cdot E^{(I \cdot \text{ArcSec}[c \cdot x])}] + (6 \cdot I) \cdot b^2 \cdot (a + b \cdot \text{ArcSec}[c \cdot x]) \cdot \text{PolyLog}[2, I \cdot E^{(I \cdot \text{ArcSec}[c \cdot x])}] + 6 \cdot b^3 \cdot \text{PolyLog}[3, (-I) \cdot E^{(I \cdot \text{ArcSec}[c \cdot x])}] - 6 \cdot b^3 \cdot \text{PolyLog}[3, I \cdot E^{(I \cdot \text{ArcSec}[c \cdot x])}]) / (6 \cdot c^3)$

**fricas** [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}(b^3 x^2 \text{arcsec}(cx)^3 + 3 ab^2 x^2 \text{arcsec}(cx)^2 + 3 a^2 b x^2 \text{arcsec}(cx) + a^3 x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^2\*arcsec(c\*x)^3 + 3\*a\*b^2\*x^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*x^2\*arcsec(c\*x) + a^3\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \text{arcsec}(cx) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^3\*x^2, x)

**maple** [B] time = 1.05, size = 687, normalized size = 2.91

$$\frac{a^3 x^3}{3} + \frac{x^3 b^3 \text{arcsec}(cx)^3}{3} - \frac{b^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} \text{arcsec}(cx)^2 x^2}{2c} + \frac{b^3 \text{arcsec}(cx) x}{c^2} - \frac{b^3 \text{arcsec}(cx)^2 \ln\left(1 - i\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x))^3,x)

[Out]  $\frac{1}{3} a^3 x^3 + \frac{1}{3} x^3 b^3 \text{arcsec}(c x)^3 - \frac{1}{2} c b^3 \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{1/2} \text{arcsec}(c x)^2 x^2 + \frac{1}{c^2} b^3 \text{arcsec}(c x) x - \frac{1}{2} c^3 b^3 \text{arcsec}(c x)^2 \ln\left(1 - I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) - \frac{1}{c^3} a b^2 \text{dilog}\left(1 + I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) - b^3 \text{polylog}\left(3, I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) / c^3 + \frac{1}{2} c^3 b^3 \text{arcsec}(c x)^2 \ln\left(1 + I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) - \frac{1}{c^3} b^3 \text{arcsec}(c x) \text{polylog}\left(2, -I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) + b^3 \text{polylog}\left(3, -I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) / c^3 + \frac{1}{c^3} b^3 \text{arcsec}(c x) \text{polylog}\left(2, I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) + b^2 x^3 a \text{arcsec}(c x)^2 - \frac{1}{c} a b^2 \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{1/2} \text{arcsec}(c x) x^2 - \frac{1}{c^3} a b^2 \text{arcsec}(c x) \ln\left(1 - I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) + \frac{1}{c^3} a b^2 \text{arcsec}(c x) \ln\left(1 + I \left(\frac{1}{c x} + I \left(1 - \frac{1}{c^2 x^2}\right)^{1/2}\right)\right) + 2 I / c^3 b^3 \text{arc}$



$x^2 \log(c^2 x^2)^2 / (c^2 x^2 - 1), x) - 12 a b^2 \int (1/4 x^2 \log(c^2 x^2) \log(x) / (c^2 x^2 - 1), x) + 12 a b^2 \int (1/4 x^2 \log(x)^2 / (c^2 x^2 - 1), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*acos(1/(c*x)))^3,x)`

[Out] `int(x^2*(a + b*acos(1/(c*x)))^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asec(c*x))**3,x)`

[Out] `Integral(x**2*(a + b*asec(c*x))**3, x)`



### 3.26 $\int x \left( a + b \sec^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=126

$$\frac{3b^2 \log\left(1 + e^{2i \sec^{-1}(cx)}\right) (a + b \sec^{-1}(cx))}{c^2} - \frac{3bx \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{2c} + \frac{3ib (a + b \sec^{-1}(cx))^2}{2c^2} + \frac{1}{2} x^2 (a +$$

[Out]  $3/2 * I * b * (a + b * \text{arcsec}(c * x))^{2/c^2 + 1/2 * x^2 * (a + b * \text{arcsec}(c * x))^{3 - 3 * b^2 * (a + b * \text{arcsec}(c * x)) * \ln(1 + (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})^2)/c^2 + 3/2 * I * b^3 * \text{polylog}(2, -(1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})^2)/c^2 - 3/2 * b * x * (a + b * \text{arcsec}(c * x))^{2 * (1 - 1/c^2/x^2)^{(1/2)}/c}$

**Rubi [A]** time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {5222, 4409, 4184, 3719, 2190, 2279, 2391}

$$\frac{3ib^3 \text{PolyLog}\left(2, -e^{2i \sec^{-1}(cx)}\right)}{2c^2} - \frac{3b^2 \log\left(1 + e^{2i \sec^{-1}(cx)}\right) (a + b \sec^{-1}(cx))}{c^2} - \frac{3bx \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{2c} + \frac{3ib}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcSec[c\*x])^3,x]

[Out]  $((3I/2) * b * (a + b * \text{ArcSec}[c * x])^2/c^2 - (3 * b * \text{Sqrt}[1 - 1/(c^2 * x^2)]) * x * (a + b * \text{ArcSec}[c * x])^2)/(2 * c) + (x^2 * (a + b * \text{ArcSec}[c * x])^3)/2 - (3 * b^2 * (a + b * \text{ArcSec}[c * x]) * \text{Log}[1 + E^{(2 * I) * \text{ArcSec}[c * x]}])/c^2 + ((3I/2) * b^3 * \text{PolyLog}[2, -E^{(2 * I) * \text{ArcSec}[c * x]}])/c^2$

#### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[(((c + d\*x)^m \* Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_))], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] - Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned}
\int x (a + b \sec^{-1}(cx))^3 dx &= \frac{\text{Subst}\left(\int (a + bx)^3 \sec^2(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2}x^2 (a + b \sec^{-1}(cx))^3 - \frac{(3b) \text{Subst}\left(\int (a + bx)^2 \sec^2(x) dx, x, \sec^{-1}(cx)\right)}{2c^2} \\
&= -\frac{3b\sqrt{1 - \frac{1}{c^2x^2}} x (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \sec^{-1}(cx))^3 + \frac{(3b^2) \text{Subst}\left(\int (a + bx) \sec^2(x) dx, x, \sec^{-1}(cx)\right)}{2c^2} \\
&= \frac{3ib (a + b \sec^{-1}(cx))^2}{2c^2} - \frac{3b\sqrt{1 - \frac{1}{c^2x^2}} x (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \sec^{-1}(cx))^3 \\
&= \frac{3ib (a + b \sec^{-1}(cx))^2}{2c^2} - \frac{3b\sqrt{1 - \frac{1}{c^2x^2}} x (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \sec^{-1}(cx))^3 \\
&= \frac{3ib (a + b \sec^{-1}(cx))^2}{2c^2} - \frac{3b\sqrt{1 - \frac{1}{c^2x^2}} x (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \sec^{-1}(cx))^3 \\
&= \frac{3ib (a + b \sec^{-1}(cx))^2}{2c^2} - \frac{3b\sqrt{1 - \frac{1}{c^2x^2}} x (a + b \sec^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \sec^{-1}(cx))^3
\end{aligned}$$

**Mathematica [A]** time = 0.50, size = 184, normalized size = 1.46

$$\frac{a \left( acx \left( acx - 3b\sqrt{1 - \frac{1}{c^2x^2}} \right) - 6b^2 \log\left(\frac{1}{cx}\right) \right) - 3b^2 \sec^{-1}(cx)^2 \left( -ac^2x^2 + b \left( cx\sqrt{1 - \frac{1}{c^2x^2}} - i \right) \right) - 3b \sec^{-1}(cx) \left( acx^2 - b \right)}{2c^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcSec[c\*x])^3,x]

[Out]  $(-3*b^2*(-(a*c^2*x^2) + b*(-I + c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x))*\text{ArcSec}[c*x]^2 + b^3*c^2*x^2*\text{ArcSec}[c*x]^3 - 3*b*\text{ArcSec}[c*x]*(a*c*x*(2*b*\text{Sqrt}[1 - 1/(c^2*x^2)] - a*c*x) + 2*b^2*\text{Log}[1 + E^{((2*I)*\text{ArcSec}[c*x])}]) + a*(a*c*x*(-3*b*\text{Sqrt}[1 - 1/(c^2*x^2)] + a*c*x) - 6*b^2*\text{Log}[1/(c*x)]) + (3*I)*b^3*\text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])}]))/(2*c^2)$

**fricas [F]** time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}(b^3x \text{arcsec}(cx)^3 + 3ab^2x \text{arcsec}(cx)^2 + 3a^2bx \text{arcsec}(cx) + a^3x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x\*arcsec(c\*x)^3 + 3\*a\*b^2\*x\*arcsec(c\*x)^2 + 3\*a^2\*b\*x\*arcsec(c\*x) + a^3\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^3\*x, x)

**maple** [A] time = 0.80, size = 285, normalized size = 2.26

$$\frac{x^2 a^3}{2} + \frac{x^2 b^3 \operatorname{arcsec}(cx)^3}{2} - \frac{3b^3 \operatorname{arcsec}(cx)^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x}{2c} + \frac{3ib^3 \operatorname{arcsec}(cx)^2}{2c^2} - \frac{3b^3 \operatorname{arcsec}(cx) \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2 x^2}}\right)^2\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x))^3,x)

[Out]  $\frac{1}{2}x^2 a^3 + \frac{1}{2}x^2 b^3 \operatorname{arcsec}(cx)^3 - \frac{3}{2} \frac{b^3 \operatorname{arcsec}(cx)^2 ((c^2 x^2 - 1)/c^2 x^2)^{(1/2)} x}{c} + \frac{3}{2} \frac{I b^3 \operatorname{arcsec}(cx)^2 - 3/c^2 b^3 \operatorname{arcsec}(cx) \ln(1 + (1/cx + I(1 - 1/c^2 x^2)^{(1/2)})^2)}{c^2} + \frac{3}{2} \frac{I b^3 \operatorname{polylog}(2, -(1/cx + I(1 - 1/c^2 x^2)^{(1/2)})^2)}{c^2} + \frac{3}{2} x^2 a^2 b^2 \operatorname{arcsec}(cx)^2 - \frac{3}{2} \frac{c a^2 b^2 ((c^2 x^2 - 1)/c^2 x^2)^{(1/2)} \operatorname{arcsec}(cx) x - 3/c^2 a^2 b^2 \ln(1/cx) + 3/2 a^2 b^2 x^2 \operatorname{arcsec}(cx) - 3/2 c a^2 b^2 / ((c^2 x^2 - 1)/c^2 x^2)^{(1/2)} x + 3/2 c^3 a^2 b^2 / ((c^2 x^2 - 1)/c^2 x^2)^{(1/2)} / x}{c^2}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2} ab^2 x^2 \operatorname{arcsec}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2} \left( x^2 \operatorname{arcsec}(cx) - \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} \right) a^2 b - 3 \left( \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1} \operatorname{arcsec}(cx)}{c} - \frac{\log(x)}{c^2} \right) ab^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $\frac{3}{2} a^2 b^2 x^2 \operatorname{arcsec}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2} (x^2 \operatorname{arcsec}(cx) - x \sqrt{-1/(c^2 x^2) + 1}/c) a^2 b - 3 (x \sqrt{-1/(c^2 x^2) + 1} \operatorname{arcsec}(cx)/c - \log(x)/c^2) a b^2 + \frac{1}{8} (4 x^2 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1})^3 - 3 x^2 a \operatorname{rctan}(\sqrt{cx + 1}) \sqrt{cx - 1} \log(c^2 x^2)^2 - 8 \operatorname{integrate}(3/8 * ((4 x^2 a$

```
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt
(c*x - 1) + 4*(2*c^2*x^3*log(c)^2 - 2*x*log(c)^2 + 2*(c^2*x^3 - x)*log(x)^2
- ((2*c^2*log(c) + c^2)*x^3 - x*(2*log(c) + 1) + 2*(c^2*x^3 - x)*log(x))*l
og(c^2*x^2) + 4*(c^2*x^3*log(c) - x*log(c))*log(x))*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)))/(c^2*x^2 - 1), x))*b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*acos(1/(c\*x)))^3,x)

[Out] int(x\*(a + b\*acos(1/(c\*x)))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{asec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asec(c\*x))\*\*3,x)

[Out] Integral(x\*(a + b\*asec(c\*x))\*\*3, x)

### 3.27 $\int (a + b \sec^{-1}(cx))^3 dx$

**Optimal.** Leaf size=158

$$-\frac{6ib^2 \text{Li}_2(-ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c} + \frac{6ib^2 \text{Li}_2(ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c} + x(a + b \sec^{-1}(cx))^3 + \frac{6ib \tan^{-1}(x)}{c}$$

[Out]  $x*(a+b*\text{arcsec}(c*x))^3+6*I*b*(a+b*\text{arcsec}(c*x))^2*\text{arctan}(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/c-6*I*b^2*(a+b*\text{arcsec}(c*x))*\text{polylog}(2,-I*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/c+6*I*b^2*(a+b*\text{arcsec}(c*x))*\text{polylog}(2,I*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/c+6*b^3*\text{polylog}(3,-I*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/c-6*b^3*\text{polylog}(3,I*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/c$

**Rubi [A]** time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {5216, 4409, 4181, 2531, 2282, 6589}

$$-\frac{6ib^2 \text{PolyLog}(2, -ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c} + \frac{6ib^2 \text{PolyLog}(2, ie^{i \sec^{-1}(cx)})(a + b \sec^{-1}(cx))}{c} + \frac{6b^3 \text{PolyLog}(3, x)}{c}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcSec[c*x])^3, x]`

[Out]  $x*(a + b*\text{ArcSec}[c*x])^3 + ((6*I)*b*(a + b*\text{ArcSec}[c*x])^2*\text{ArcTan}[E^{(I*\text{ArcSec}[c*x])}])/c - ((6*I)*b^2*(a + b*\text{ArcSec}[c*x])*\text{PolyLog}[2, (-I)*E^{(I*\text{ArcSec}[c*x])}])/c + ((6*I)*b^2*(a + b*\text{ArcSec}[c*x])*\text{PolyLog}[2, I*E^{(I*\text{ArcSec}[c*x])}])/c + (6*b^3*\text{PolyLog}[3, (-I)*E^{(I*\text{ArcSec}[c*x])}])/c - (6*b^3*\text{PolyLog}[3, I*E^{(I*\text{ArcSec}[c*x])}])/c$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
```

, g, n}, x] && GtQ[m, 0]

### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4409

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Sec[a + b\*x]^n)/(b\*n), x] - Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sec[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 5216

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))^(n\_), x\_Symbol] :> Dist[1/c, Subst[Int[(a + b\*x)^n\*Sec[x]\*Tan[x], x], x, ArcSec[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[n, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int (a + b \sec^{-1}(cx))^3 dx &= \frac{\text{Subst}\left(\int (a + bx)^3 \sec(x) \tan(x) dx, x, \sec^{-1}(cx)\right)}{c} \\
&= x (a + b \sec^{-1}(cx))^3 - \frac{(3b) \text{Subst}\left(\int (a + bx)^2 \sec(x) dx, x, \sec^{-1}(cx)\right)}{c} \\
&= x (a + b \sec^{-1}(cx))^3 + \frac{6ib (a + b \sec^{-1}(cx))^2 \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} + \frac{(6b^2) \text{Subst}\left(\int (a + b \sec^{-1}(cx)) dx, x, \sec^{-1}(cx)\right)}{c} \\
&= x (a + b \sec^{-1}(cx))^3 + \frac{6ib (a + b \sec^{-1}(cx))^2 \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{6ib^2 (a + b \sec^{-1}(cx))}{c} \\
&= x (a + b \sec^{-1}(cx))^3 + \frac{6ib (a + b \sec^{-1}(cx))^2 \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{6ib^2 (a + b \sec^{-1}(cx))}{c} \\
&= x (a + b \sec^{-1}(cx))^3 + \frac{6ib (a + b \sec^{-1}(cx))^2 \tan^{-1}\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{6ib^2 (a + b \sec^{-1}(cx))}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 289, normalized size = 1.83

$$a^3 cx - 3a^2 b \log\left(cx \left(\sqrt{1 - \frac{1}{c^2 x^2}} + 1\right)\right) + 3a^2 b cx \sec^{-1}(cx) - 6ib^2 \text{Li}_2\left(-ie^{i \sec^{-1}(cx)}\right) (a + b \sec^{-1}(cx)) + 6ib^2 \text{Li}_2\left(ie^{i \sec^{-1}(cx)}\right) (a + b \sec^{-1}(cx))$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])^3,x]

[Out] (a^3\*c\*x + 3\*a^2\*b\*c\*x\*ArcSec[c\*x] + 3\*a\*b^2\*c\*x\*ArcSec[c\*x]^2 + b^3\*c\*x\*ArcSec[c\*x]^3 - 6\*a\*b^2\*ArcSec[c\*x]\*Log[1 - I\*E^(I\*ArcSec[c\*x])] - 3\*b^3\*ArcSec[c\*x]^2\*Log[1 - I\*E^(I\*ArcSec[c\*x])] + 6\*a\*b^2\*ArcSec[c\*x]\*Log[1 + I\*E^(I\*ArcSec[c\*x])] + 3\*b^3\*ArcSec[c\*x]^2\*Log[1 + I\*E^(I\*ArcSec[c\*x])] - 3\*a^2\*b\*Log[c\*(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x] - (6\*I)\*b^2\*(a + b\*ArcSec[c\*x])\*PolyLog[2, (-I)\*E^(I\*ArcSec[c\*x])] + (6\*I)\*b^2\*(a + b\*ArcSec[c\*x])\*PolyLog[2, I\*E^(I\*ArcSec[c\*x])] + 6\*b^3\*PolyLog[3, (-I)\*E^(I\*ArcSec[c\*x])] - 6\*b^3\*PolyLog[3, I\*E^(I\*ArcSec[c\*x])])/c

**fricas [F]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3 \text{arcsec}(cx)^3 + 3ab^2 \text{arcsec}(cx)^2 + 3a^2b \text{arcsec}(cx) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3,x, algorithm="fricas")



[Out] integral(b^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*arcsec(c\*x) + a^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^3, x)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^3,x)

[Out] int((a+b\*arcsec(c\*x))^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $-3/2*a*b^2*c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3)*\log(c)^2 - 12*b^3*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))/((c^2*x^2 - 1), x)*\log(c)^2 + b^3*x*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})^3 - 3/4*b^3*x*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})*\log(c^2*x^2)^2 + 12*b^3*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) - 24*b^3*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*\log(x)/(c^2*x^2 - 1), x)*\log(c) + 12*a*b^2*c^2*\int(1/4*x^2*\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) - 24*a*b^2*c^2*\int(1/4*x^2*\log(x)/(c^2*x^2 - 1), x)*\log(c) + 12*b^3*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*\log(c^2*x^2)*\log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*\log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))^2/(c^2*x^2 - 1), x) + 12*b^3*c^2*\int(1/4*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*\log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*\int(1/4*x^2*\log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*\int(1/4*x^2*\log(c^2*x^2)*\log(x)/(c^2*x$

$\wedge 2 - 1), x) - 12*a*b^2*c^2*\text{integrate}(1/4*x^2*\log(x)^2/(c^2*x^2 - 1), x) - 3/2*a*b^2*(\log(c*x + 1)/c - \log(c*x - 1)/c)*\log(c)^2 + 12*b^3*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})/(c^2*x^2 - 1), x)*\log(c)^2 - 12*b^3*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) + 24*b^3*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(x)/(c^2*x^2 - 1), x)*\log(c) - 12*a*b^2*\text{integrate}(1/4*\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) + 24*a*b^2*\text{integrate}(1/4*\log(x)/(c^2*x^2 - 1), x)*\log(c) + a^3*x - 12*b^3*\text{integrate}(1/4*\sqrt{c*x + 1})*\sqrt{c*x - 1}*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2/(c^2*x^2 - 1), x) + 3*b^3*\text{integrate}(1/4*\sqrt{c*x + 1})*\sqrt{c*x - 1}*\log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c^2*x^2)*\log(x)/(c^2*x^2 - 1), x) + 12*b^3*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(x)^2/(c^2*x^2 - 1), x) - 12*a*b^2*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2/(c^2*x^2 - 1), x) - 12*b^3*\text{integrate}(1/4*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c^2*x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*\text{integrate}(1/4*\log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*a*b^2*\text{integrate}(1/4*\log(c^2*x^2)*\log(x)/(c^2*x^2 - 1), x) + 12*a*b^2*\text{integrate}(1/4*\log(x)^2/(c^2*x^2 - 1), x) + 3/2*(2*c*x*\text{arcsec}(c*x) - \log(\sqrt{-1/(c^2*x^2) + 1} + 1) + \log(-\sqrt{-1/(c^2*x^2) + 1} + 1))*a^2*b/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^3,x)

[Out] int((a + b\*acos(1/(c\*x)))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*3,x)

[Out] Integral((a + b\*asec(c\*x))\*\*3, x)

$$3.28 \quad \int \frac{(a+b \sec^{-1}(cx))^3}{x} dx$$

**Optimal.** Leaf size=128

$$-\frac{3}{2}b^2\text{Li}_3\left(-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))+\frac{3}{2}ib\text{Li}_2\left(-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2+\frac{i(a+b\sec^{-1}(cx))^4}{4b}-\log\left(1+\right.$$

```
[Out] 1/4*I*(a+b*arcsec(c*x))^4/b-(a+b*arcsec(c*x))^3*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+3/2*I*b*(a+b*arcsec(c*x))^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-3/2*b^2*(a+b*arcsec(c*x))*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-3/4*I*b^3*polylog(4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

**Rubi [A]** time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5222, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))+\frac{3}{2}ib\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2-\frac{3}{4}ib^3\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSec[c*x])^3/x, x]
```

```
[Out] ((I/4)*(a + b*ArcSec[c*x])^4)/b - (a + b*ArcSec[c*x])^3*Log[1 + E^((2*I)*ArcSec[c*x])] + ((3*I)/2)*b*(a + b*ArcSec[c*x])^2*PolyLog[2, -E^((2*I)*ArcSec[c*x])] - (3*b^2*(a + b*ArcSec[c*x])*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/2 - ((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*ArcSec[c*x])]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_))^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec^{-1}(cx))^3}{x} dx &= \text{Subst} \left( \int (a + bx)^3 \tan(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^4}{4b} - 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)^3}{1 + e^{2ix}} dx, x, \sec^{-1}(cx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^4}{4b} - (a + b \sec^{-1}(cx))^3 \log(1 + e^{2i \sec^{-1}(cx)}) + (3b) \text{Subst} \left( \int (a + bx) \right) \\
&= \frac{i(a + b \sec^{-1}(cx))^4}{4b} - (a + b \sec^{-1}(cx))^3 \log(1 + e^{2i \sec^{-1}(cx)}) + \frac{3}{2} ib (a + b \sec^{-1}(cx)) \\
&= \frac{i(a + b \sec^{-1}(cx))^4}{4b} - (a + b \sec^{-1}(cx))^3 \log(1 + e^{2i \sec^{-1}(cx)}) + \frac{3}{2} ib (a + b \sec^{-1}(cx)) \\
&= \frac{i(a + b \sec^{-1}(cx))^4}{4b} - (a + b \sec^{-1}(cx))^3 \log(1 + e^{2i \sec^{-1}(cx)}) + \frac{3}{2} ib (a + b \sec^{-1}(cx)) \\
&= \frac{i(a + b \sec^{-1}(cx))^4}{4b} - (a + b \sec^{-1}(cx))^3 \log(1 + e^{2i \sec^{-1}(cx)}) + \frac{3}{2} ib (a + b \sec^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 204, normalized size = 1.59

$$\frac{1}{4} \left( 4a^3 \log(cx) + 6ia^2b \sec^{-1}(cx)^2 - 12a^2b \sec^{-1}(cx) \log(1 + e^{2i \sec^{-1}(cx)}) - 6b^2 \text{Li}_3(-e^{2i \sec^{-1}(cx)}) (a + b \sec^{-1}(cx)) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])^3/x, x]

[Out] ((6\*I)\*a^2\*b\*ArcSec[c\*x]^2 + (4\*I)\*a\*b^2\*ArcSec[c\*x]^3 + I\*b^3\*ArcSec[c\*x]^4 - 12\*a^2\*b\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - 12\*a\*b^2\*ArcSec[c\*x]^2\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - 4\*b^3\*ArcSec[c\*x]^3\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] + 4\*a^3\*Log[c\*x] + (6\*I)\*b\*(a + b\*ArcSec[c\*x])^2\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])] - 6\*b^2\*(a + b\*ArcSec[c\*x])\*PolyLog[3, -E^((2\*I)\*ArcSec[c\*x])] - (3\*I)\*b^3\*PolyLog[4, -E^((2\*I)\*ArcSec[c\*x])])/4

**fricas [F]** time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^3 \text{arcsec}(cx)^3 + 3ab^2 \text{arcsec}(cx)^2 + 3a^2b \text{arcsec}(cx) + a^3}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x,x, algorithm="fricas")

[Out] integral((b^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*arcsec(c\*x) + a^3)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^3/x, x)

**maple** [B] time = 0.26, size = 390, normalized size = 3.05

$$a^3 \ln(cx) + \frac{ib^3 \operatorname{arcsec}(cx)^4}{4} - b^3 \operatorname{arcsec}(cx)^3 \ln \left( 1 + \left( \frac{1}{cx} + i \sqrt{1 - \frac{1}{c^2 x^2}} \right)^2 \right) + \frac{3ib^3 \operatorname{arcsec}(cx)^2 \operatorname{polylog} \left( 2, - \left( \frac{1}{cx} + i \sqrt{1 - \frac{1}{c^2 x^2}} \right) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^3/x,x)

[Out] a^3\*ln(c\*x)+1/4\*I\*b^3\*arcsec(c\*x)^4-b^3\*arcsec(c\*x)^3\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+3/2\*I\*b^3\*arcsec(c\*x)^2\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)-3/2\*b^3\*arcsec(c\*x)\*polylog(3,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)-3/4\*I\*b^3\*polylog(4,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+I\*a\*b^2\*arcsec(c\*x)^3-3\*a\*b^2\*arcsec(c\*x)^2\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+3\*I\*a\*b^2\*arcsec(c\*x)\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)-3/2\*a\*b^2\*polylog(3,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+3/2\*I\*a^2\*b\*arcsec(c\*x)^2-3\*a^2\*b\*arcsec(c\*x)\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+3/2\*I\*a^2\*b\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x,x, algorithm="maxima")

[Out] -3/2\*a\*b^2\*c^2\*(log(c\*x + 1)/c^2 + log(c\*x - 1)/c^2)\*log(c)^2 - 12\*b^3\*c^2\*integrate(1/4\*x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(c^2\*x^3 - x), x)\*log(c)^2 + 12\*b^3\*c^2\*integrate(1/4\*x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*lo

$g(c^2x^2)/(c^2x^3 - x), x) \log(c) - 24b^3c^2 \int (1/4x^2 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(x)/(c^2x^3 - x), x) \log(c) + 12ab^2c^2 \int (1/4x^2 \log(c^2x^2)/(c^2x^3 - x), x) \log(c) - 24a^2b^2c^2 \int (1/4x^2 \log(x)/(c^2x^3 - x), x) \log(c) + b^3 \arctan(\sqrt{cx+1}) \sqrt{cx-1}^3 \log(x) - 3/4b^3 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(c^2x^2)^2 \log(x) + 24b^3c^2 \int (1/4x^2 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(c^2x^2) \log(x)/(c^2x^3 - x), x) - 12b^3c^2 \int (1/4x^2 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(x)^2/(c^2x^3 - x), x) + 12a^2b^2c^2 \int (1/4x^2 \arctan(\sqrt{cx+1}) \sqrt{cx-1})^2/(c^2x^3 - x), x) - 3a^2b^2c^2 \int (1/4x^2 \log(c^2x^2)^2/(c^2x^3 - x), x) + 12a^2b^2c^2 \int (1/4x^2 \log(c^2x^2) \log(x)/(c^2x^3 - x), x) - 12a^2b^2c^2 \int (1/4x^2 \log(x)^2/(c^2x^3 - x), x) + 12a^2b^2c^2 \int (1/4x^2 \arctan(\sqrt{cx+1}) \sqrt{cx-1})/(c^2x^3 - x), x) + 3/2a^2b^2(\log(cx+1) + \log(cx-1) - 2\log(x)) \log(c)^2 + 12b^3 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1})/(c^2x^3 - x), x) \log(c)^2 - 12b^3 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(c^2x^2)/(c^2x^3 - x), x) \log(c) + 24b^3 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(x)/(c^2x^3 - x), x) \log(c) - 12ab^2 \int (1/4 \log(c^2x^2)/(c^2x^3 - x), x) \log(c) + 24ab^2 \int (1/4 \log(x)/(c^2x^3 - x), x) \log(c) - 12b^3 \int (1/4 \sqrt{cx+1}) \sqrt{cx-1} \arctan(\sqrt{cx+1}) \sqrt{cx-1})^2 \log(x)/(c^2x^3 - x), x) + 3b^3 \int (1/4 \sqrt{cx+1}) \sqrt{cx-1} \log(c^2x^2)^2 \log(x)/(c^2x^3 - x), x) - 24b^3 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(c^2x^2) \log(x)/(c^2x^3 - x), x) + 12b^3 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1}) \log(x)^2/(c^2x^3 - x), x) - 12ab^2 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1})^2/(c^2x^3 - x), x) + 3a^2b^2 \int (1/4 \log(c^2x^2)^2/(c^2x^3 - x), x) - 12a^2b^2 \int (1/4 \log(c^2x^2) \log(x)/(c^2x^3 - x), x) + 12a^2b^2 \int (1/4 \log(x)^2/(c^2x^3 - x), x) - 12a^2b^2 \int (1/4 \arctan(\sqrt{cx+1}) \sqrt{cx-1})/(c^2x^3 - x), x) + a^3 \log(x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^3/x,x)

[Out] int((a + b\*acos(1/(c\*x)))^3/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))**3/x,x)
```

```
[Out] Integral((a + b*asec(c*x))**3/x, x)
```



$$3.29 \quad \int \frac{(a+b \sec^{-1}(cx))^3}{x^2} dx$$

Optimal. Leaf size=80

$$\frac{6b^2(a+b \sec^{-1}(cx))}{x} + 3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))^2 - \frac{(a+b \sec^{-1}(cx))^3}{x} - 6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

[Out]  $6*b^2*(a+b*\text{arcsec}(c*x))/x-(a+b*\text{arcsec}(c*x))^3/x-6*b^3*c*(1-1/c^2/x^2)^(1/2)+3*b*c*(a+b*\text{arcsec}(c*x))^2*(1-1/c^2/x^2)^(1/2)$

**Rubi** [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5222, 3296, 2637}

$$\frac{6b^2(a+b \sec^{-1}(cx))}{x} + 3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))^2 - \frac{(a+b \sec^{-1}(cx))^3}{x} - 6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^3/x^2, x]

[Out]  $-6*b^3*c*\text{Sqrt}[1-1/(c^2*x^2)]+(6*b^2*(a+b*\text{ArcSec}[c*x]))/x+3*b*c*\text{Sqrt}[1-1/(c^2*x^2)]*(a+b*\text{ArcSec}[c*x])^2-(a+b*\text{ArcSec}[c*x])^3/x$

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m+1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m+1)\*Tan[x], x], x, ArcSec[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec^{-1}(cx))^3}{x^2} dx &= c \operatorname{Subst} \left( \int (a + bx)^3 \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{(a + b \sec^{-1}(cx))^3}{x} + (3bc) \operatorname{Subst} \left( \int (a + bx)^2 \cos(x) dx, x, \sec^{-1}(cx) \right) \\
&= 3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2 - \frac{(a + b \sec^{-1}(cx))^3}{x} - (6b^2 c) \operatorname{Subst} \left( \int (a + bx) \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{6b^2 (a + b \sec^{-1}(cx))}{x} + 3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2 - \frac{(a + b \sec^{-1}(cx))^3}{x} - (6b^3 c) \operatorname{Subst} \left( \int \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= -6b^3 c \sqrt{1 - \frac{1}{c^2 x^2}} + \frac{6b^2 (a + b \sec^{-1}(cx))}{x} + 3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2 - \frac{(a + b \sec^{-1}(cx))^3}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 141, normalized size = 1.76

$$\frac{-a^3 + 3b \sec^{-1}(cx) \left( -a^2 + 2abcx \sqrt{1 - \frac{1}{c^2 x^2}} + 2b^2 \right) + 3a^2 bcx \sqrt{1 - \frac{1}{c^2 x^2}} + 3b^2 \sec^{-1}(cx)^2 \left( bcx \sqrt{1 - \frac{1}{c^2 x^2}} - a \right) + 6ab^3}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^3/x^2,x]

[Out]  $(-a^3 + 6a^2 b^2 + 3a^2 b^2 c \sqrt{1 - 1/(c^2 x^2)})x - 6b^3 c \sqrt{1 - 1/(c^2 x^2)} + 3b^2 (-a^2 + 2abcx \sqrt{1 - 1/(c^2 x^2)} + 2b^2) \operatorname{ArcSec}[cx] + 3b^2 (-a + bcx \sqrt{1 - 1/(c^2 x^2)}) \operatorname{ArcSec}[cx]^2 - b^3 \operatorname{ArcSec}[cx]^3)/x$

**fricas [A]** time = 0.69, size = 98, normalized size = 1.22

$$\frac{b^3 \operatorname{arcsec}(cx)^3 + 3ab^2 \operatorname{arcsec}(cx)^2 + a^3 - 6ab^2 + 3(a^2 b - 2b^3) \operatorname{arcsec}(cx) - 3(b^3 \operatorname{arcsec}(cx)^2 + 2ab^2 \operatorname{arcsec}(cx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^2,x, algorithm="fricas")

[Out]  $-(b^3 \operatorname{arcsec}(cx)^3 + 3a^2 b^2 \operatorname{arcsec}(cx)^2 + a^3 - 6a^2 b^2 + 3(a^2 b - 2b^3) \operatorname{arcsec}(cx) - 3(b^3 \operatorname{arcsec}(cx)^2 + 2ab^2 \operatorname{arcsec}(cx)) \sqrt{c^2 x^2 - 1})/x$

**giac [B]** time = 0.19, size = 196, normalized size = 2.45

$$\left( 3b^3 \sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right)^2 + 6ab^2 \sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right) - \frac{b^3 \arccos\left(\frac{1}{cx}\right)^3}{cx} + 3a^2b \sqrt{-\frac{1}{c^2x^2} + 1} - 6b^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^2,x, algorithm="giac")

[Out] (3\*b^3\*sqrt(-1/(c^2\*x^2) + 1)\*arccos(1/(c\*x))^2 + 6\*a\*b^2\*sqrt(-1/(c^2\*x^2) + 1)\*arccos(1/(c\*x)) - b^3\*arccos(1/(c\*x))^3/(c\*x) + 3\*a^2\*b\*sqrt(-1/(c^2\*x^2) + 1) - 6\*b^3\*sqrt(-1/(c^2\*x^2) + 1) - 3\*a\*b^2\*arccos(1/(c\*x))^2/(c\*x) - 3\*a^2\*b\*arccos(1/(c\*x))/(c\*x) + 6\*b^3\*arccos(1/(c\*x))/(c\*x) - a^3/(c\*x) + 6\*a\*b^2/(c\*x))\*c

**maple [B]** time = 0.18, size = 198, normalized size = 2.48

$$c \left( -\frac{a^3}{cx} + b^3 \left( -\frac{\operatorname{arcsec}(cx)^3}{cx} + 3\operatorname{arcsec}(cx)^2 \sqrt{\frac{c^2x^2-1}{c^2x^2}} - 6\sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{6\operatorname{arcsec}(cx)}{cx} \right) + 3ab^2 \left( -\frac{\operatorname{arcsec}(cx)^2}{cx} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))^3/x^2,x)

[Out] c\*(-a^3/c/x+b^3\*(-1/c/x\*arcsec(c\*x)^3+3\*arcsec(c\*x)^2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)-6\*((c^2\*x^2-1)/c^2/x^2)^(1/2)+6/c/x\*arcsec(c\*x))+3\*a\*b^2\*(-1/c/x\*arcsec(c\*x)^2+2/c/x+2\*arcsec(c\*x)\*((c^2\*x^2-1)/c^2/x^2)^(1/2))+3\*a^2\*b\*(-1/c/x\*arcsec(c\*x)+1/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2\*(c^2\*x^2-1)))

**maxima [A]** time = 0.33, size = 146, normalized size = 1.82

$$-\frac{b^3 \operatorname{arcsec}(cx)^3}{x} + 3 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsec}(cx)}{x} \right) a^2 b + 6 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} \operatorname{arcsec}(cx) + \frac{1}{x} \right) a b^2 + 3 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^2,x, algorithm="maxima")

[Out] -b^3\*arcsec(c\*x)^3/x + 3\*(c\*sqrt(-1/(c^2\*x^2) + 1) - arcsec(c\*x)/x)\*a^2\*b + 6\*(c\*sqrt(-1/(c^2\*x^2) + 1)\*arcsec(c\*x) + 1/x)\*a\*b^2 + 3\*(c\*sqrt(-1/(c^2\*x^2) + 1)\*arcsec(c\*x)^2 - 2\*c\*sqrt(-1/(c^2\*x^2) + 1) + 2\*arcsec(c\*x)/x)\*b^3 - 3\*a\*b^2\*arcsec(c\*x)^2/x - a^3/x

**mupad [B]** time = 0.80, size = 156, normalized size = 1.95

$$\frac{b^3 \left( 6 \operatorname{acos} \left( \frac{1}{cx} \right) - \operatorname{acos} \left( \frac{1}{cx} \right)^3 \right)}{x} - \frac{a^3}{x} + 3a^2bc \left( \sqrt{1 - \frac{1}{c^2x^2}} - \frac{\operatorname{acos} \left( \frac{1}{cx} \right)}{cx} \right) + b^3c \sqrt{1 - \frac{1}{c^2x^2}} \left( 3 \operatorname{acos} \left( \frac{1}{cx} \right)^2 - 6 \right) + 3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^3/x^2,x)

[Out] (b^3\*(6\*acos(1/(c\*x)) - acos(1/(c\*x))^3))/x - a^3/x + 3\*a^2\*b\*c\*((1 - 1/(c^2\*x^2))^(1/2) - acos(1/(c\*x))/(c\*x)) + b^3\*c\*(1 - 1/(c^2\*x^2))^(1/2)\*(3\*acos(1/(c\*x))^2 - 6) + 3\*a\*b^2\*c\*(2\*acos(1/(c\*x))\*(1 - 1/(c^2\*x^2))^(1/2) - (acos(1/(c\*x))^2 - 2)/(c\*x))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*3/x\*\*2,x)

[Out] Integral((a + b\*asec(c\*x))\*\*3/x\*\*2, x)

$$3.30 \quad \int \frac{(a+b \sec^{-1}(cx))^3}{x^3} dx$$

Optimal. Leaf size=137

$$-\frac{3}{4}b^2 \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx)) + \frac{3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{4x} + \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^3 - \frac{1}{4}c^2 (a + b \sec^{-1}(cx))$$

[Out] 3/8\*b^3\*c^2\*arcsec(c\*x)-3/4\*b^2\*(c^2-1/x^2)\*(a+b\*arcsec(c\*x))-1/4\*c^2\*(a+b\*arcsec(c\*x))^3+1/2\*(c^2-1/x^2)\*(a+b\*arcsec(c\*x))^3-3/8\*b^3\*c\*(1-1/c^2/x^2)^(1/2)/x+3/4\*b\*c\*(a+b\*arcsec(c\*x))^2\*(1-1/c^2/x^2)^(1/2)/x

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4404, 3311, 32, 2635, 8}

$$-\frac{3}{4}b^2 \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx)) + \frac{3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{4x} + \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^3 - \frac{1}{4}c^2 (a + b \sec^{-1}(cx))$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^3/x^3, x]

[Out] (-3\*b^3\*c\*sqrt[1 - 1/(c^2\*x^2)])/(8\*x) + (3\*b^3\*c^2\*ArcSec[c\*x])/8 - (3\*b^2\*(c^2 - x^(-2))\*(a + b\*ArcSec[c\*x]))/4 + (3\*b\*c\*sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcSec[c\*x])^2)/(4\*x) - (c^2\*(a + b\*ArcSec[c\*x])^3)/4 + ((c^2 - x^(-2))\*(a + b\*ArcSec[c\*x])^3)/2

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*cos[c + d\*x])\*(b\*sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*SIN[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*SIN[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*SIN[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*SIN[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*SIN[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m + 1)\*Tan[x], x], x, ArcSec[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sec^{-1}(cx))^3}{x^3} dx &= c^2 \text{Subst} \left( \int (a + bx)^3 \cos(x) \sin(x) dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^3 - \frac{1}{2} (3bc^2) \text{Subst} \left( \int (a + bx)^2 \sin^2(x) dx, x, \sec^{-1}(cx) \right) \\
 &= -\frac{3}{4} b^2 \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx)) + \frac{3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{4x} + \frac{1}{2} \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx))^3 \\
 &= -\frac{3b^3 c \sqrt{1 - \frac{1}{c^2 x^2}}}{8x} - \frac{3}{4} b^2 \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx)) + \frac{3bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{4x} \\
 &= -\frac{3b^3 c \sqrt{1 - \frac{1}{c^2 x^2}}}{8x} + \frac{3}{8} b^3 c^2 \sec^{-1}(cx) - \frac{3}{4} b^2 \left( c^2 - \frac{1}{x^2} \right) (a + b \sec^{-1}(cx)) + \frac{3bc \sqrt{1 - \frac{1}{c^2 x^2}}}{4x}
 \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 185, normalized size = 1.35

$$\frac{-4a^3 + 3bc^2x^2(b^2 - 2a^2)\sin^{-1}\left(\frac{1}{cx}\right) + 6b\sec^{-1}(cx)\left(-2a^2 + 2abcx\sqrt{1 - \frac{1}{c^2x^2}} + b^2\right) + 6a^2bcx\sqrt{1 - \frac{1}{c^2x^2}} + 6b^2\sec^{-1}\left(\frac{1}{cx}\right)}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^3/x^3,x]

[Out]  $(-4a^3 + 6ab^2 + 6a^2bc\sqrt{1 - 1/(c^2x^2)}x - 3b^3c\sqrt{1 - 1/(c^2x^2)}x + 6b(-2a^2 + b^2 + 2abc\sqrt{1 - 1/(c^2x^2)}x)\text{ArcSec}[cx] + 6b^2(bc\sqrt{1 - 1/(c^2x^2)}x + a(-2 + c^2x^2))\text{ArcSec}[cx]^2 + 2b^3(-2 + c^2x^2)\text{ArcSec}[cx]^3 + 3b(-2a^2 + b^2)c^2x^2\text{ArcSin}[1/(cx)])/(8x^2)$

**fricas [A]** time = 0.67, size = 150, normalized size = 1.09

$$\frac{2(b^3c^2x^2 - 2b^3)\text{arcsec}(cx)^3 - 4a^3 + 6ab^2 + 6(ab^2c^2x^2 - 2ab^2)\text{arcsec}(cx)^2 + 3((2a^2b - b^3)c^2x^2 - 4a^2b + 2b^3)\text{arcsec}(cx) + 3(2a^2b - b^3)\sqrt{c^2x^2 - 1}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^3,x, algorithm="fricas")

[Out]  $1/8*(2*(b^3c^2x^2 - 2b^3)*\text{arcsec}(cx)^3 - 4a^3 + 6ab^2 + 6(a^2b^2c^2x^2 - 2a^2b^2)*\text{arcsec}(cx)^2 + 3*((2a^2b - b^3)*c^2x^2 - 4a^2b + 2b^3)*\text{arcsec}(cx) + 3*(2b^3*\text{arcsec}(cx)^2 + 4a^2b*\text{arcsec}(cx) + 2a^2b - b^3)*\sqrt{c^2x^2 - 1})/x^2$

**giac [B]** time = 0.18, size = 278, normalized size = 2.03

$$\frac{1}{8}\left(2b^3c\arccos\left(\frac{1}{cx}\right)^3 + 6ab^2c\arccos\left(\frac{1}{cx}\right)^2 + 6a^2bc\arccos\left(\frac{1}{cx}\right) - 3b^3c\arccos\left(\frac{1}{cx}\right) + \frac{6b^3\sqrt{-\frac{1}{c^2x^2} + 1}\arccos\left(\frac{1}{cx}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^3,x, algorithm="giac")

[Out]  $1/8*(2b^3c*\arccos(1/(cx))^3 + 6a^2b^2c*\arccos(1/(cx))^2 + 6a^2b^2c*\arccos(1/(cx)) - 3b^3c*\arccos(1/(cx)) + 6b^3*\sqrt{-1/(c^2x^2) + 1}*\arccos(1/(cx))^2/x - 3a^2b^2c + 12a^2b^2*\sqrt{-1/(c^2x^2) + 1}*\arccos(1/(cx)))/x - 4b^3*\arccos(1/(cx))^3/(cx^2) + 6a^2b*\sqrt{-1/(c^2x^2) + 1}/x - 3b^3*\sqrt{-1/(c^2x^2) + 1}/x - 12a^2b^2*\arccos(1/(cx))^2/(cx^2) - 12a^2b^2*\arccos(1/(cx))$

$\int (2bx \arccos(1/(cx)) / (cx^2) + 6b^3 \arccos(1/(cx)) / (cx^2) - 4a^3 / (cx^2) + 6a^2b / (cx^2)) dx$

**maple [B]** time = 0.37, size = 324, normalized size = 2.36

$$\frac{a^3}{2x^2} - \frac{b^3 \operatorname{arcsec}(cx)^3}{2x^2} + \frac{c^2 b^3 \operatorname{arcsec}(cx)^3}{4} + \frac{3c b^3 \operatorname{arcsec}(cx)^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{4x} + \frac{3b^3 \operatorname{arcsec}(cx)}{4x^2} - \frac{3c b^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{8x} - \frac{3b^3 c^2 \operatorname{arcsec}(cx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))^3/x^3,x)`

[Out] 
$$-1/2*a^3/x^2 - 1/2*b^3/x^2*arcsec(c*x)^3 + 1/4*c^2*b^3*arcsec(c*x)^3 + 3/4*c*b^3*arcsec(c*x)^2/x*((c^2*x^2-1)/c^2/x^2)^{(1/2)} + 3/4*b^3/x^2*arcsec(c*x) - 3/8*c*b^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x - 3/8*b^3*c^2*arcsec(c*x) - 3/2*a*b^2/x^2*arcsec(c*x)^2 + 3/4*c^2*a*b^2*arcsec(c*x)^2 + 3/2*c*a*b^2*arcsec(c*x)/x*((c^2*x^2-1)/c^2/x^2)^{(1/2)} - 3/4*c^2*a*b^2 + 3/4*a*b^2/x^2 - 3/2*a^2*b/x^2*arcsec(c*x) - 3/4*c*a^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*arctan(1/(c^2*x^2-1)^{(1/2)}) + 3/4*c*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x - 3/4/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^3$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="maxima")`

[Out] 
$$-3/4*a^2*b*((c^4*x*\sqrt{-1/(c^2*x^2)+1})/(c^2*x^2*(1/(c^2*x^2)-1)-1) - c^3*arctan(c*x*\sqrt{-1/(c^2*x^2)+1}))/c + 2*arcsec(c*x)/x^2 - 1/2*a^3/x^2 - 1/8*(4*b^3*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})^3 - 3*b^3*arctan(\sqrt{c*x+1})*\sqrt{c*x-1}*\log(c^2*x^2)^2 + 12*(a*b^2*c^2*(\log(c*x+1)+\log(c*x-1)-2*\log(x))*\log(c)^2 + 16*b^3*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})/(c^2*x^5-x^3),x)*\log(c)^2 - 16*b^3*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c^2*x^2)/(c^2*x^5-x^3),x)*\log(c) + 32*b^3*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})*\log(x)/(c^2*x^5-x^3),x)*\log(c) - 16*a*b^2*c^2*\int(1/8*x^2*\log(c^2*x^2)/(c^2*x^5-x^3),x)*\log(c) + 32*a*b^2*c^2*\int(1/8*x^2*\log(x)/(c^2*x^5-x^3),x)*\log(c) - 16*b^3*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c^2*x^2)*\log(x)/(c^2*x^5-x^3),x) + 16*b^3*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})*\log(x)^2/(c^2*x^5-x^3),x) - 16*a*b^2*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})^2/(c^2*x^5-x^3),x) + 8*b^3*c^2*\int(1/8*x^2*arctan(\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c^2*x^2)/(c^2*x^5-x^3),x) + 4*a*b^2*c^2*\int(1/8*x^2*\log(c^2*x^2)/(c^2*x^5-x^3),x)$$



$$g(c^2x^2)^2/(c^2x^5 - x^3), x) - 16ab^2c^2 \int (1/8x^2 \log(c^2x^2) \log(x)/(c^2x^5 - x^3), x) + 16ab^2c^2 \int (1/8x^2 \log(x)^2/(c^2x^5 - x^3), x) - (c^2 \log(cx + 1) + c^2 \log(cx - 1) - 2c^2 \log(x) + 1/x^2)ab^2 \log(c)^2 - 16b^3 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1})/(c^2x^5 - x^3), x) \log(c)^2 + 16b^3 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1}) \log(c^2x^2)/(c^2x^5 - x^3), x) \log(c) - 32b^3 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1}) \log(x)/(c^2x^5 - x^3), x) \log(c) + 16ab^2 \int (1/8 \log(c^2x^2)/(c^2x^5 - x^3), x) \log(c) - 32ab^2 \int (1/8 \log(x)/(c^2x^5 - x^3), x) \log(c) - 8b^3 \int (1/8 \sqrt{cx + 1} \sqrt{cx - 1} \arctan(\sqrt{cx + 1}) \sqrt{cx - 1})^2/(c^2x^5 - x^3), x) + 2b^3 \int (1/8 \sqrt{cx + 1} \sqrt{cx - 1} \log(c^2x^2)^2/(c^2x^5 - x^3), x) + 16b^3 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1}) \log(c^2x^2) \log(x)/(c^2x^5 - x^3), x) - 16b^3 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1}) \log(x)^2/(c^2x^5 - x^3), x) + 16ab^2 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1})^2/(c^2x^5 - x^3), x) - 8b^3 \int (1/8 \arctan(\sqrt{cx + 1}) \sqrt{cx - 1}) \log(c^2x^2)/(c^2x^5 - x^3), x) - 4ab^2 \int (1/8 \log(c^2x^2)^2/(c^2x^5 - x^3), x) + 16ab^2 \int (1/8 \log(x)^2/(c^2x^5 - x^3), x))x^2/x^2$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^3/x^3,x)

[Out] int((a + b\*acos(1/(c\*x)))^3/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*\*3/x\*\*3,x)

[Out] Integral((a + b\*asec(c\*x))\*\*3/x\*\*3, x)

$$3.31 \quad \int \frac{(a+b \sec^{-1}(cx))^3}{x^4} dx$$

**Optimal.** Leaf size=170

$$\frac{4b^2c^2(a+b \sec^{-1}(cx))}{3x} + \frac{2b^2(a+b \sec^{-1}(cx))}{9x^3} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))^2}{3x^2} + \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))$$

[Out]  $2/27*b^3*c^3*(1-1/c^2/x^2)^{(3/2)}+2/9*b^2*(a+b*\text{arcsec}(c*x))/x^3+4/3*b^2*c^2*(a+b*\text{arcsec}(c*x))/x-1/3*(a+b*\text{arcsec}(c*x))^3/x^3-14/9*b^3*c^3*(1-1/c^2/x^2)^{(1/2)}+2/3*b*c^3*(a+b*\text{arcsec}(c*x))^2*(1-1/c^2/x^2)^{(1/2)}+1/3*b*c*(a+b*\text{arcsec}(c*x))^2*(1-1/c^2/x^2)^{(1/2)}/x^2$

**Rubi [A]** time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4405, 3311, 3296, 2637, 2633}

$$\frac{4b^2c^2(a+b \sec^{-1}(cx))}{3x} + \frac{2b^2(a+b \sec^{-1}(cx))}{9x^3} + \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^3/x^4, x]

[Out]  $(-14*b^3*c^3*\text{Sqrt}[1-1/(c^2*x^2)]/9 + (2*b^3*c^3*(1-1/(c^2*x^2))^{(3/2)})/27 + (2*b^2*(a+b*\text{ArcSec}[c*x]))/(9*x^3) + (4*b^2*c^2*(a+b*\text{ArcSec}[c*x]))/(3*x) + (2*b*c^3*\text{Sqrt}[1-1/(c^2*x^2)]*(a+b*\text{ArcSec}[c*x])^2)/3 + (b*c*\text{Sqrt}[1-1/(c^2*x^2)]*(a+b*\text{ArcSec}[c*x])^2)/(3*x^2) - (a+b*\text{ArcSec}[c*x])^3/(3*x^3)$

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3311

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)} * \{(b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol]$   $\rightarrow$   $\text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{(n-2)}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m-2)}*(b*\sin[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{(n-1)}]/(f*n), x]) /;$   
 $\text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

### Rule 4405

$\text{Int}[\cos[(a_.) + (b_.)*(x_.)]^{(n_.)} * \{(c_.) + (d_.)*(x_.)\}^{(m_.)} * \sin[(a_.) + (b_.)*(x_.)], x\_Symbol]$   $\rightarrow$   $-\text{Simp}[(c + d*x)^m*\cos[a + b*x]^{(n+1)}]/(b*(n+1)), x] + \text{Dist}[(d*m)/(b*(n+1)), \text{Int}[(c + d*x)^{(m-1)}*\cos[a + b*x]^{(n+1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[n, -1]$

### Rule 5222

$\text{Int}[\{(a_.) + \text{ArcSec}[(c_.)*(x_.)]*(b_.)\}^{(n_.)} * (x_.)^{(m_.)}, x\_Symbol]$   $\rightarrow$   $\text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\sec[x]^{(m+1)}*\tan[x], x], x, \text{ArcSec}[c*x]], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{LtQ}[m, -1])$

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec^{-1}(cx))^3}{x^4} dx &= c^3 \text{Subst} \left( \int (a + bx)^3 \cos^2(x) \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{(a + b \sec^{-1}(cx))^3}{3x^3} + (bc^3) \text{Subst} \left( \int (a + bx)^2 \cos^3(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{2b^2 (a + b \sec^{-1}(cx))}{9x^3} + \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2}{3x^2} - \frac{(a + b \sec^{-1}(cx))^3}{3x^3} + \frac{1}{3} (2bc^3) \\
&= \frac{2b^2 (a + b \sec^{-1}(cx))}{9x^3} + \frac{2}{3} bc^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))^2 + \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec^{-1}(cx))}{3x^2} \\
&= -\frac{2}{9} b^3 c^3 \sqrt{1 - \frac{1}{c^2 x^2}} + \frac{2}{27} b^3 c^3 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} + \frac{2b^2 (a + b \sec^{-1}(cx))}{9x^3} + \frac{4b^2 c^2 (a + b \sec^{-1}(cx))}{3x} \\
&= -\frac{14}{9} b^3 c^3 \sqrt{1 - \frac{1}{c^2 x^2}} + \frac{2}{27} b^3 c^3 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} + \frac{2b^2 (a + b \sec^{-1}(cx))}{9x^3} + \frac{4b^2 c^2 (a + b \sec^{-1}(cx))}{3x}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 204, normalized size = 1.20

$$\frac{-9a^3 + 3b \sec^{-1}(cx) \left(-9a^2 + 6abcx \sqrt{1 - \frac{1}{c^2 x^2}} (2c^2 x^2 + 1) + 2b^2 (6c^2 x^2 + 1)\right) + 9a^2 bcx \sqrt{1 - \frac{1}{c^2 x^2}} (2c^2 x^2 + 1) + 6a^2 bcx \sqrt{1 - \frac{1}{c^2 x^2}} (2c^2 x^2 + 1)}{27}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^3/x^4,x]

[Out]  $(-9a^3 + 9a^2 b c \sqrt{1 - 1/(c^2 x^2)}) x (1 + 2c^2 x^2) + 6a^2 b^2 (1 + 6c^2 x^2) - 2b^3 c \sqrt{1 - 1/(c^2 x^2)} x (1 + 20c^2 x^2) + 3b^2 (-9a^2 + 6a^2 b c \sqrt{1 - 1/(c^2 x^2)}) x (1 + 2c^2 x^2) + 2b^2 (1 + 6c^2 x^2) \text{ArcSec}[c x] + 9b^2 (-3a + b c \sqrt{1 - 1/(c^2 x^2)}) x (1 + 2c^2 x^2) \text{ArcSec}[c x]^2 - 9b^3 \text{ArcSec}[c x]^3 / (27x^3)$

**fricas [A]** time = 0.71, size = 172, normalized size = 1.01

$$\frac{36 ab^2 c^2 x^2 - 9 b^3 \text{arcsec}(cx)^3 - 27 ab^2 \text{arcsec}(cx)^2 - 9 a^3 + 6 ab^2 + 3 (12 b^3 c^2 x^2 - 9 a^2 b + 2 b^3) \text{arcsec}(cx) + (2 \dots)}{27 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^4,x, algorithm="fricas")

[Out]  $\frac{1}{27}*(36*a*b^2*c^2*x^2 - 9*b^3*\text{arcsec}(c*x))^3 - 27*a*b^2*\text{arcsec}(c*x)^2 - 9*a^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*\text{arcsec}(c*x) + (2*(9*a^2*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*\text{arcsec}(c*x))^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*\text{arcsec}(c*x)*\sqrt{c^2*x^2 - 1}/x^3$

**giac** [B] time = 0.19, size = 336, normalized size = 1.98

$$\frac{1}{27} \left( 18b^3c^2 \sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right)^2 + 36ab^2c^2 \sqrt{-\frac{1}{c^2x^2} + 1} \arccos\left(\frac{1}{cx}\right) + 18a^2bc^2 \sqrt{-\frac{1}{c^2x^2} + 1} - 40b^3c^2 \sqrt{-\frac{1}{c^2x^2} + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="giac")`

[Out]  $\frac{1}{27}*(18*b^3*c^2*\sqrt{-1/(c^2*x^2) + 1}*\arccos(1/(c*x))^2 + 36*a*b^2*c^2*\sqrt{-1/(c^2*x^2) + 1}*\arccos(1/(c*x)) + 18*a^2*b*c^2*\sqrt{-1/(c^2*x^2) + 1} - 40*b^3*c^2*\sqrt{-1/(c^2*x^2) + 1} + 36*b^3*c*\arccos(1/(c*x))/x + 9*b^3*\sqrt{-1/(c^2*x^2) + 1}*\arccos(1/(c*x))^2/x^2 + 36*a*b^2*c/x + 18*a*b^2*\sqrt{-1/(c^2*x^2) + 1}*\arccos(1/(c*x))/x^2 - 9*b^3*\arccos(1/(c*x))^3/(c*x^3) + 9*a^2*b*\sqrt{-1/(c^2*x^2) + 1}/x^2 - 2*b^3*\sqrt{-1/(c^2*x^2) + 1}/x^2 - 27*a*a*b^2*\arccos(1/(c*x))^2/(c*x^3) - 27*a^2*b*\arccos(1/(c*x))/(c*x^3) + 6*b^3*\arccos(1/(c*x))/(c*x^3) - 9*a^3/(c*x^3) + 6*a*b^2/(c*x^3))*c$

**maple** [B] time = 0.59, size = 299, normalized size = 1.76

$$c^3 \left( -\frac{a^3}{3c^3x^3} + b^3 \left( -\frac{\text{arcsec}(cx)^3}{3c^3x^3} + \frac{\text{arcsec}(cx)^2 (2c^2x^2 + 1) \sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2} - \frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} + \frac{4 \text{arcsec}(cx)}{3cx} + \frac{2 \text{arcsec}(cx)}{9c^3x^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))^3/x^4,x)`

[Out]  $c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*\text{arcsec}(c*x))^3+1/3*\text{arcsec}(c*x)^2*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-4/3*((c^2*x^2-1)/c^2/x^2)^(1/2)+4/3/c/x*\text{arcsec}(c*x)+2/9/c^3/x^3*\text{arcsec}(c*x)-2/27*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*\text{arcsec}(c*x))^2+2/9*\text{arcsec}(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+3*a^2*b*(-1/3/c^3/x^3*\text{arcsec}(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)^(1/2))/c^4/x^4)$

**maxima** [B] time = 0.98, size = 575, normalized size = 3.38

$$-\frac{1}{216} \left( \frac{72 \left( c^4 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3 c^4 \sqrt{-\frac{1}{c^2 x^2} + 1} \right) \operatorname{arcsec}(cx)^2}{c} + \frac{72 c^4 \left( \frac{c^2 \arcsin\left(\frac{1}{c|x|}\right) + \frac{2\sqrt{c^2 x^2 - 1}c - \sqrt{c^2 x^2 - 1}}{x}}{c} - \frac{c^2 \arcsin\left(\frac{1}{c|x|}\right) - 2\sqrt{c^2 x^2 - 1}}{c} \right)}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^4,x, algorithm="maxima")

[Out] 
$$-\frac{1}{216} \left( 72 \left( c^4 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3 c^4 \sqrt{-\frac{1}{c^2 x^2} + 1} \right) \operatorname{arcsec}(cx)^2 + \left( \frac{c^2 \arcsin\left(\frac{1}{c|x|}\right) + \frac{2\sqrt{c^2 x^2 - 1}c - \sqrt{c^2 x^2 - 1}}{x}}{c} - \frac{c^2 \arcsin\left(\frac{1}{c|x|}\right) - 2\sqrt{c^2 x^2 - 1}}{c} \right) \right) \frac{1}{c}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))^3/x^4,x)

[Out] int((a + b\*acos(1/(c\*x)))^3/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*asec(c*x))**3/x**4, x)
```

$$3.32 \quad \int \frac{(a+b \sec^{-1}(cx))^3}{x^5} dx$$

Optimal. Leaf size=208

$$\frac{9b^2c^2(a+b \sec^{-1}(cx))}{32x^2} + \frac{3b^2(a+b \sec^{-1}(cx))}{32x^4} + \frac{3}{32}c^4(a+b \sec^{-1}(cx))^3 + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))^2}{16x^3} + \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}$$

[Out]  $-45/256*b^3*c^4*arcsec(c*x)+3/32*b^2*(a+b*arcsec(c*x))/x^4+9/32*b^2*c^2*(a+b*arcsec(c*x))/x^2+3/32*c^4*(a+b*arcsec(c*x))^3-1/4*(a+b*arcsec(c*x))^3/x^4-3/128*b^3*c*(1-1/c^2/x^2)^{(1/2)}/x^3-45/256*b^3*c^3*(1-1/c^2/x^2)^{(1/2)}/x^3+1/16*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^{(1/2)}/x^3+9/32*b*c^3*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4405, 3311, 32, 2635, 8}

$$\frac{9b^2c^2(a+b \sec^{-1}(cx))}{32x^2} + \frac{3b^2(a+b \sec^{-1}(cx))}{32x^4} + \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))^2}{32x} + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \sec^{-1}(cx))}{16x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])^3/x^5,x]

[Out]  $(-3*b^3*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(128*x^3) - (45*b^3*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(256*x) - (45*b^3*c^4*\text{ArcSec}[c*x])/256 + (3*b^2*(a + b*\text{ArcSec}[c*x]))/(32*x^4) + (9*b^2*c^2*(a + b*\text{ArcSec}[c*x]))/(32*x^2) + (3*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*(a + b*\text{ArcSec}[c*x])^2)/(16*x^3) + (9*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*(a + b*\text{ArcSec}[c*x])^2)/(32*x) + (3*c^4*(a + b*\text{ArcSec}[c*x])^3)/32 - (a + b*\text{ArcSec}[c*x])^3/(4*x^4)$

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*cos[c + d\*x])\*(b\*sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*sin[c



+ d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*SIN[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*SIN[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*SIN[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 4405

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[a + b\*x]^(n + 1))/(b\*(n + 1)), x] + Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Cos[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m + 1)\*Tan[x], x], x, ArcSec[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec^{-1}(cx))^3}{x^5} dx &= c^4 \text{Subst} \left( \int (a + bx)^3 \cos^3(x) \sin(x) dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{(a + b \sec^{-1}(cx))^3}{4x^4} + \frac{1}{4} (3bc^4) \text{Subst} \left( \int (a + bx)^2 \cos^4(x) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{3b^2 (a + b \sec^{-1}(cx))}{32x^4} + \frac{3bc \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))^2}{16x^3} - \frac{(a + b \sec^{-1}(cx))^3}{4x^4} + \frac{1}{16} (9) \\
&= -\frac{3b^3c \sqrt{1 - \frac{1}{c^2x^2}}}{128x^3} + \frac{3b^2 (a + b \sec^{-1}(cx))}{32x^4} + \frac{9b^2c^2 (a + b \sec^{-1}(cx))}{32x^2} + \frac{3bc \sqrt{1 - \frac{1}{c^2x^2}} (a + b \sec^{-1}(cx))}{16x^3} \\
&= -\frac{3b^3c \sqrt{1 - \frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3 \sqrt{1 - \frac{1}{c^2x^2}}}{256x} + \frac{3b^2 (a + b \sec^{-1}(cx))}{32x^4} + \frac{9b^2c^2 (a + b \sec^{-1}(cx))}{32x^2} \\
&= -\frac{3b^3c \sqrt{1 - \frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3 \sqrt{1 - \frac{1}{c^2x^2}}}{256x} - \frac{45}{256} b^3c^4 \sec^{-1}(cx) + \frac{3b^2 (a + b \sec^{-1}(cx))}{32x^4} + \frac{9}{32} b^2c^2 \sec^{-1}(cx)
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 283, normalized size = 1.36

$$-64a^3 + 9bc^4x^4 (5b^2 - 8a^2) \sin^{-1} \left( \frac{1}{cx} \right) + 24b \sec^{-1}(cx) \left( -8a^2 + 2abcx \sqrt{1 - \frac{1}{c^2x^2}} (3c^2x^2 + 2) + b^2 (3c^2x^2 + 1) \right) + 4$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])^3/x^5,x]

[Out] (-64\*a^3 + 24\*a\*b^2 + 48\*a^2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x - 6\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + 72\*a\*b^2\*c^2\*x^2 + 72\*a^2\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3 - 45\*b^3\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3 + 24\*b\*(-8\*a^2 + b^2\*(1 + 3\*c^2\*x^2) + 2\*a\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(2 + 3\*c^2\*x^2))\*ArcSec[c\*x] + 24\*b^2\*(b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(2 + 3\*c^2\*x^2) + a\*(-8 + 3\*c^4\*x^4))\*ArcSec[c\*x]^2 + 8\*b^3\*(-8 + 3\*c^4\*x^4)\*ArcSec[c\*x]^3 + 9\*b\*(-8\*a^2 + 5\*b^2)\*c^4\*x^4\*ArcSin[1/(c\*x)])/(256\*x^4)

**fricas [A]** time = 0.72, size = 225, normalized size = 1.08

$$72 ab^2c^2x^2 + 8 (3 b^3c^4x^4 - 8 b^3) \operatorname{arcsec}(cx)^3 - 64 a^3 + 24 ab^2 + 24 (3 ab^2c^4x^4 - 8 ab^2) \operatorname{arcsec}(cx)^2 + 3 (3 (8 a^2b -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))^3/x^5,x, algorithm="fricas")

[Out]  $\frac{1}{256}*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*\text{arcsec}(c*x)^3 - 64*a^3 + 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*\text{arcsec}(c*x)^2 + 3*(3*(8*a^2*b - 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*\text{arcsec}(c*x) + 3*(3*(8*a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*\text{arcsec}(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*\text{arcsec}(c*x))*\text{sqrt}(c^2*x^2 - 1))/x^4$

**giac** [B] time = 0.19, size = 427, normalized size = 2.05

$$\frac{1}{256} \left( 24 b^3 c^3 \arccos\left(\frac{1}{cx}\right)^3 + 72 ab^2 c^3 \arccos\left(\frac{1}{cx}\right)^2 + 72 a^2 b c^3 \arccos\left(\frac{1}{cx}\right) - 45 b^3 c^3 \arccos\left(\frac{1}{cx}\right) + \frac{72 b^3 c^2 \sqrt{-1}}{c^2 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="giac")`

[Out]  $\frac{1}{256}*(24*b^3*c^3*\arccos(1/(c*x))^3 + 72*a*b^2*c^3*\arccos(1/(c*x))^2 + 72*a^2*b*c^3*\arccos(1/(c*x)) - 45*b^3*c^3*\arccos(1/(c*x)) + 72*b^3*c^2*\text{sqrt}(-1/(c^2*x^2) + 1)*\arccos(1/(c*x))^2/x - 45*a*b^2*c^3 + 144*a*b^2*c^2*\text{sqrt}(-1/(c^2*x^2) + 1)*\arccos(1/(c*x))/x + 72*a^2*b*c^2*\text{sqrt}(-1/(c^2*x^2) + 1)/x - 45*b^3*c^2*\text{sqrt}(-1/(c^2*x^2) + 1)/x + 72*b^3*c*\arccos(1/(c*x))/x^2 + 48*b^3*\text{sqrt}(-1/(c^2*x^2) + 1)*\arccos(1/(c*x))^2/x^3 + 72*a*b^2*c/x^2 + 96*a*b^2*\text{sqrt}(-1/(c^2*x^2) + 1)*\arccos(1/(c*x))/x^3 - 64*b^3*\arccos(1/(c*x))^3/(c*x^4) + 48*a^2*b*\text{sqrt}(-1/(c^2*x^2) + 1)/x^3 - 6*b^3*\text{sqrt}(-1/(c^2*x^2) + 1)/x^3 - 192*a*b^2*\arccos(1/(c*x))^2/(c*x^4) - 192*a^2*b*\arccos(1/(c*x))/(c*x^4) + 24*b^3*\arccos(1/(c*x))/(c*x^4) - 64*a^3/(c*x^4) + 24*a*b^2/(c*x^4))*c$

**maple** [B] time = 0.64, size = 472, normalized size = 2.27

$$\frac{a^3}{4x^4} - \frac{b^3 \text{arcsec}(cx)^3}{4x^4} + \frac{3c^4 b^3 \text{arcsec}(cx)^3}{32} + \frac{9c^3 b^3 \text{arcsec}(cx)^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{32x} + \frac{3c b^3 \text{arcsec}(cx)^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{16x^3} + \frac{3b^3 \text{arcsec}(cx)}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))^3/x^5,x)`

[Out]  $-1/4*a^3/x^4 - 1/4*b^3/x^4*\text{arcsec}(c*x)^3 + 3/32*c^4*b^3*\text{arcsec}(c*x)^3 + 9/32*c^3*b^3*\text{arcsec}(c*x)^2/x*((c^2*x^2-1)/c^2/x^2)^(1/2) + 3/16*c*b^3*\text{arcsec}(c*x)^2/x^3*((c^2*x^2-1)/c^2/x^2)^(1/2) + 3/32*b^3/x^4*\text{arcsec}(c*x) - 45/256*c^3*b^3*((c^2*x^2-1)/c^2/x^2)^(1/2)/x - 3/128*c*b^3/x^3*((c^2*x^2-1)/c^2/x^2)^(1/2) - 45/256*b^3*c^4*\text{arcsec}(c*x) + 9/32*c^2*b^3/x^2*\text{arcsec}(c*x) - 3/4*a*b^2/x^4*\text{arcsec}(c*x)^2 + 9/32*c^4*a*b^2*\text{arcsec}(c*x)^2 + 9/16*c^3*a*b^2*\text{arcsec}(c*x)/x*((c^2*x^2-1)/c^2/x^2)^(1/2) + 3/8*c*a*b^2*\text{arcsec}(c*x)/x^3*((c^2*x^2-1)/c^2/x^2)^(1/2) + 3/32*$



```
e(1/16*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^7 - x^5), x)
- 16*a*b^2*integrate(1/16*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*in
tegrate(1/16*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 64*a*b^2*integrate(1
/16*log(x)^2/(c^2*x^7 - x^5), x))*x^4)/x^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))^3/x^5, x)
```

```
[Out] int((a + b*acos(1/(c*x)))^3/x^5, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))**3/x**5, x)
```

```
[Out] Integral((a + b*asec(c*x))**3/x**5, x)
```

$$3.33 \quad \int \frac{x}{a+b \sec^{-1}(cx)} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x}{a+b \sec^{-1}(cx)}, x\right)$$

[Out] Unintegrable(x/(a+b\*arcsec(c\*x)), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x}{a+b \sec^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int][x/(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int \frac{x}{a+b \sec^{-1}(cx)} dx = \int \frac{x}{a+b \sec^{-1}(cx)} dx$$

Mathematica [A] time = 2.59, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b \sec^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[x/(a + b\*ArcSec[c\*x]), x]

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b \text{arcsec}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] integral(x/(b\*arcsec(c\*x) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \operatorname{arcsec}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate(x/(b\*arcsec(c\*x) + a), x)

**maple** [A] time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{arcsec}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsec(c\*x)),x)

[Out] int(x/(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \operatorname{arcsec}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(x/(b\*arcsec(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{a + b \arccos\left(\frac{1}{cx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*acos(1/(c\*x))),x)

[Out] int(x/(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{asec}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asec(c*x)),x)
```

```
[Out] Integral(x/(a + b*asec(c*x)), x)
```



$$3.34 \quad \int \frac{1}{a+b \sec^{-1}(cx)} dx$$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{a+b \sec^{-1}(cx)}, x\right)$$

[Out] Unintegrable(1/(a+b\*arcsec(c\*x)), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{a+b \sec^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])^(-1), x]

[Out] Defer[Int][(a + b\*ArcSec[c\*x])^(-1), x]

Rubi steps

$$\int \frac{1}{a+b \sec^{-1}(cx)} dx = \int \frac{1}{a+b \sec^{-1}(cx)} dx$$

Mathematica [A] time = 2.31, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b \sec^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])^(-1), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])^(-1), x]

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b \operatorname{arcsec}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*arcsec(c\*x) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arcsec}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate(1/(b\*arcsec(c\*x) + a), x)

**maple** [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{arcsec}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsec(c\*x)),x)

[Out] int(1/(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arcsec}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(1/(b\*arcsec(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{a + b \operatorname{acos}\left(\frac{1}{cx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*acos(1/(c\*x))),x)

[Out] int(1/(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{asec}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asec(c*x)),x)
```

```
[Out] Integral(1/(a + b*asec(c*x)), x)
```

$$3.35 \quad \int \frac{1}{x(a+b \sec^{-1}(cx))} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x(a+b \sec^{-1}(cx))}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*arcsec(c\*x)), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSec[c\*x])), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSec[c\*x])), x]

Rubi steps

$$\int \frac{1}{x(a+b \sec^{-1}(cx))} dx = \int \frac{1}{x(a+b \sec^{-1}(cx))} dx$$

**Mathematica [A]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSec[c\*x])), x]

[Out] Integrate[1/(x\*(a + b\*ArcSec[c\*x])), x]

**fricas [A]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx \text{arcsec}(cx) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x\*arcsec(c\*x) + a\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arcsec(c\*x) + a)\*x), x)

**maple** [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{arcsec}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsec(c\*x)),x)

[Out] int(1/x/(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsec(c\*x) + a)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*acos(1/(c\*x))))),x)

[Out] int(1/(x\*(a + b\*acos(1/(c\*x))))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{asec}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*asec(c\*x)),x)

[Out] Integral(1/(x\*(a + b\*asec(c\*x))), x)

$$3.36 \quad \int \frac{1}{x^2(a+b \sec^{-1}(cx))} dx$$

Optimal. Leaf size=46

$$\frac{c \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b} - \frac{c \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b}$$

[Out] c\*cos(a/b)\*Si(a/b+arcsec(c\*x))/b-c\*Ci(a/b+arcsec(c\*x))\*sin(a/b)/b

**Rubi [A]** time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5222, 3303, 3299, 3302}

$$\frac{c \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b} - \frac{c \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*ArcSec[c\*x])),x]

[Out] -((c\*CosIntegral[a/b + ArcSec[c\*x]]\*Sin[a/b])/b) + (c\*Cos[a/b]\*SinIntegral[a/b + ArcSec[c\*x]])/b

Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*(x\_)^(m\_), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m + 1)\*Tan[x], x], x, ArcSec[c\*x]]

], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \sec^{-1}(cx))} dx &= c \operatorname{Subst} \left( \int \frac{\sin(x)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\ &= \left( c \cos \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\sin \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \sec^{-1}(cx) \right) - \left( c \sin \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cos \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\ &= -\frac{c \operatorname{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) \sin \left( \frac{a}{b} \right)}{b} + \frac{c \cos \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right)}{b} \end{aligned}$$

**Mathematica** [A] time = 0.08, size = 43, normalized size = 0.93

$$\frac{c \left( \cos \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right) - \sin \left( \frac{a}{b} \right) \operatorname{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*ArcSec[c\*x])),x]

[Out] (c\*(-(CosIntegral[a/b + ArcSec[c\*x]]\*Sin[a/b]) + Cos[a/b]\*SinIntegral[a/b + ArcSec[c\*x]]))/b

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{1}{bx^2 \operatorname{arcsec}(cx) + ax^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x^2\*arcsec(c\*x) + a\*x^2), x)

**giac** [A] time = 0.15, size = 55, normalized size = 1.20

$$-c \left( \frac{\operatorname{Ci} \left( \frac{a}{b} + \arccos \left( \frac{1}{cx} \right) \right) \sin \left( \frac{a}{b} \right)}{b} - \frac{\cos \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \arccos \left( \frac{1}{cx} \right) \right)}{b} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] -c\*(cos\_integral(a/b + arccos(1/(c\*x)))\*sin(a/b)/b - cos(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/b)

**maple** [A] time = 0.14, size = 47, normalized size = 1.02

$$c \left( \frac{\text{Si}\left(\frac{a}{b} + \text{arcsec}(cx)\right) \cos\left(\frac{a}{b}\right)}{b} - \frac{\text{Ci}\left(\frac{a}{b} + \text{arcsec}(cx)\right) \sin\left(\frac{a}{b}\right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsec(c\*x)),x)

[Out] c\*(Si(a/b+arcsec(c\*x))\*cos(a/b)/b-Ci(a/b+arcsec(c\*x))\*sin(a/b)/b)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsec(c\*x) + a)\*x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*acos(1/(c\*x))))) ,x)

[Out] int(1/(x^2\*(a + b\*acos(1/(c\*x))))) , x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asec}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asec(c\*x)),x)

[Out] Integral(1/(x\*\*2\*(a + b\*asec(c\*x))), x)

$$3.37 \quad \int \frac{1}{x^3(a+b \sec^{-1}(cx))} dx$$

Optimal. Leaf size=63

$$\frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{2b} - \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{2b}$$

[Out]  $1/2*c^2*\cos(2*a/b)*\text{Si}(2*a/b+2*\text{arcsec}(c*x))/b-1/2*c^2*\text{Ci}(2*a/b+2*\text{arcsec}(c*x))*\sin(2*a/b)/b$

**Rubi [A]** time = 0.14, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4406, 12, 3303, 3299, 3302}

$$\frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{2b} - \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(a + b*\text{ArcSec}[c*x])), x]$

[Out]  $-(c^2*\text{CosIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]]*\text{Sin}[(2*a)/b])/(2*b) + (c^2*\text{Cos}[(2*a)/b]*\text{SinIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]])/(2*b)$

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 3299

$\text{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

### Rule 3302

$\text{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

### Rule 3303

$\text{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)$

)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b  
\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x  
]^(n)\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG  
tQ[p, 0]

### Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1  
/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m + 1)\*Tan[x], x], x, ArcSec[c\*x  
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||  
LtQ[m, -1])

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a + b \sec^{-1}(cx))} dx &= c^2 \text{Subst} \left( \int \frac{\cos(x) \sin(x)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\
 &= c^2 \text{Subst} \left( \int \frac{\sin(2x)}{2(a + bx)} dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{1}{2} c^2 \text{Subst} \left( \int \frac{\sin(2x)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{1}{2} \left( c^2 \cos \left( \frac{2a}{b} \right) \right) \text{Subst} \left( \int \frac{\sin \left( \frac{2a}{b} + 2x \right)}{a + bx} dx, x, \sec^{-1}(cx) \right) - \frac{1}{2} \left( c^2 \sin \left( \frac{2a}{b} \right) \right) \text{Subst} \\
 &= -\frac{c^2 \text{Ci} \left( \frac{2a}{b} + 2 \sec^{-1}(cx) \right) \sin \left( \frac{2a}{b} \right)}{2b} + \frac{c^2 \cos \left( \frac{2a}{b} \right) \text{Si} \left( \frac{2a}{b} + 2 \sec^{-1}(cx) \right)}{2b}
 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 56, normalized size = 0.89

$$\frac{c^2 \left( \cos \left( \frac{2a}{b} \right) \text{Si} \left( \frac{2a}{b} + 2 \sec^{-1}(cx) \right) - \sin \left( \frac{2a}{b} \right) \text{Ci} \left( \frac{2a}{b} + 2 \sec^{-1}(cx) \right) \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*ArcSec[c\*x])),x]

[Out]  $(c^2 * (-(\text{CosIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]] * \text{Sin}[(2*a)/b]) + \text{Cos}[(2*a)/b] * \text{SinIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]])) / (2*b)$

**fricas** [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^3 \text{arcsec}(cx) + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="fricas")`

[Out] `integral(1/(b*x^3*arcsec(c*x) + a*x^3), x)`

**giac** [A] time = 0.13, size = 95, normalized size = 1.51

$$-\frac{1}{2} \left( \frac{2c \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \arccos\left(\frac{1}{cx}\right)\right) \sin\left(\frac{a}{b}\right)}{b} - \frac{2c \cos\left(\frac{a}{b}\right)^2 \text{Si}\left(\frac{2a}{b} + 2 \arccos\left(\frac{1}{cx}\right)\right)}{b} + \frac{c \text{Si}\left(\frac{2a}{b} + 2 \arccos\left(\frac{1}{cx}\right)\right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="giac")`

[Out] `-1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/b - 2*c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/b + c*sin_integral(2*a/b + 2*arccos(1/(c*x)))/b)*c`

**maple** [A] time = 0.10, size = 58, normalized size = 0.92

$$c^2 \left( \frac{\text{Si}\left(\frac{2a}{b} + 2 \text{arcsec}(cx)\right) \cos\left(\frac{2a}{b}\right)}{2b} - \frac{\text{Ci}\left(\frac{2a}{b} + 2 \text{arcsec}(cx)\right) \sin\left(\frac{2a}{b}\right)}{2b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(a+b*arcsec(c*x)),x)`

[Out] `c^2*(1/2*Si(2*a/b+2*arcsec(c*x))*cos(2*a/b)/b-1/2*Ci(2*a/b+2*arcsec(c*x))*sin(2*a/b)/b)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \text{arcsec}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsec(c\*x) + a)\*x^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \left( a + b \operatorname{arcsec} \left( \frac{1}{cx} \right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*(a + b\*acos(1/(c\*x))))),x)

[Out] int(1/(x^3\*(a + b\*acos(1/(c\*x))))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{asec}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*asec(c\*x)),x)

[Out] Integral(1/(x\*\*3\*(a + b\*asec(c\*x))), x)

$$3.38 \quad \int \frac{1}{x^4(a+b \sec^{-1}(cx))} dx$$

**Optimal.** Leaf size=117

$$\frac{c^3 \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b} - \frac{c^3 \sin\left(\frac{3a}{b}\right) \text{Ci}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{4b} + \frac{c^3 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b} + \frac{c^3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{4b}$$

[Out]  $1/4*c^3*\cos(a/b)*\text{Si}(a/b+\text{arcsec}(c*x))/b+1/4*c^3*\cos(3*a/b)*\text{Si}(3*a/b+3*\text{arcsec}(c*x))/b-1/4*c^3*\text{Ci}(a/b+\text{arcsec}(c*x))*\sin(a/b)/b-1/4*c^3*\text{Ci}(3*a/b+3*\text{arcsec}(c*x))*\sin(3*a/b)/b$

**Rubi [A]** time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5222, 4406, 3303, 3299, 3302}

$$\frac{c^3 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b} - \frac{c^3 \sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{4b} + \frac{c^3 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*ArcSec[c\*x])),x]

[Out]  $-(c^3*\text{CosIntegral}[a/b + \text{ArcSec}[c*x]]*\text{Sin}[a/b])/(4*b) - (c^3*\text{CosIntegral}[(3*a)/b + 3*\text{ArcSec}[c*x]]*\text{Sin}[(3*a)/b])/(4*b) + (c^3*\text{Cos}[a/b]*\text{SinIntegral}[a/b + \text{ArcSec}[c*x]])/(4*b) + (c^3*\text{Cos}[(3*a)/b]*\text{SinIntegral}[(3*a)/b + 3*\text{ArcSec}[c*x]])/(4*b)$

#### Rule 3299

Int[sin[(e.) + (f.)\*(x\_)]/((c.) + (d.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e.) + (f.)\*(x\_)]/((c.) + (d.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rule 3303

Int[sin[(e.) + (f.)\*(x\_)]/((c.) + (d.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d\*e - c\*f, 0]

### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^(n)\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5222

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b\*x)^n\*Sec[x]^(m + 1)\*Tan[x], x], x, ArcSec[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 (a + b \sec^{-1}(cx))} dx &= c^3 \text{Subst} \left( \int \frac{\cos^2(x) \sin(x)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\
 &= c^3 \text{Subst} \left( \int \left( \frac{\sin(x)}{4(a + bx)} + \frac{\sin(3x)}{4(a + bx)} \right) dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{1}{4} c^3 \text{Subst} \left( \int \frac{\sin(x)}{a + bx} dx, x, \sec^{-1}(cx) \right) + \frac{1}{4} c^3 \text{Subst} \left( \int \frac{\sin(3x)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\
 &= \frac{1}{4} \left( c^3 \cos \left( \frac{a}{b} \right) \right) \text{Subst} \left( \int \frac{\sin \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \sec^{-1}(cx) \right) + \frac{1}{4} \left( c^3 \cos \left( \frac{3a}{b} \right) \right) \text{Subst} \left( \int \frac{\sin \left( \frac{3a}{b} + x \right)}{a + bx} dx, x, \sec^{-1}(cx) \right) \\
 &= -\frac{c^3 \text{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) \sin \left( \frac{a}{b} \right)}{4b} - \frac{c^3 \text{Ci} \left( \frac{3a}{b} + 3 \sec^{-1}(cx) \right) \sin \left( \frac{3a}{b} \right)}{4b} + \frac{c^3 \cos \left( \frac{a}{b} \right) \text{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right)}{4b} + \frac{c^3 \cos \left( \frac{3a}{b} \right) \text{Si} \left( \frac{3a}{b} + 3 \sec^{-1}(cx) \right)}{4b}
 \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 91, normalized size = 0.78

$$\frac{c^3 \left( \sin \left( \frac{a}{b} \right) \left( -\text{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) \right) - \sin \left( \frac{3a}{b} \right) \text{Ci} \left( 3 \left( \frac{a}{b} + \sec^{-1}(cx) \right) \right) + \cos \left( \frac{a}{b} \right) \text{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right) + \cos \left( \frac{3a}{b} \right) \text{Si} \left( 3 \left( \frac{a}{b} + \sec^{-1}(cx) \right) \right) \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*ArcSec[c\*x])),x]

[Out]  $(c^3 * (-(\text{CosIntegral}[a/b + \text{ArcSec}[c*x]] * \text{Sin}[a/b]) - \text{CosIntegral}[3*(a/b + \text{ArcSec}[c*x]]) * \text{Sin}[(3*a)/b] + \text{Cos}[a/b] * \text{SinIntegral}[a/b + \text{ArcSec}[c*x]] + \text{Cos}[(3*a)/b] * \text{SinIntegral}[3*(a/b + \text{ArcSec}[c*x])])) / (4*b)$

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^4 \text{arcsec}(cx) + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="fricas")`

[Out] `integral(1/(b*x^4*arcsec(c*x) + a*x^4), x)`

**giac** [A] time = 0.14, size = 199, normalized size = 1.70

$$-\frac{1}{4} \left( \frac{4c^2 \cos\left(\frac{a}{b}\right)^2 \text{Ci}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right) \sin\left(\frac{a}{b}\right)}{b} - \frac{4c^2 \cos\left(\frac{a}{b}\right)^3 \text{Si}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right)}{b} - \frac{c^2 \text{Ci}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="giac")`

[Out] `-1/4*(4*c^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b - 4*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(c*x)))/b - c^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b + c^2*cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/b + 3*c^2*cos(a/b)*sin_integral(3*a/b + 3*arccos(1/(c*x)))/b - c^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/b)*c`

**maple** [A] time = 0.11, size = 102, normalized size = 0.87

$$c^3 \left( \frac{\text{Si}\left(\frac{3a}{b} + 3 \text{arcsec}(cx)\right) \cos\left(\frac{3a}{b}\right)}{4b} - \frac{\text{Ci}\left(\frac{3a}{b} + 3 \text{arcsec}(cx)\right) \sin\left(\frac{3a}{b}\right)}{4b} + \frac{\text{Si}\left(\frac{a}{b} + \text{arcsec}(cx)\right) \cos\left(\frac{a}{b}\right)}{4b} - \frac{\text{Ci}\left(\frac{a}{b} + \text{arcsec}(cx)\right) \sin\left(\frac{a}{b}\right)}{4b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(a+b*arcsec(c*x)),x)`

[Out] `c^3*(1/4*Si(3*a/b+3*arcsec(c*x))*cos(3*a/b)/b-1/4*Ci(3*a/b+3*arcsec(c*x))*sin(3*a/b)/b+1/4*Si(a/b+arcsec(c*x))*cos(a/b)/b-1/4*Ci(a/b+arcsec(c*x))*sin(a/b)/b)`



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arcsec(c\*x) + a)\*x^4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4\*(a + b\*acos(1/(c\*x))))),x)

[Out] int(1/(x^4\*(a + b\*acos(1/(c\*x))))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \operatorname{asec}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*asec(c\*x)),x)

[Out] Integral(1/(x\*\*4\*(a + b\*asec(c\*x))), x)

$$3.39 \quad \int \frac{x}{(a+b \sec^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=15

$$\text{Int} \left( \frac{x}{(a + b \sec^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable(x/(a+b\*arcsec(c\*x))^2,x)

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x}{(a + b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b\*ArcSec[c\*x])^2,x]

[Out] Defer[Int][x/(a + b\*ArcSec[c\*x])^2, x]

Rubi steps

$$\int \frac{x}{(a + b \sec^{-1}(cx))^2} dx = \int \frac{x}{(a + b \sec^{-1}(cx))^2} dx$$

**Mathematica [A]** time = 10.30, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b\*ArcSec[c\*x])^2,x]

[Out] Integrate[x/(a + b\*ArcSec[c\*x])^2, x]

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x}{b^2 \text{arcsec}(cx)^2 + 2ab \text{arcsec}(cx) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(x/(b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \operatorname{arcsec}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate(x/(b\*arcsec(c\*x) + a)^2, x)

maple [A] time = 2.24, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arcsec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsec(c\*x))^2,x)

[Out] int(x/(a+b\*arcsec(c\*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$4(bx^2 \arctan(\sqrt{cx+1}\sqrt{cx-1}) + ax^2)\sqrt{cx+1}\sqrt{cx-1} + 4(4b^3 \arctan(\sqrt{cx+1}\sqrt{cx-1})^2 + b^3 \log(c^2x^2))^2$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] -(4\*(b\*x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + a\*x^2)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - (4\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + b^3\*log(c^2\*x^2)^2 + 4\*b^3\*log(c)^2 + 8\*b^3\*log(c)\*log(x) + 4\*b^3\*log(x)^2 + 8\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*a^2\*b - 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))\*integrate(-4\*(3\*a\*c^2\*x^3 - 2\*a\*x + (3\*b\*c^2\*x^3 - 2\*b\*x)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)))\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/(4\*b^3\*log(c)^2 + 4\*a^2\*b - 4\*(b^3\*c^2\*log(c)^2 + a^2\*b\*c^2)\*x^2 - 4\*(b^3\*c^2\*x^2 - b^3)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 - (b^3\*c^2\*x^2 - b^3)\*log(c^2\*x^2)^2 - 4\*(b^3\*c^2\*x^2 - b^3)\*log(x)^2 - 8\*(a\*b^2\*c^2\*x^2 - a\*b^2)\*arctan(sqrt(c\*x + 1

```
) * sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)
*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4
*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log
(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*acos(1/(c*x)))^2,x)
```

```
[Out] int(x/(a + b*acos(1/(c*x)))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{asec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(x/(a + b*asec(c*x))**2, x)
```

$$3.40 \quad \int \frac{1}{(a+b \sec^{-1}(cx))^2} dx$$

Optimal. Leaf size=13

$$\text{Int} \left( \frac{1}{(a + b \sec^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable(1/(a+b\*arcsec(c\*x))^2, x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a + b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])^(-2), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])^(-2), x]

Rubi steps

$$\int \frac{1}{(a + b \sec^{-1}(cx))^2} dx = \int \frac{1}{(a + b \sec^{-1}(cx))^2} dx$$

Mathematica [A] time = 22.77, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])^(-2), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])^(-2), x]

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 \text{arcsec}(cx)^2 + 2ab \text{arcsec}(cx) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^(-2), x)

**maple** [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsec(c\*x))^2,x)

[Out] int(1/(a+b\*arcsec(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$4 \left( bx \arctan(\sqrt{cx+1} \sqrt{cx-1}) + ax \right) \sqrt{cx+1} \sqrt{cx-1} + 4 \left( 4b^3 \arctan(\sqrt{cx+1} \sqrt{cx-1})^2 + b^3 \log(c^2x^2) \right)^2 + 4$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] -(4\*(b\*x\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + a\*x)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - (4\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)))^2 + b^3\*log(c^2\*x^2)^2 + 4\*b^3\*log(c)^2 + 8\*b^3\*log(c)\*log(x) + 4\*b^3\*log(x)^2 + 8\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*a^2\*b - 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))\*integrate(-4\*(2\*a\*c^2\*x^2 + (2\*b\*c^2\*x^2 - b)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - a)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/(4\*b^3\*log(c)^2 + 4\*a^2\*b - 4\*(b^3\*c^2\*log(c)^2 + a^2\*b\*c^2)\*x^2 - 4\*(b^3\*c^2\*x^2 - b^3)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 - (b^3\*c^2\*x^2 - b^3)\*log(c^2\*x^2)^2 - 4\*(b^3\*c^2\*x^2 - b^3)\*log(x)^2 - 8\*(a\*b^2\*c^2\*x^2 - a\*b^2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))

1)) + 4\*(b^3\*c^2\*x^2\*log(c) - b^3\*log(c) + (b^3\*c^2\*x^2 - b^3)\*log(x))\*log(c^2\*x^2) - 8\*(b^3\*c^2\*x^2\*log(c) - b^3\*log(c))\*log(x)), x)/(4\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + b^3\*log(c^2\*x^2)^2 + 4\*b^3\*log(c)^2 + 8\*b^3\*log(c)\*log(x) + 4\*b^3\*log(x)^2 + 8\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*a^2\*b - 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))

**mupad [A]** time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*acos(1/(c\*x)))^2,x)

[Out] int(1/(a + b\*acos(1/(c\*x)))^2, x)

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*asec(c\*x))\*\*2,x)

[Out] Integral((a + b\*asec(c\*x))\*\*(-2), x)

$$3.41 \quad \int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx$$

Optimal. Leaf size=17

$$\text{Int} \left( \frac{1}{x(a+b \sec^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*arcsec(c\*x))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSec[c\*x])^2), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSec[c\*x])^2), x]

Rubi steps

$$\int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx = \int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx$$

Mathematica [A] time = 3.47, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSec[c\*x])^2), x]

[Out] Integrate[1/(x\*(a + b\*ArcSec[c\*x])^2), x]

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2x \text{arcsec}(cx)^2 + 2abx \text{arcsec}(cx) + a^2x}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*arcsec(c\*x)^2 + 2\*a\*b\*x\*arcsec(c\*x) + a^2\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate(1/((b\*arcsec(c\*x) + a)^2\*x), x)

**maple** [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{arcsec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsec(c\*x))^2,x)

[Out] int(1/x/(a+b\*arcsec(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$4 \sqrt{cx+1} \sqrt{cx-1} (b \arctan(\sqrt{cx+1} \sqrt{cx-1}) + a) + 4 \left( 4b^3 \arctan(\sqrt{cx+1} \sqrt{cx-1})^2 + b^3 \log(c^2 x^2) \right)^2 + 4$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] -(4\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*(b\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + a) - (4\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + b^3\*log(c^2\*x^2))^2 + 4\*b^3\*log(c)^2 + 8\*b^3\*log(c)\*log(x) + 4\*b^3\*log(x)^2 + 8\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*a^2\*b - 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))\* integrate(-4\*(b\*c^2\*x\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + a\*c^2\*x)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/(4\*b^3\*log(c)^2 + 4\*a^2\*b - 4\*(b^3\*c^2\*log(c)^2 + a^2\*b\*c^2)\*x^2 - 4\*(b^3\*c^2\*x^2 - b^3)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 - (b^3\*c^2\*x^2 - b^3)\*log(c^2\*x^2)^2 - 4\*(b^3\*c^2\*x^2 - b^3)\*log(x)^2 - 8\*(a\*b^2\*c^2\*x^2 - a\*b^2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*(b^3\*c^2\*x^2\*

```
log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2
*x^2*log(c) - b^3*log(c))*log(x)), x)/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x
- 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^
3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3
*log(c) + b^3*log(x))*log(c^2*x^2))
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acos(1/(c*x))))^2), x)
```

```
[Out] int(1/(x*(a + b*acos(1/(c*x))))^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x (a + b \operatorname{asec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asec(c*x))**2, x)
```

```
[Out] Integral(1/(x*(a + b*asec(c*x))**2), x)
```

$$3.42 \quad \int \frac{1}{x^2(a+b \sec^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=75

$$\frac{c \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b^2} + \frac{c \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b^2} - \frac{c\sqrt{1 - \frac{1}{c^2x^2}}}{b(a + b \sec^{-1}(cx))}$$

[Out]  $c \text{Ci}(a/b + \text{arcsec}(c*x)) * \cos(a/b) / b^2 + c \text{Si}(a/b + \text{arcsec}(c*x)) * \sin(a/b) / b^2 - c * (1 - 1/c^2/x^2)^{(1/2)} / b / (a + b * \text{arcsec}(c*x))$

**Rubi [A]** time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5222, 3297, 3303, 3299, 3302}

$$\frac{c \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b^2} + \frac{c \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{b^2} - \frac{c\sqrt{1 - \frac{1}{c^2x^2}}}{b(a + b \sec^{-1}(cx))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(a + b*\text{ArcSec}[c*x])^2), x]$

[Out]  $-((c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(b*(a + b*\text{ArcSec}[c*x]))) + (c*\text{Cos}[a/b]*\text{CosIntegral}[a/b + \text{ArcSec}[c*x]])/b^2 + (c*\text{Sin}[a/b]*\text{SinIntegral}[a/b + \text{ArcSec}[c*x]])/b^2$

**Rule 3297**

$\text{Int}[\left((c_{.}) + (d_{.})*(x_{.})\right)^{(m_{.})} * \sin\left[(e_{.}) + (f_{.})*(x_{.})\right], x\_Symbol] \rightarrow \text{Simp}[\left(\left(c + d*x\right)^{(m + 1)} * \text{Sin}[e + f*x] / (d*(m + 1)), x\right) - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)} * \text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

**Rule 3299**

$\text{Int}[\sin\left[(e_{.}) + (f_{.})*(x_{.})\right] / \left((c_{.}) + (d_{.})*(x_{.})\right), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x] / d, x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

**Rule 3302**

$\text{Int}[\sin\left[(e_{.}) + (f_{.})*(x_{.})\right] / \left((c_{.}) + (d_{.})*(x_{.})\right), x\_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x] / d, x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^2} dx &= c \operatorname{Subst} \left( \int \frac{\sin(x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{c \sqrt{1 - \frac{1}{c^2 x^2}}}{b (a + b \sec^{-1}(cx))} + \frac{c \operatorname{Subst} \left( \int \frac{\cos(x)}{a + bx} dx, x, \sec^{-1}(cx) \right)}{b} \\
&= -\frac{c \sqrt{1 - \frac{1}{c^2 x^2}}}{b (a + b \sec^{-1}(cx))} + \frac{\left( c \cos \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cos \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \sec^{-1}(cx) \right)}{b} + \frac{\left( c \sin \left( \frac{a}{b} \right) \right)}{b} \\
&= -\frac{c \sqrt{1 - \frac{1}{c^2 x^2}}}{b (a + b \sec^{-1}(cx))} + \frac{c \cos \left( \frac{a}{b} \right) \operatorname{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right)}{b^2} + \frac{c \sin \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.28, size = 69, normalized size = 0.92

$$\frac{c \left( -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}}}{a + b \sec^{-1}(cx)} + \cos \left( \frac{a}{b} \right) \operatorname{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) + \sin \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right) \right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcSec[c*x])^2), x]
```

```
[Out] (c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a + b*ArcSec[c*x])) + Cos[a/b]*CosIntegral
[a/b + ArcSec[c*x]] + Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]]))/b^2
```

**fricas** [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x^2 \operatorname{arcsec}(cx)^2 + 2abx^2 \operatorname{arcsec}(cx) + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*arcsec(c\*x)^2 + 2\*a\*b\*x^2\*arcsec(c\*x) + a^2\*x^2), x)

**giac** [B] time = 0.13, size = 226, normalized size = 3.01

$$\left(\frac{b \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{b \arccos\left(\frac{1}{cx}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{a \cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] (b\*arccos(1/(c\*x))\*cos(a/b)\*cos\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + b\*arccos(1/(c\*x))\*sin(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + a\*cos(a/b)\*cos\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + a\*sin(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - b\*sqrt(-1/(c^2\*x^2) + 1)/(b^3\*arccos(1/(c\*x)) + a\*b^2))\*c

**maple** [A] time = 0.15, size = 78, normalized size = 1.04

$$c \left( \frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a + b \operatorname{arcsec}(cx))b} + \frac{\operatorname{Si}\left(\frac{a}{b} + \operatorname{arcsec}(cx)\right) \sin\left(\frac{a}{b}\right) + \operatorname{Ci}\left(\frac{a}{b} + \operatorname{arcsec}(cx)\right) \cos\left(\frac{a}{b}\right)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsec(c\*x))^2,x)

[Out] c\*(-((c^2\*x^2-1)/c^2/x^2)^(1/2)/(a+b\*arcsec(c\*x))/b+(Si(a/b+arcsec(c\*x))\*sin(a/b)+Ci(a/b+arcsec(c\*x))\*cos(a/b))/b^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4\sqrt{cx+1}\sqrt{cx-1}\left(b \arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right) + a\right) - 4\left(4b^3x \arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)^2 + b^3x \log\left(c^2x^2\right)^2 + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] 
$$-(4\sqrt{c*x + 1}\sqrt{c*x - 1}(b*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}) + a) - (4*b^3*x*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}))^2 + b^3*x*\log(c^2*x^2)^2 + 8*b^3*x*\log(c)*\log(x) + 4*b^3*x*\log(x)^2 + 8*a*b^2*x*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}) + 4*(b^3*\log(c)^2 + a^2*b)*x - 4*(b^3*x*\log(c) + b^3*x*\log(x))*\log(c^2*x^2))*\int(4*\sqrt{c*x + 1}\sqrt{c*x - 1}(b*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}) + a)/(4*(b^3*c^2*\log(c)^2 + a^2*b*c^2)*x^4 - 4*(b^3*\log(c)^2 + a^2*b)*x^2 + 4*(b^3*c^2*x^4 - b^3*x^2)*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}))^2 + (b^3*c^2*x^4 - b^3*x^2)*\log(c^2*x^2)^2 + 4*(b^3*c^2*x^4 - b^3*x^2)*\log(x)^2 + 8*(a*b^2*c^2*x^4 - a*b^2*x^2)*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}) - 4*(b^3*c^2*x^4*\log(c) - b^3*x^2*\log(c) + (b^3*c^2*x^4 - b^3*x^2)*\log(x))*\log(c^2*x^2) + 8*(b^3*c^2*x^4*\log(c) - b^3*x^2*\log(c))*\log(x)), x)/(4*b^3*x*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}))^2 + b^3*x*\log(c^2*x^2)^2 + 8*b^3*x*\log(c)*\log(x) + 4*b^3*x*\log(x)^2 + 8*a*b^2*x*\arctan(\sqrt{c*x + 1}\sqrt{c*x - 1}) + 4*(b^3*\log(c)^2 + a^2*b)*x - 4*(b^3*x*\log(c) + b^3*x*\log(x))*\log(c^2*x^2))$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*acos(1/(c\*x)))^2),x)

[Out] int(1/(x^2\*(a + b\*acos(1/(c\*x)))^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asec(c\*x))\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*asec(c\*x))\*\*2), x)

$$3.43 \quad \int \frac{1}{x^3(a+b \sec^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=84

$$\frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^2} + \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^2} - \frac{c^2 \sin\left(2 \sec^{-1}(cx)\right)}{2b(a + b \sec^{-1}(cx))}$$

[Out]  $c^2 \text{Ci}(2*a/b+2*\text{arcsec}(c*x))*\cos(2*a/b)/b^2+c^2 \text{Si}(2*a/b+2*\text{arcsec}(c*x))*\sin(2*a/b)/b^2-1/2*c^2*\sin(2*\text{arcsec}(c*x))/b/(a+b*\text{arcsec}(c*x))$

**Rubi [A]** time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5222, 4406, 12, 3297, 3303, 3299, 3302}

$$\frac{c^2 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^2} + \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^2} - \frac{c^2 \sin\left(2 \sec^{-1}(cx)\right)}{2b(a + b \sec^{-1}(cx))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(a + b*\text{ArcSec}[c*x])^2), x]$

[Out]  $(c^2*\text{Cos}[(2*a)/b]*\text{CosIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]])/b^2 - (c^2*\text{Sin}[2*\text{ArcSec}[c*x]])/(2*b*(a + b*\text{ArcSec}[c*x])) + (c^2*\text{Sin}[(2*a)/b]*\text{SinIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]])/b^2$

**Rule 12**

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[Q[u, (b_)*(v_)] /; \text{FreeQ}[b, x]]$

**Rule 3297**

$\text{Int}[(c_*) + (d_)*(x_)]^{(m_)}*\sin[(e_*) + (f_)*(x_)], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\text{Sin}[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{LtQ}[m, -1]$

**Rule 3299**

$\text{Int}[\sin[(e_*) + (f_)*(x_)]/((c_*) + (d_)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

Rubi steps



$$\begin{aligned}
\int \frac{1}{x^3 (a + b \sec^{-1}(cx))^2} dx &= c^2 \text{Subst} \left( \int \frac{\cos(x) \sin(x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right) \\
&= c^2 \text{Subst} \left( \int \frac{\sin(2x)}{2(a + bx)^2} dx, x, \sec^{-1}(cx) \right) \\
&= \frac{1}{2} c^2 \text{Subst} \left( \int \frac{\sin(2x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{c^2 \sin(2 \sec^{-1}(cx))}{2b(a + b \sec^{-1}(cx))} + \frac{c^2 \text{Subst} \left( \int \frac{\cos(2x)}{a+bx} dx, x, \sec^{-1}(cx) \right)}{b} \\
&= -\frac{c^2 \sin(2 \sec^{-1}(cx))}{2b(a + b \sec^{-1}(cx))} + \frac{\left( c^2 \cos\left(\frac{2a}{b}\right) \right) \text{Subst} \left( \int \frac{\cos\left(\frac{2a}{b} + 2x\right)}{a+bx} dx, x, \sec^{-1}(cx) \right)}{b} + \frac{c^2 \sin\left(\frac{2a}{b}\right)}{b} \\
&= \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^2} - \frac{c^2 \sin(2 \sec^{-1}(cx))}{2b(a + b \sec^{-1}(cx))} + \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 80, normalized size = 0.95

$$\frac{c \left( -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}}}{ax + bx \sec^{-1}(cx)} + c \cos\left(\frac{2a}{b}\right) \text{Ci}\left(2\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right) + c \sin\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*ArcSec[c\*x])^2), x]

[Out] (c\*(-((b\*Sqrt[1 - 1/(c^2\*x^2)])/(a\*x + b\*x\*ArcSec[c\*x])) + c\*Cos[(2\*a)/b]\*CosIntegral[2\*(a/b + ArcSec[c\*x])] + c\*Sin[(2\*a)/b]\*SinIntegral[2\*(a/b + ArcSec[c\*x])]))/b^2

**fricas [F]** time = 0.67, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 x^3 \text{arcsec}(cx)^2 + 2 abx^3 \text{arcsec}(cx) + a^2 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^3\*arcsec(c\*x)^2 + 2\*a\*b\*x^3\*arcsec(c\*x) + a^2\*x^3), x)

**giac** [B] time = 0.16, size = 357, normalized size = 4.25

$$\left( \frac{2bc \operatorname{arccos}\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b} + 2 \operatorname{arccos}\left(\frac{1}{cx}\right)\right)}{b^3 \operatorname{arccos}\left(\frac{1}{cx}\right) + ab^2} + \frac{2bc \operatorname{arccos}\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2 \operatorname{arccos}\left(\frac{1}{cx}\right)\right)}{b^3 \operatorname{arccos}\left(\frac{1}{cx}\right) + ab^2} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] (2\*b\*c\*arccos(1/(c\*x))\*cos(a/b)^2\*cos\_integral(2\*a/b + 2\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + 2\*b\*c\*arccos(1/(c\*x))\*cos(a/b)\*sin(a/b)\*sin\_integral(2\*a/b + 2\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + 2\*a\*c\*cos(a/b)^2\*cos\_integral(2\*a/b + 2\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + 2\*a\*c\*cos(a/b)\*sin(a/b)\*sin\_integral(2\*a/b + 2\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - b\*c\*arccos(1/(c\*x))\*cos\_integral(2\*a/b + 2\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - a\*c\*cos\_integral(2\*a/b + 2\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - b\*sqrt(-1/(c^2\*x^2) + 1)/((b^3\*arccos(1/(c\*x)) + a\*b^2)\*x))\*c

**maple** [A] time = 0.10, size = 77, normalized size = 0.92

$$c^2 \left( -\frac{\sin(2 \operatorname{arcsec}(cx))}{2(a + b \operatorname{arcsec}(cx))b} + \frac{\operatorname{Si}\left(\frac{2a}{b} + 2 \operatorname{arcsec}(cx)\right) \sin\left(\frac{2a}{b}\right) + \operatorname{Ci}\left(\frac{2a}{b} + 2 \operatorname{arcsec}(cx)\right) \cos\left(\frac{2a}{b}\right)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b\*arcsec(c\*x))^2,x)

[Out] c^2\*(-1/2\*sin(2\*arcsec(c\*x))/(a+b\*arcsec(c\*x))/b+(Si(2\*a/b+2\*arcsec(c\*x))\*sin(2\*a/b)+Ci(2\*a/b+2\*arcsec(c\*x))\*cos(2\*a/b))/b^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4\sqrt{cx+1}\sqrt{cx-1}\left(b\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)+a\right)+4\left(4b^3x^2\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)^2+b^3x^2\log\left(c^2x^2\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

```
[Out] -(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)
+ (4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2)^
2 + 8*b^3*x^2*log(c)*log(x) + 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log(c)
+ b^3*x^2*log(x))*log(c^2*x^2))*integrate(4*(a*c^2*x^2 + (b*c^2*x^2 - 2*b)*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*(
b^3*c^2*log(c)^2 + a^2*b*c^2)*x^5 - 4*(b^3*log(c)^2 + a^2*b)*x^3 + 4*(b^3*c
^2*x^5 - b^3*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*c^2*x^5 - b^
3*x^3)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^5 - b^3*x^3)*log(x)^2 + 8*(a*b^2*c^2*x
^5 - a*b^2*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*c^2*x^5*log(c)
- b^3*x^3*log(c) + (b^3*c^2*x^5 - b^3*x^3)*log(x))*log(c^2*x^2) + 8*(b^3*c
^2*x^5*log(c) - b^3*x^3*log(c))*log(x)), x)/(4*b^3*x^2*arctan(sqrt(c*x + 1)
)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2)^2 + 8*b^3*x^2*log(c)*log(x) + 4*b
^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*
log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log(c) + b^3*x^2*log(x))*log(c^2*x^2))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*acos(1/(c*x)))^2), x)
```

```
[Out] int(1/(x^3*(a + b*acos(1/(c*x)))^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{asec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*asec(c*x))**2, x)
```

```
[Out] Integral(1/(x**3*(a + b*asec(c*x))**2), x)
```

$$3.44 \quad \int \frac{1}{x^4(a+b \sec^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=178

$$\frac{c^3 \cos\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3 \cos\left(\frac{3a}{b}\right) \text{Ci}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{4b^2} + \frac{c^3 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3 \sin\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{4b^2}$$

[Out]  $1/4*c^3*\text{Ci}(a/b+\text{arcsec}(c*x))*\cos(a/b)/b^2+3/4*c^3*\text{Ci}(3*a/b+3*\text{arcsec}(c*x))*\cos(3*a/b)/b^2+1/4*c^3*\text{Si}(a/b+\text{arcsec}(c*x))*\sin(a/b)/b^2+3/4*c^3*\text{Si}(3*a/b+3*\text{arcsec}(c*x))*\sin(3*a/b)/b^2-1/4*c^3*\sin(3*\text{arcsec}(c*x))/b/(a+b*\text{arcsec}(c*x))-1/4*c^3*(1-1/c^2/x^2)^{(1/2)}/b/(a+b*\text{arcsec}(c*x))$

**Rubi [A]** time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4406, 3297, 3303, 3299, 3302}

$$\frac{c^3 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{4b^2} + \frac{c^3 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*ArcSec[c\*x])^2), x]

[Out]  $-(c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*b*(a + b*\text{ArcSec}[c*x])) + (c^3*\text{Cos}[a/b]*\text{CosIntegral}[a/b + \text{ArcSec}[c*x]])/(4*b^2) + (3*c^3*\text{Cos}[(3*a)/b]*\text{CosIntegral}[(3*a)/b + 3*\text{ArcSec}[c*x]])/(4*b^2) - (c^3*\text{Sin}[3*\text{ArcSec}[c*x]])/(4*b*(a + b*\text{ArcSec}[c*x])) + (c^3*\text{Sin}[a/b]*\text{SinIntegral}[a/b + \text{ArcSec}[c*x]])/(4*b^2) + (3*c^3*\text{Sin}[(3*a)/b]*\text{SinIntegral}[(3*a)/b + 3*\text{ArcSec}[c*x]])/(4*b^2)$

**Rule 3297**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

**Rule 3299**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 3302**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n * Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n * Sec[x]^(m + 1) * Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^2} dx &= c^3 \text{Subst} \left( \int \frac{\cos^2(x) \sin(x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right) \\
&= c^3 \text{Subst} \left( \int \left( \frac{\sin(x)}{4(a + bx)^2} + \frac{\sin(3x)}{4(a + bx)^2} \right) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{1}{4} c^3 \text{Subst} \left( \int \frac{\sin(x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right) + \frac{1}{4} c^3 \text{Subst} \left( \int \frac{\sin(3x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{4b (a + b \sec^{-1}(cx))} - \frac{c^3 \sin(3 \sec^{-1}(cx))}{4b (a + b \sec^{-1}(cx))} + \frac{c^3 \text{Subst} \left( \int \frac{\cos(x)}{a + bx} dx, x, \sec^{-1}(cx) \right)}{4b} \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{4b (a + b \sec^{-1}(cx))} - \frac{c^3 \sin(3 \sec^{-1}(cx))}{4b (a + b \sec^{-1}(cx))} + \frac{(c^3 \cos(\frac{a}{b})) \text{Subst} \left( \int \frac{\cos(\frac{a}{b} + x)}{a + bx} dx, x, \sec^{-1}(cx) \right)}{4b} \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{4b (a + b \sec^{-1}(cx))} + \frac{c^3 \cos(\frac{a}{b}) \text{Ci}(\frac{a}{b} + \sec^{-1}(cx))}{4b^2} + \frac{3c^3 \cos(\frac{3a}{b}) \text{Ci}(\frac{3a}{b} + 3 \sec^{-1}(cx))}{4b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.48, size = 223, normalized size = 1.25

$$\frac{c^3 x^2 \cos\left(\frac{a}{b}\right) (a + b \sec^{-1}(cx)) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right) + 3c^3 x^2 \cos\left(\frac{3a}{b}\right) (a + b \sec^{-1}(cx)) \text{Ci}\left(3\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right) + ac^3 x^2}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*ArcSec[c\*x])^2), x]

[Out] (-4\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)] + c^3\*x^2\*(a + b\*ArcSec[c\*x])\*Cos[a/b]\*CosIntegral[a/b + ArcSec[c\*x]] + 3\*c^3\*x^2\*(a + b\*ArcSec[c\*x])\*Cos[(3\*a)/b]\*CosIntegral[3\*(a/b + ArcSec[c\*x])] + a\*c^3\*x^2\*Sin[a/b]\*SinIntegral[a/b + ArcSec[c\*x]] + b\*c^3\*x^2\*ArcSec[c\*x]\*Sin[a/b]\*SinIntegral[a/b + ArcSec[c\*x]] + 3\*a\*c^3\*x^2\*Sin[(3\*a)/b]\*SinIntegral[3\*(a/b + ArcSec[c\*x])] + 3\*b\*c^3\*x^2\*ArcSec[c\*x]\*Sin[(3\*a)/b]\*SinIntegral[3\*(a/b + ArcSec[c\*x])])/(4\*b^2\*x^2\*(a + b\*ArcSec[c\*x]))

**fricas [F]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 x^4 \text{arcsec}(cx)^2 + 2 ab x^4 \text{arcsec}(cx) + a^2 x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^4\*arcsec(c\*x)^2 + 2\*a\*b\*x^4\*arcsec(c\*x) + a^2\*x^4), x)

**giac** [B] time = 0.22, size = 694, normalized size = 3.90

$$\frac{1}{4} \left( \frac{12bc^2 \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{12bc^2 \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] 1/4\*(12\*b\*c^2\*arccos(1/(c\*x))\*cos(a/b)^3\*cos\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + 12\*b\*c^2\*arccos(1/(c\*x))\*cos(a/b)^2\*sin(a/b)\*sin\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + 12\*a\*c^2\*cos(a/b)^3\*cos\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + 12\*a\*c^2\*cos(a/b)^2\*sin(a/b)\*sin\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - 9\*b\*c^2\*arccos(1/(c\*x))\*cos(a/b)\*cos\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + b\*c^2\*arccos(1/(c\*x))\*cos(a/b)\*cos\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - 3\*b\*c^2\*arccos(1/(c\*x))\*sin(a/b)\*sin\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + b\*c^2\*arccos(1/(c\*x))\*sin(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - 9\*a\*c^2\*cos(a/b)\*cos\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + a\*c^2\*cos(a/b)\*cos\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - 3\*a\*c^2\*sin(a/b)\*sin\_integral(3\*a/b + 3\*arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) + a\*c^2\*sin(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^3\*arccos(1/(c\*x)) + a\*b^2) - 4\*b\*sqrt(-1/(c^2\*x^2) + 1)/((b^3\*arccos(1/(c\*x)) + a\*b^2)\*x^2))\*c

**maple** [A] time = 0.11, size = 153, normalized size = 0.86

$$c^3 \left( -\frac{\sin(3 \operatorname{arcsec}(cx))}{4(a+b \operatorname{arcsec}(cx))b} + \frac{3 \operatorname{Si}\left(\frac{3a}{b} + 3 \operatorname{arcsec}(cx)\right) \sin\left(\frac{3a}{b}\right)}{4} + \frac{3 \operatorname{Ci}\left(\frac{3a}{b} + 3 \operatorname{arcsec}(cx)\right) \cos\left(\frac{3a}{b}\right)}{4b^2} - \frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4(a+b \operatorname{arcsec}(cx))b} + \frac{\operatorname{Si}\left(\frac{3a}{b} + 3 \operatorname{arcsec}(cx)\right)}{4b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a+b\*arcsec(c\*x))^2,x)

[Out] c^3\*(-1/4\*sin(3\*arcsec(c\*x))/(a+b\*arcsec(c\*x))/b+3/4\*(Si(3\*a/b+3\*arcsec(c\*x))\*sin(3\*a/b)+Ci(3\*a/b+3\*arcsec(c\*x))\*cos(3\*a/b))/b^2-1/4\*((c^2\*x^2-1)/c^2/

$x^2)^{(1/2)/(a+b*\text{arcsec}(c*x))/b+1/4*(\text{Si}(a/b+\text{arcsec}(c*x))*\sin(a/b)+\text{Ci}(a/b+\text{arcsec}(c*x))*\cos(a/b))/b^2)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4\sqrt{cx+1}\sqrt{cx-1}\left(b\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)+a\right)+4\left(4b^3x^3\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)^2+b^3x^3\log\left(c^2x^2\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out]  $-(4*\sqrt{c*x+1}*\sqrt{c*x-1}*(b*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}))+a)+ (4*b^3*x^3*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}))^2 + b^3*x^3*\log(c^2*x^2)^2 + 8*b^3*x^3*\log(c)*\log(x) + 4*b^3*x^3*\log(x)^2 + 8*a*b^2*x^3*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}) + 4*(b^3*\log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*\log(c) + b^3*x^3*\log(x))*\log(c^2*x^2)) * \text{integrate}(4*(2*a*c^2*x^2 + (2*b*c^2*x^2 - 3*b)*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1})) - 3*a)*\sqrt{c*x+1}*\sqrt{c*x-1} / (4*(b^3*c^2*\log(c)^2 + a^2*b*c^2)*x^6 - 4*(b^3*\log(c)^2 + a^2*b)*x^4 + 4*(b^3*c^2*x^6 - b^3*x^4)*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}))^2 + (b^3*c^2*x^6 - b^3*x^4)*\log(c^2*x^2)^2 + 4*(b^3*c^2*x^6 - b^3*x^4)*\log(x)^2 + 8*(a*b^2*c^2*x^6 - a*b^2*x^4)*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}) - 4*(b^3*c^2*x^6*\log(c) - b^3*x^4*\log(c) + (b^3*c^2*x^6 - b^3*x^4)*\log(x))*\log(c^2*x^2) + 8*(b^3*c^2*x^6*\log(c) - b^3*x^4*\log(c))*\log(x)), x) / (4*b^3*x^3*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}))^2 + b^3*x^3*\log(c^2*x^2)^2 + 8*b^3*x^3*\log(c)*\log(x) + 4*b^3*x^3*\log(x)^2 + 8*a*b^2*x^3*\arctan(\sqrt{c*x+1}*\sqrt{c*x-1}) + 4*(b^3*\log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*\log(c) + b^3*x^3*\log(x))*\log(c^2*x^2))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4\*(a + b\*acos(1/(c\*x))))^2,x)

[Out] int(1/(x^4\*(a + b\*acos(1/(c\*x))))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \operatorname{asec}(cx))^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

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[In] integrate(1/x**4/(a+b*asec(c*x))**2,x)
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[Out] Integral(1/(x**4*(a + b*asec(c*x))**2), x)
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$$3.45 \quad \int \frac{x}{(a+b \sec^{-1}(cx))^3} dx$$

Optimal. Leaf size=15

$$\text{Int} \left( \frac{x}{(a+b \sec^{-1}(cx))^3}, x \right)$$

[Out] Unintegrable(x/(a+b\*arcsec(c\*x))^3,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x}{(a+b \sec^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b\*ArcSec[c\*x])^3,x]

[Out] Defer[Int][x/(a + b\*ArcSec[c\*x])^3, x]

Rubi steps

$$\int \frac{x}{(a+b \sec^{-1}(cx))^3} dx = \int \frac{x}{(a+b \sec^{-1}(cx))^3} dx$$

Mathematica [A] time = 3.68, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b \sec^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b\*ArcSec[c\*x])^3,x]

[Out] Integrate[x/(a + b\*ArcSec[c\*x])^3, x]

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x}{b^3 \text{arcsec}(cx)^3 + 3ab^2 \text{arcsec}(cx)^2 + 3a^2b \text{arcsec}(cx) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(x/(b^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*arcsec(c\*x) + a^3), x)

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] Timed out

**maple** [A] time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arcsec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arcsec(c\*x))^3,x)

[Out] int(x/(a+b\*arcsec(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $-(24*(a*b^2*c^2*\log(c)^2 + a^3*c^2)*x^4 + 8*(3*b^3*c^2*x^4 - 2*b^3*x^2)*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))^3 - 16*(a*b^2*\log(c)^2 + a^3)*x^2 + 24*(3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})^2 + 2*(3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*\log(c^2*x^2)^2 + 8*(3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*\log(x)^2 + 2*(4*b^3*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))^2 - b^3*x^2*\log(c^2*x^2)^2 - 8*b^3*x^2*\log(c)*\log(x) - 4*b^3*x^2*\log(x)^2 + 8*a*b^2*x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}) - 4*(b^3*\log(c)^2 - a^2*b)*x^2 + 4*(b^3*x^2*\log(c) + b^3*x^2*\log(x))*\log(c^2*x^2)*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 2*(12*(b^3*c^2*\log(c)^2 + 3*a^2*b*c^2)*x^4 - 8*(b^3*\log(c)^2 + 3*a^2*b)*x^2 + (3*b^3*c^2*x^4 - 2*b^3*x^2)*\log(c^2*x^2)^2 + 4*(3*b^3*c^2*x^4 - 2*b^3*x^2)*\log(x)^2 - 4*(3*b^3*c^2*x^4*\log(c) - 2*b^3*x^2*\log(c) + (3*b^3*c^2*x^4 - 2*b^3*x^2)*\log(x))*\log(c^2*x^2) + 8*(3*b^3*c^2*x^4*\log(c) - 2*b^3*x^2*\log$

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(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c
)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2
*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*
b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x)
+ 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*
log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log
(x) + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^
2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(
c)*log(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*lo
g(x))*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*log(x))*integrate(8
*(3*a*c^2*x^3 - a*x + (3*b*c^2*x^3 - b*x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
)))/(4*b^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^4*log(c^2*x^2)^2 + 4*b
^4*log(c)^2 + 8*b^4*log(c)*log(x) + 4*b^4*log(x)^2 + 8*a*b^3*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)) + 4*a^2*b^2 - 4*(b^4*log(c) + b^4*log(x))*log(c^2*x^2
)), x) - 8*(3*a*b^2*c^2*x^4*log(c) - 2*a*b^2*x^2*log(c) + (3*a*b^2*c^2*x^4
- 2*a*b^2*x^2)*log(x))*log(c^2*x^2) + 16*(3*a*b^2*c^2*x^4*log(c) - 2*a*b^2*
x^2*log(c))*log(x))/(16*b^6*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log
(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 +
64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a
^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2
+ 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(
b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^
2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log
(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^
2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3*
b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*log(x)^2 + b^6*log(x)^3 + a^
2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x))*log(c^2*x^2) + 64*(b^6*lo
g(c)^3 + a^2*b^4*log(c))*log(x))
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**mupad [A]** time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*acos(1/(c\*x)))^3,x)

[Out] int(x/(a + b\*acos(1/(c\*x)))^3, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{asec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*asec(c\*x))\*\*3,x)

[Out] Integral(x/(a + b\*asec(c\*x))\*\*3, x)

$$3.46 \quad \int \frac{1}{(a+b \sec^{-1}(cx))^3} dx$$

Optimal. Leaf size=13

$$\text{Int} \left( \frac{1}{(a+b \sec^{-1}(cx))^3}, x \right)$$

[Out] Unintegrable(1/(a+b\*arcsec(c\*x))^3, x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(a+b \sec^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])^(-3), x]

[Out] Defer[Int][(a + b\*ArcSec[c\*x])^(-3), x]

Rubi steps

$$\int \frac{1}{(a+b \sec^{-1}(cx))^3} dx = \int \frac{1}{(a+b \sec^{-1}(cx))^3} dx$$

Mathematica [A] time = 12.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \sec^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])^(-3), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])^(-3), x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^3 \text{arcsec}(cx)^3 + 3ab^2 \text{arcsec}(cx)^2 + 3a^2b \text{arcsec}(cx) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*arcsec(c\*x) + a^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^(-3), x)

**maple** [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arcsec(c\*x))^3,x)

[Out] int(1/(a+b\*arcsec(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $-(16*(a*b^2*c^2*\log(c)^2 + a^3*c^2)*x^3 + 8*(2*b^3*c^2*x^3 - b^3*x)*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))^3 + 24*(2*a*b^2*c^2*x^3 - a*b^2*x)*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})^2 + 2*(2*a*b^2*c^2*x^3 - a*b^2*x)*\log(c^2*x^2)^2 + 8*(2*a*b^2*c^2*x^3 - a*b^2*x)*\log(x)^2 + 2*(4*b^3*x*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})^2 - b^3*x*\log(c^2*x^2)^2 - 8*b^3*x*\log(c)*\log(x) - 4*b^3*x*\log(x)^2 + 8*a*b^2*x*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}) - 4*(b^3*\log(c)^2 - a^2*b)*x + 4*(b^3*x*\log(c) + b^3*x*\log(x))*\log(c^2*x^2))*\sqrt{c*x + 1}*\sqrt{c*x - 1} - 8*(a*b^2*\log(c)^2 + a^3)*x + 2*(8*(b^3*c^2*\log(c)^2 + 3*a^2*b*c^2)*x^3 + (2*b^3*c^2*x^3 - b^3*x)*\log(c^2*x^2)^2 + 4*(2*b^3*c^2*x^3 - b^3*x)*\log(x)^2 - 4*(b^3*\log(c)^2 + 3*a^2*b)*x - 4*(2*b^3*c^2*x^3*\log(c) - b^3*x*\log(c) + (2*b^3*c^2*x^3 - b^3*x)*\log(x))*\log(c^2*x^2) + 8*(2*b^3*c^2*x^3*1$

```

og(c) - b^3*x*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)
^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^
6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*1
og(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*lo
g(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b
^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log
(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8
*a*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b
^5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(
c)^3 + 3*b^6*log(c)*log(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)
)^2 + a^2*b^4)*log(x))*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*lo
g(x))*integrate(2*(6*a*c^2*x^2 + (6*b*c^2*x^2 - b)*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1)) - a)/(4*b^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^4*log(c^2
*x^2)^2 + 4*b^4*log(c)^2 + 8*b^4*log(c)*log(x) + 4*b^4*log(x)^2 + 8*a*b^3*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b^2 - 4*(b^4*log(c) + b^4*log(x)
)*log(c^2*x^2)), x) - 8*(2*a*b^2*c^2*x^3*log(c) - a*b^2*x*log(c) + (2*a*b^2
*c^2*x^3 - a*b^2*x)*log(x))*log(c^2*x^2) + 16*(2*a*b^2*c^2*x^3*log(c) - a*b
^2*x*log(c))*log(x))/(16*b^6*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*lo
g(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 +
64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*
a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^
2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*
(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*lo
g(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x
^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3
*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*log(x)^2 + b^6*log(x)^3 + a
^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x))*log(c^2*x^2) + 64*(b^6*1
og(c)^3 + a^2*b^4*log(c))*log(x))

```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*acos(1/(c\*x)))^3,x)

[Out] int(1/(a + b\*acos(1/(c\*x)))^3, x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*asec(c\*x))\*\*3,x)

[Out] Integral((a + b\*asec(c\*x))\*\*(-3), x)

$$3.47 \quad \int \frac{1}{x(a+b \sec^{-1}(cx))^3} dx$$

Optimal. Leaf size=17

$$\text{Int} \left( \frac{1}{x(a+b \sec^{-1}(cx))^3}, x \right)$$

[Out] Unintegrable(1/x/(a+b\*arcsec(c\*x))^3,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b \sec^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcSec[c\*x])^3), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcSec[c\*x])^3), x]

Rubi steps

$$\int \frac{1}{x(a+b \sec^{-1}(cx))^3} dx = \int \frac{1}{x(a+b \sec^{-1}(cx))^3} dx$$

Mathematica [A] time = 1.79, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b \sec^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcSec[c\*x])^3), x]

[Out] Integrate[1/(x\*(a + b\*ArcSec[c\*x])^3), x]

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^3 x \operatorname{arcsec}(cx)^3 + 3 a b^2 x \operatorname{arcsec}(cx)^2 + 3 a^2 b x \operatorname{arcsec}(cx) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*x\*arcsec(c\*x)^3 + 3\*a\*b^2\*x\*arcsec(c\*x)^2 + 3\*a^2\*b\*x\*arcsec(c\*x) + a^3\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate(1/((b\*arcsec(c\*x) + a)^3\*x), x)

**maple** [A] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{arcsec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arcsec(c\*x))^3,x)

[Out] int(1/x/(a+b\*arcsec(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $-(8*b^3*c^2*x^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^3 + 24*a*b^2*c^2*x^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 + 2*a*b^2*c^2*x^2*\log(c^2*x^2)^2 + 16*a*b^2*c^2*x^2*\log(c)*\log(x) + 8*a*b^2*c^2*x^2*\log(x)^2 + 8*(a*b^2*c^2*\log(c)^2 + a^3*c^2)*x^2 + 2*(4*b^3*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - b^3*\log(c^2*x^2)^2 - 4*b^3*\log(c)^2 - 8*b^3*\log(c)*\log(x) - 4*b^3*\log(x)^2 + 8*a*b^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) + 4*a^2*b + 4*(b^3*\log(c) + b^3*\log(x))*\log(c^2*x^2))*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 2*(b^3*c^2*x^2*\log(c^2*x^2)^2 + 8*b^3*c^2*x^2*\log(c)*\log(x) + 4*b^3*c^2*x^2*\log(x)^2 + 4*(b^3*c^2*\log(c)^2 + 3*a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2*\log(c) + b^3*c^2*x^2*\log(x))*\log(c^2*x^2))*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) - (16*b^6*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^4 + b^6*\log(c^2*x^2)^4 + 16*b^6*\log(c)^4 + 64*b^6*\log(c)*1$

```

og(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3
+ 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^
2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6
*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3
*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log
(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x)
+ 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^
2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*
log(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x)
)*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*log(x))*integrate(4*(b
*c^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*c^2*x)/(4*b^4*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1))^2 + b^4*log(c^2*x^2)^2 + 4*b^4*log(c)^2 + 8*b^4*log(c
)*log(x) + 4*b^4*log(x)^2 + 8*a*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4
*a^2*b^2 - 4*(b^4*log(c) + b^4*log(x))*log(c^2*x^2)), x) - 8*(a*b^2*c^2*x^2
*log(c) + a*b^2*c^2*x^2*log(x))*log(c^2*x^2))/(16*b^6*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*log
(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 +
32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^2)
^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*1
og(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b
^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)
^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x) +
4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^2
))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*lo
g(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x)
)*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*log(x))

```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*acos(1/(c\*x)))^3),x)

[Out] int(1/(x\*(a + b\*acos(1/(c\*x)))^3), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x (a + b \operatorname{asec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asec(c*x))**3,x)
```

```
[Out] Integral(1/(x*(a + b*asec(c*x))**3), x)
```

$$3.48 \quad \int \frac{1}{x^2(a+b \sec^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=103

$$\frac{c \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{2b^3} - \frac{c \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{2b^3} - \frac{1}{2b^2 x (a + b \sec^{-1}(cx))} - \frac{c \sqrt{1 - \frac{1}{c^2 x^2}}}{2b (a + b \sec^{-1}(cx))^2}$$

[Out]  $-1/2/b^2/x/(a+b*\text{arcsec}(c*x))-1/2*c*\cos(a/b)*\text{Si}(a/b+\text{arcsec}(c*x))/b^3+1/2*c*\text{Ci}(a/b+\text{arcsec}(c*x))*\sin(a/b)/b^3-1/2*c*(1-1/c^2/x^2)^{(1/2)}/b/(a+b*\text{arcsec}(c*x))^2$

**Rubi [A]** time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5222, 3297, 3303, 3299, 3302}

$$\frac{c \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{2b^3} - \frac{c \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{2b^3} - \frac{1}{2b^2 x (a + b \sec^{-1}(cx))} - \frac{c \sqrt{1 - \frac{1}{c^2 x^2}}}{2b (a + b \sec^{-1}(cx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(a + b*\text{ArcSec}[c*x])^3), x]$

[Out]  $-(c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(2*b*(a + b*\text{ArcSec}[c*x])^2) - 1/(2*b^2*x*(a + b*\text{ArcSec}[c*x])) + (c*\text{CosIntegral}[a/b + \text{ArcSec}[c*x]]*\text{Sin}[a/b])/(2*b^3) - (c*\text{Cos}[a/b]*\text{SinIntegral}[a/b + \text{ArcSec}[c*x]])/(2*b^3)$

**Rule 3297**

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\sin\left[(e_.) + (f_.)*(x_.)\right], x\_Symbol] \rightarrow \text{Simp}[\left((c + d*x)^{(m + 1)}*\text{Sin}[e + f*x]\right)/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[\left((c + d*x)^{(m + 1)}*\text{Cos}[e + f*x], x\right), x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

**Rule 3299**

$\text{Int}[\sin\left[(e_.) + (f_.)*(x_.)\right]/\left((c_.) + (d_.)*(x_.)\right), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

**Rule 3302**

$\text{Int}[\sin\left[(e_.) + (f_.)*(x_.)\right]/\left((c_.) + (d_.)*(x_.)\right), x\_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{EqQ}[d*(e - \text{Pi}/2) -$

$c*f, 0]$

### Rule 3303

$\text{Int}[\sin[(e\_.) + (f\_.)*(x\_)]/((c\_.) + (d\_.)*(x\_)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{NeQ}[d*e - c*f, 0]$

### Rule 5222

$\text{Int}[(a\_.) + \text{ArcSec}[(c\_.)*(x\_)]*(b\_.)]^{(n\_)}*(x\_)^{(m\_)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x]^{(m+1)}*\text{Tan}[x], x], x, \text{ArcSec}[c*x]], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[m] \&\& (\text{GtQ}[n, 0] \parallel \text{LtQ}[m, -1])$

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \sec^{-1}(cx))^3} dx &= c \text{Subst} \left( \int \frac{\sin(x)}{(a + bx)^3} dx, x, \sec^{-1}(cx) \right) \\ &= -\frac{c\sqrt{1 - \frac{1}{c^2x^2}}}{2b(a + b \sec^{-1}(cx))^2} + \frac{c \text{Subst} \left( \int \frac{\cos(x)}{(a+bx)^2} dx, x, \sec^{-1}(cx) \right)}{2b} \\ &= -\frac{c\sqrt{1 - \frac{1}{c^2x^2}}}{2b(a + b \sec^{-1}(cx))^2} - \frac{1}{2b^2x(a + b \sec^{-1}(cx))} - \frac{c \text{Subst} \left( \int \frac{\sin(x)}{a+bx} dx, x, \sec^{-1}(cx) \right)}{2b^2} \\ &= -\frac{c\sqrt{1 - \frac{1}{c^2x^2}}}{2b(a + b \sec^{-1}(cx))^2} - \frac{1}{2b^2x(a + b \sec^{-1}(cx))} - \frac{(c \cos(\frac{a}{b})) \text{Subst} \left( \int \frac{\sin(\frac{a}{b}+x)}{a+bx} dx \right)}{2b^2} \\ &= -\frac{c\sqrt{1 - \frac{1}{c^2x^2}}}{2b(a + b \sec^{-1}(cx))^2} - \frac{1}{2b^2x(a + b \sec^{-1}(cx))} + \frac{c \text{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) \sin \left( \frac{a}{b} \right)}{2b^3} \end{aligned}$$

**Mathematica [A]** time = 0.39, size = 88, normalized size = 0.85

$$\frac{b \left( a + bcx \sqrt{1 - \frac{1}{c^2x^2}} + b \sec^{-1}(cx) \right)}{x(a + b \sec^{-1}(cx))^2} - c \sin \left( \frac{a}{b} \right) \text{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right) + c \cos \left( \frac{a}{b} \right) \text{Si} \left( \frac{a}{b} + \sec^{-1}(cx) \right)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*ArcSec[c\*x]))^3,x]

[Out] -1/2\*((b\*(a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + b\*ArcSec[c\*x]))/(x\*(a + b\*ArcSec[c\*x]))^2 - c\*CosIntegral[a/b + ArcSec[c\*x]]\*Sin[a/b] + c\*Cos[a/b]\*SinIntegral[a/b + ArcSec[c\*x]])/b^3

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^2 \operatorname{arcsec}(cx)^3 + 3ab^2x^2 \operatorname{arcsec}(cx)^2 + 3a^2bx^2 \operatorname{arcsec}(cx) + a^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*x^2\*arcsec(c\*x)^3 + 3\*a\*b^2\*x^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*x^2\*arcsec(c\*x) + a^3\*x^2), x)

**giac** [B] time = 0.14, size = 580, normalized size = 5.63

$$\frac{1}{2} \left( \frac{b^2 \arccos\left(\frac{1}{cx}\right)^2 \operatorname{Ci}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right) \sin\left(\frac{a}{b}\right) - b^2 \arccos\left(\frac{1}{cx}\right)^2 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^5 \arccos\left(\frac{1}{cx}\right)^2 + 2ab^4 \arccos\left(\frac{1}{cx}\right) + a^2b^3} - \frac{b^2 \arccos\left(\frac{1}{cx}\right)^2 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b^5 \arccos\left(\frac{1}{cx}\right)^2 + 2ab^4 \arccos\left(\frac{1}{cx}\right) + a^2b^3} + \frac{2ab \arccos\left(\frac{1}{cx}\right)}{b^5 \arccos\left(\frac{1}{cx}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] 1/2\*(b^2\*arccos(1/(c\*x))^2\*cos\_integral(a/b + arccos(1/(c\*x)))\*sin(a/b)/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) - b^2\*arccos(1/(c\*x))^2\*cos(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) + 2\*a\*b\*arccos(1/(c\*x))\*cos\_integral(a/b + arccos(1/(c\*x)))\*sin(a/b)/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) - 2\*a\*b\*arccos(1/(c\*x))\*cos(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) + a^2\*cos\_integral(a/b + arccos(1/(c\*x)))\*sin(a/b)/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) - a^2\*cos(a/b)\*sin\_integral(a/b + arccos(1/(c\*x)))/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) - b^2\*sqrt(-1/(c^2\*x^2) + 1)/(b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3) - b^2\*arccos(1/(c\*x))/((b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3)\*c\*x) - a\*b/((b^5\*arccos(1/(c\*x))^2 + 2\*a\*b^4\*arccos(1/(c\*x)) + a^2\*b^3)\*c\*x))\*c



**maple** [A] time = 0.16, size = 154, normalized size = 1.50

$$c \left( \frac{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{2(a + b \operatorname{arcsec}(cx))^2 b} - \frac{\operatorname{arcsec}(cx) \operatorname{Si}\left(\frac{a}{b} + \operatorname{arcsec}(cx)\right) \cos\left(\frac{a}{b}\right) cxb - \operatorname{arcsec}(cx) \operatorname{Ci}\left(\frac{a}{b} + \operatorname{arcsec}(cx)\right) \sin\left(\frac{a}{b}\right)}{2cx(a + b \operatorname{arcsec}(cx))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arcsec(c\*x))^3,x)

[Out] c\*(-1/2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)/(a+b\*arcsec(c\*x))^2/b-1/2\*(arcsec(c\*x)\*Si(a/b+arcsec(c\*x))\*cos(a/b)\*c\*x\*b-arcsec(c\*x)\*Ci(a/b+arcsec(c\*x))\*sin(a/b)\*c\*x\*b+Si(a/b+arcsec(c\*x))\*cos(a/b)\*c\*x\*a-Ci(a/b+arcsec(c\*x))\*sin(a/b)\*c\*x\*(a+b)/c/x/(a+b\*arcsec(c\*x))/b^3)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out] -(8\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^3 + 24\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + 2\*a\*b^2\*log(c^2\*x^2)^2 + 8\*a\*b^2\*log(c)^2 + 16\*a\*b^2\*log(c)\*log(x) + 8\*a\*b^2\*log(x)^2 + 8\*a^3 + 2\*(4\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 - b^3\*log(c^2\*x^2)^2 - 4\*b^3\*log(c)^2 - 8\*b^3\*log(c)\*log(x) - 4\*b^3\*log(x)^2 + 8\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*a^2\*b + 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) + 2\*(b^3\*log(c^2\*x^2)^2 + 4\*b^3\*log(c)^2 + 8\*b^3\*log(c)\*log(x) + 4\*b^3\*log(x)^2 + 12\*a^2\*b - 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + (16\*b^6\*x\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^4 + b^6\*x\*log(c^2\*x^2)^4 + 64\*b^6\*x\*log(c)\*log(x)^3 + 16\*b^6\*x\*log(x)^4 + 64\*a\*b^5\*x\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^3 - 8\*(b^6\*x\*log(c) + b^6\*x\*log(x))\*log(c^2\*x^2)^3 + 32\*(3\*b^6\*log(c)^2 + a^2\*b^4)\*x\*log(x)^2 + 8\*(b^6\*x\*log(c^2\*x^2)^2 + 8\*b^6\*x\*log(c)\*log(x) + 4\*b^6\*x\*log(x)^2 + 4\*(b^6\*log(c)^2 + 3\*a^2\*b^4)\*x - 4\*(b^6\*x\*log(c) + b^6\*x\*log(x))\*log(c^2\*x^2))\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + 8\*(6\*b^6\*x\*log(c)\*log(x) + 3\*b^6\*x\*log(x)^2 + (3\*b^6\*log(c)^2 + a^2\*b^4)\*x)\*log(c^2\*x^2)^2 + 64\*(b^6\*log(c)^3 + a^2\*b^4\*log(c))\*x\*log(x) + 16\*(b^6\*log(c)^4 + 2\*a^2\*b^4\*log(c)^2 + a^4\*b^2)\*x + 16\*(a\*b^5\*x\*log(c^2\*x^2)^2 + 8\*a\*b^5\*x\*log(c)\*log(x) + 4\*a\*b^5\*x\*log(x)^2 + 4\*(a\*b^5\*log(c)^2 + a^3\*b^3)\*x - 4\*(a\*b^5\*x\*log(c) + a\*b^5\*x\*log(x))\*log(c^2\*x^2))\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - 32\*(3\*b^6\*x\*log(c)\*log(x)^2 + b^6\*x\*log(x)^3 + (3\*b^6\*log(c)^2 + a^2\*b^4)\*x\*log(x) + (b^6\*log(c)^3 + a^2\*b^4\*log(c))\*x)\*log(c^2\*x^2))\*integrate(2\*(b\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + a)/(4\*b^4\*x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + b^4\*x^2\*log(c^2\*x^2)^2 + 8

```

*b^4*x^2*log(c)*log(x) + 4*b^4*x^2*log(x)^2 + 8*a*b^3*x^2*arctan(sqrt(c*x +
1)*sqrt(c*x - 1)) + 4*(b^4*log(c)^2 + a^2*b^2)*x^2 - 4*(b^4*x^2*log(c) + b
^4*x^2*log(x))*log(c^2*x^2), x) - 8*(a*b^2*log(c) + a*b^2*log(x))*log(c^2*
x^2))/(16*b^6*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x*log(c^2*x^2)^
4 + 64*b^6*x*log(c)*log(x)^3 + 16*b^6*x*log(x)^4 + 64*a*b^5*x*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))^3 - 8*(b^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2)^3 +
32*(3*b^6*log(c)^2 + a^2*b^4)*x*log(x)^2 + 8*(b^6*x*log(c^2*x^2)^2 + 8*b^6
*x*log(c)*log(x) + 4*b^6*x*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x - 4*(b
^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))^2 + 8*(6*b^6*x*log(c)*log(x) + 3*b^6*x*log(x)^2 + (3*b^6*log(c)^2 + a^2*
b^4)*x)*log(c^2*x^2)^2 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x*log(x) + 16*(
b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x + 16*(a*b^5*x*log(c^2*x^2)^2
+ 8*a*b^5*x*log(c)*log(x) + 4*a*b^5*x*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b
^3)*x - 4*(a*b^5*x*log(c) + a*b^5*x*log(x))*log(c^2*x^2))*arctan(sqrt(c*x +
1)*sqrt(c*x - 1)) - 32*(3*b^6*x*log(c)*log(x)^2 + b^6*x*log(x)^3 + (3*b^6*
log(c)^2 + a^2*b^4)*x*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x)*log(c^2*x
^2))

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*acos(1/(c\*x)))^3), x)

[Out] int(1/(x^2\*(a + b\*acos(1/(c\*x)))^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{asec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*asec(c\*x))\*\*3, x)

[Out] Integral(1/(x\*\*2\*(a + b\*asec(c\*x))\*\*3), x)

$$3.49 \quad \int \frac{1}{x^3(a+b \sec^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=112

$$\frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Ci}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^3} - \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^3} - \frac{c^2 \cos\left(2 \sec^{-1}(cx)\right)}{2b^2(a+b \sec^{-1}(cx))} - \frac{c^2 \sin\left(2 \sec^{-1}(cx)\right)}{4b(a+b \sec^{-1}(cx))^2}$$

[Out]  $-1/2*c^2*\cos(2*\text{arcsec}(c*x))/b^2/(a+b*\text{arcsec}(c*x))-c^2*\cos(2*a/b)*\text{Si}(2*a/b+2*\text{arcsec}(c*x))/b^3+c^2*\text{Ci}(2*a/b+2*\text{arcsec}(c*x))*\sin(2*a/b)/b^3-1/4*c^2*\sin(2*\text{arcsec}(c*x))/b/(a+b*\text{arcsec}(c*x))^2$

**Rubi [A]** time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5222, 4406, 12, 3297, 3303, 3299, 3302}

$$\frac{c^2 \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^3} - \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right)}{b^3} - \frac{c^2 \cos\left(2 \sec^{-1}(cx)\right)}{2b^2(a+b \sec^{-1}(cx))} - \frac{c^2 \sin\left(2 \sec^{-1}(cx)\right)}{4b(a+b \sec^{-1}(cx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(a + b*\text{ArcSec}[c*x])^3), x]$

[Out]  $-(c^2*\text{Cos}[2*\text{ArcSec}[c*x]])/(2*b^2*(a + b*\text{ArcSec}[c*x])) + (c^2*\text{CosIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]]*\text{Sin}[(2*a)/b])/b^3 - (c^2*\text{Sin}[2*\text{ArcSec}[c*x]])/(4*b*(a + b*\text{ArcSec}[c*x])^2) - (c^2*\text{Cos}[(2*a)/b]*\text{SinIntegral}[(2*a)/b + 2*\text{ArcSec}[c*x]])/b^3$

**Rule 12**

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_) /; \text{FreeQ}[b, x]]$

**Rule 3297**

$\text{Int}[(c_*) + (d_*)*(x_*)]^{(m_*)}*\sin[(e_*) + (f_*)*(x_)], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\text{Sin}[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

**Rule 3299**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

### Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^(n)*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (a + b \sec^{-1}(cx))^3} dx &= c^2 \text{Subst} \left( \int \frac{\cos(x) \sin(x)}{(a + bx)^3} dx, x, \sec^{-1}(cx) \right) \\
&= c^2 \text{Subst} \left( \int \frac{\sin(2x)}{2(a + bx)^3} dx, x, \sec^{-1}(cx) \right) \\
&= \frac{1}{2} c^2 \text{Subst} \left( \int \frac{\sin(2x)}{(a + bx)^3} dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{c^2 \sin(2 \sec^{-1}(cx))}{4b (a + b \sec^{-1}(cx))^2} + \frac{c^2 \text{Subst} \left( \int \frac{\cos(2x)}{(a+bx)^2} dx, x, \sec^{-1}(cx) \right)}{2b} \\
&= -\frac{c^2 \cos(2 \sec^{-1}(cx))}{2b^2 (a + b \sec^{-1}(cx))} - \frac{c^2 \sin(2 \sec^{-1}(cx))}{4b (a + b \sec^{-1}(cx))^2} - \frac{c^2 \text{Subst} \left( \int \frac{\sin(2x)}{a+bx} dx, x, \sec^{-1}(cx) \right)}{b^2} \\
&= -\frac{c^2 \cos(2 \sec^{-1}(cx))}{2b^2 (a + b \sec^{-1}(cx))} - \frac{c^2 \sin(2 \sec^{-1}(cx))}{4b (a + b \sec^{-1}(cx))^2} - \frac{\left( c^2 \cos\left(\frac{2a}{b}\right) \right) \text{Subst} \left( \int \frac{\sin\left(\frac{2a}{b} + 2x\right)}{a+bx} dx \right)}{b^2} \\
&= -\frac{c^2 \cos(2 \sec^{-1}(cx))}{2b^2 (a + b \sec^{-1}(cx))} + \frac{c^2 \text{Ci}\left(\frac{2a}{b} + 2 \sec^{-1}(cx)\right) \sin\left(\frac{2a}{b}\right)}{b^3} - \frac{c^2 \sin(2 \sec^{-1}(cx))}{4b (a + b \sec^{-1}(cx))^2}
\end{aligned}$$

**Mathematica [A]** time = 0.38, size = 114, normalized size = 1.02

$$\frac{-\frac{b^2 c \sqrt{1 - \frac{1}{c^2 x^2}}}{x(a + b \sec^{-1}(cx))^2} + 2c^2 \left( \sin\left(\frac{2a}{b}\right) \text{Ci}\left(2\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right) - \cos\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right) \right) + \frac{b(c^2 x^2 - 2)}{x^2(a + b \sec^{-1}(cx))}}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*ArcSec[c\*x])^3), x]

[Out]  $\left( -\frac{(b^2 c \sqrt{1 - 1/(c^2 x^2)})}{x(a + b \text{ArcSec}[c x])^2} + (b(-2 + c^2 x^2))/(x^2(a + b \text{ArcSec}[c x])) + 2c^2(\text{CosIntegral}[2(a/b + \text{ArcSec}[c x])] * \text{Sin}[(2a)/b] - \text{Cos}[(2a)/b] * \text{SinIntegral}[2(a/b + \text{ArcSec}[c x])]) \right) / (2b^3)$

**fricas [F]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^3 x^3 \text{arcsec}(cx)^3 + 3ab^2 x^3 \text{arcsec}(cx)^2 + 3a^2 b x^3 \text{arcsec}(cx) + a^3 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*x^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*x^3\*arcsec(c\*x)^2 + 3\*a^2\*b\*x^3\*arcsec(c\*x) + a^3\*x^3), x)

**giac** [B] time = 0.14, size = 929, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] 
$$\frac{1}{2} \cdot (4b^2c \arccos(1/(cx))^2 \cos(a/b) \cos\_integral(2a/b + 2\arccos(1/(cx))) \sin(a/b) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) - 4b^2c \arccos(1/(cx))^2 \cos(a/b)^2 \sin\_integral(2a/b + 2\arccos(1/(cx))) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + 8ab^2c \arccos(1/(cx)) \cos(a/b) \cos\_integral(2a/b + 2\arccos(1/(cx))) \sin(a/b) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) - 8ab^2c \arccos(1/(cx)) \cos(a/b)^2 \sin\_integral(2a/b + 2\arccos(1/(cx))) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + 4a^2c \cos(a/b) \cos\_integral(2a/b + 2\arccos(1/(cx))) \sin(a/b) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + 2b^2c \arccos(1/(cx))^2 \sin\_integral(2a/b + 2\arccos(1/(cx))) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) - 4a^2c \cos(a/b)^2 \sin\_integral(2a/b + 2\arccos(1/(cx))) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + 4ab^2c \arccos(1/(cx)) \sin\_integral(2a/b + 2\arccos(1/(cx))) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + b^2c \arccos(1/(cx)) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + 2a^2c \sin\_integral(2a/b + 2\arccos(1/(cx))) / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) + abc / (b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3) - b^2 \sqrt{-1/(c^2x^2 + 1)} / ((b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3)x) - 2b^2 \arccos(1/(cx)) / ((b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3)cx^2) - 2ab / ((b^5 \arccos(1/(cx))^2 + 2ab^4 \arccos(1/(cx)) + a^2b^3)cx^2)) * c$$

**maple** [A] time = 0.11, size = 157, normalized size = 1.40

$$c^2 \left[ \frac{\sin(2 \operatorname{arcsec}(cx))}{4(a+b \operatorname{arcsec}(cx))^2 b} - \frac{2 \operatorname{arcsec}(cx) \operatorname{Si}\left(\frac{2a}{b} + 2 \operatorname{arcsec}(cx)\right) \cos\left(\frac{2a}{b}\right) b - 2 \operatorname{arcsec}(cx) \operatorname{Ci}\left(\frac{2a}{b} + 2 \operatorname{arcsec}(cx)\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/x^3/(a+b*\text{arcsec}(c*x))^3,x)$

[Out]  $c^2*(-1/4*\sin(2*\text{arcsec}(c*x))/(a+b*\text{arcsec}(c*x))^2/b-1/2*(2*\text{arcsec}(c*x)*\text{Si}(2*a/b+2*\text{arcsec}(c*x))*\cos(2*a/b)*b-2*\text{arcsec}(c*x)*\text{Ci}(2*a/b+2*\text{arcsec}(c*x))*\sin(2*a/b)*b+2*\text{Si}(2*a/b+2*\text{arcsec}(c*x))*\cos(2*a/b)*a-2*\text{Ci}(2*a/b+2*\text{arcsec}(c*x))*\sin(2*a/b)*a+\cos(2*\text{arcsec}(c*x))*b)/(a+b*\text{arcsec}(c*x))/b^3)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/x^3/(a+b*\text{arcsec}(c*x))^3,x, \text{algorithm}="maxima")$

[Out]  $-(16*a*b^2*\log(c)^2 - 8*(b^3*c^2*x^2 - 2*b^3)*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^3 + 16*a^3 - 8*(a*b^2*c^2*\log(c)^2 + a^3*c^2)*x^2 - 24*(a*b^2*c^2*x^2 - 2*a*b^2)*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - 2*(a*b^2*c^2*x^2 - 2*a*b^2)*\log(c^2*x^2)^2 - 8*(a*b^2*c^2*x^2 - 2*a*b^2)*\log(x)^2 + 2*(4*b^3*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - b^3*\log(c^2*x^2)^2 - 4*b^3*\log(c)^2 - 8*b^3*\log(c)*\log(x) - 4*b^3*\log(x)^2 + 8*a*b^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) + 4*a^2*b + 4*(b^3*\log(c) + b^3*\log(x))*\log(c^2*x^2))*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 2*(8*b^3*\log(c)^2 + 24*a^2*b - 4*(b^3*c^2*\log(c)^2 + 3*a^2*b*c^2)*x^2 - (b^3*c^2*x^2 - 2*b^3)*\log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - 2*b^3)*\log(x)^2 + 4*(b^3*c^2*x^2*\log(c) - 2*b^3*\log(c) + (b^3*c^2*x^2 - 2*b^3)*\log(x))*\log(c^2*x^2) - 8*(b^3*c^2*x^2*\log(c) - 2*b^3*\log(c))*\log(x))*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) + (16*b^6*x^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^4 + b^6*x^2*\log(c^2*x^2)^4 + 64*b^6*x^2*\log(c)*\log(x)^3 + 16*b^6*x^2*\log(x)^4 + 64*a*b^5*x^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^3 + 32*(3*b^6*\log(c)^2 + a^2*b^4)*x^2*\log(x)^2 - 8*(b^6*x^2*\log(c) + b^6*x^2*\log(x))*\log(c^2*x^2)^3 + 64*(b^6*\log(c)^3 + a^2*b^4*\log(c))*x^2*\log(x) + 16*(b^6*\log(c)^4 + 2*a^2*b^4*\log(c)^2 + a^4*b^2)*x^2 + 8*(b^6*x^2*\log(c^2*x^2)^2 + 8*b^6*x^2*\log(c)*\log(x) + 4*b^6*x^2*\log(x)^2 + 4*(b^6*\log(c)^2 + 3*a^2*b^4)*x^2 - 4*(b^6*x^2*\log(c) + b^6*x^2*\log(x))*\log(c^2*x^2))*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 + 8*(6*b^6*x^2*\log(c)*\log(x) + 3*b^6*x^2*\log(x)^2 + (3*b^6*\log(c)^2 + a^2*b^4)*x^2)*\log(c^2*x^2)^2 + 16*(a*b^5*x^2*\log(c^2*x^2)^2 + 8*a*b^5*x^2*\log(c)*\log(x) + 4*a*b^5*x^2*\log(x)^2 + 4*(a*b^5*\log(c)^2 + a^3*b^3)*x^2 - 4*(a*b^5*x^2*\log(c) + a*b^5*x^2*\log(x))*\log(c^2*x^2))*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) - 32*(3*b^6*x^2*\log(c)*\log(x)^2 + b^6*x^2*\log(x)^3 + (3*b^6*\log(c)^2 + a^2*b^4)*x^2*\log(x) + (b^6*\log(c)^3 + a^2*b^4*\log(c))*x^2)*\log(c^2*x^2))*\text{integrate}(8*(b*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) + a)/(4*b^4*x^3*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 + b^4*x^3*\log(c^2*x^2)^2 + 8*b^4*x^3*\log(c)*\log(x) + 4*b^4*x^3*\log(x)^2 + 8*a*b^3*x^3*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) + 4*(b^4*\log(c)^2 + a^2*b^2)*x^3 - 4*(b^4*x^3*\log(c) + b^4*x^3*\log(x))*\log(c^2*x^2)), x) + 8*(a*b^2*c^2*x^2*\log(c) - 2*a*b^2*\log(c) + (a*b^2*c^2*x^2 - 2*a*b^2)*\log(x))*\log(c^2*x^2) - 16*(a*b^2*c^2*x^2*\log(c) - 2$

```

*a*b^2*log(c))*log(x))/(16*b^6*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 +
b^6*x^2*log(c^2*x^2)^4 + 64*b^6*x^2*log(c)*log(x)^3 + 16*b^6*x^2*log(x)^4 +
64*a*b^5*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*(3*b^6*log(c)^2 +
a^2*b^4)*x^2*log(x)^2 - 8*(b^6*x^2*log(c) + b^6*x^2*log(x))*log(c^2*x^2)^3
+ 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^2*log(x) + 16*(b^6*log(c)^4 + 2*a^2*
b^4*log(c)^2 + a^4*b^2)*x^2 + 8*(b^6*x^2*log(c^2*x^2))^2 + 8*b^6*x^2*log(c)*
log(x) + 4*b^6*x^2*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x^2 - 4*(b^6*x^2
*log(c) + b^6*x^2*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
^2 + 8*(6*b^6*x^2*log(c)*log(x) + 3*b^6*x^2*log(x)^2 + (3*b^6*log(c)^2 + a^
2*b^4)*x^2)*log(c^2*x^2)^2 + 16*(a*b^5*x^2*log(c^2*x^2)^2 + 8*a*b^5*x^2*log
(c)*log(x) + 4*a*b^5*x^2*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x^2 - 4*(a
*b^5*x^2*log(c) + a*b^5*x^2*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt
(c*x - 1)) - 32*(3*b^6*x^2*log(c)*log(x)^2 + b^6*x^2*log(x)^3 + (3*b^6*log(
c)^2 + a^2*b^4)*x^2*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x^2)*log(c^2*x
^2))

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*(a + b\*acos(1/(c\*x)))^3), x)

[Out] int(1/(x^3\*(a + b\*acos(1/(c\*x)))^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{asec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*asec(c\*x))\*\*3, x)

[Out] Integral(1/(x\*\*3\*(a + b\*asec(c\*x))\*\*3), x)



$$3.50 \quad \int \frac{1}{x^4(a+b \sec^{-1}(cx))^3} dx$$

**Optimal.** Leaf size=228

$$\frac{c^3 \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{8b^3} + \frac{9c^3 \sin\left(\frac{3a}{b}\right) \text{Ci}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{8b^3} - \frac{c^3 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{8b^3} - \frac{9c^3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{8b^3}$$

[Out]  $-1/8*c^2/b^2/x/(a+b*\text{arcsec}(c*x))-3/8*c^3*\cos(3*\text{arcsec}(c*x))/b^2/(a+b*\text{arcsec}(c*x))-1/8*c^3*\cos(a/b)*\text{Si}(a/b+\text{arcsec}(c*x))/b^3-9/8*c^3*\cos(3*a/b)*\text{Si}(3*a/b+3*\text{arcsec}(c*x))/b^3+1/8*c^3*\text{Ci}(a/b+\text{arcsec}(c*x))*\sin(a/b)/b^3+9/8*c^3*\text{Ci}(3*a/b+3*\text{arcsec}(c*x))*\sin(3*a/b)/b^3-1/8*c^3*\sin(3*\text{arcsec}(c*x))/b/(a+b*\text{arcsec}(c*x))^2-1/8*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*\text{arcsec}(c*x))^2$

**Rubi [A]** time = 0.31, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5222, 4406, 3297, 3303, 3299, 3302}

$$\frac{c^3 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{8b^3} + \frac{9c^3 \sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{8b^3} - \frac{c^3 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right)}{8b^3} - \frac{9c^3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3a}{b} + 3 \sec^{-1}(cx)\right)}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*ArcSec[c\*x])^3), x]

[Out]  $-(c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(8*b*(a + b*\text{ArcSec}[c*x])^2) - c^2/(8*b^2*x*(a + b*\text{ArcSec}[c*x])) - (3*c^3*\text{Cos}[3*\text{ArcSec}[c*x]])/(8*b^2*(a + b*\text{ArcSec}[c*x])) + (c^3*\text{CosIntegral}[a/b + \text{ArcSec}[c*x]]*\text{Sin}[a/b])/(8*b^3) + (9*c^3*\text{CosIntegral}[(3*a)/b + 3*\text{ArcSec}[c*x]]*\text{Sin}[(3*a)/b])/(8*b^3) - (c^3*\text{Sin}[3*\text{ArcSec}[c*x]])/(8*b*(a + b*\text{ArcSec}[c*x])^2) - (c^3*\text{Cos}[a/b]*\text{SinIntegral}[a/b + \text{ArcSec}[c*x]])/(8*b^3) - (9*c^3*\text{Cos}[(3*a)/b]*\text{SinIntegral}[(3*a)/b + 3*\text{ArcSec}[c*x]])/(8*b^3)$

Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

### Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^(n)*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^3} dx &= c^3 \text{Subst} \left( \int \frac{\cos^2(x) \sin(x)}{(a + bx)^3} dx, x, \sec^{-1}(cx) \right) \\
&= c^3 \text{Subst} \left( \int \left( \frac{\sin(x)}{4(a + bx)^3} + \frac{\sin(3x)}{4(a + bx)^3} \right) dx, x, \sec^{-1}(cx) \right) \\
&= \frac{1}{4} c^3 \text{Subst} \left( \int \frac{\sin(x)}{(a + bx)^3} dx, x, \sec^{-1}(cx) \right) + \frac{1}{4} c^3 \text{Subst} \left( \int \frac{\sin(3x)}{(a + bx)^3} dx, x, \sec^{-1}(cx) \right) \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{8b (a + b \sec^{-1}(cx))^2} - \frac{c^3 \sin(3 \sec^{-1}(cx))}{8b (a + b \sec^{-1}(cx))^2} + \frac{c^3 \text{Subst} \left( \int \frac{\cos(x)}{(a + bx)^2} dx, x, \sec^{-1}(cx) \right)}{8b} \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{8b (a + b \sec^{-1}(cx))^2} - \frac{c^2}{8b^2 x (a + b \sec^{-1}(cx))} - \frac{3c^3 \cos(3 \sec^{-1}(cx))}{8b^2 (a + b \sec^{-1}(cx))} - \frac{c^3 \sin(3 \sec^{-1}(cx))}{8b (a + b \sec^{-1}(cx))} \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{8b (a + b \sec^{-1}(cx))^2} - \frac{c^2}{8b^2 x (a + b \sec^{-1}(cx))} - \frac{3c^3 \cos(3 \sec^{-1}(cx))}{8b^2 (a + b \sec^{-1}(cx))} - \frac{c^3 \sin(3 \sec^{-1}(cx))}{8b (a + b \sec^{-1}(cx))} \\
&= -\frac{c^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{8b (a + b \sec^{-1}(cx))^2} - \frac{c^2}{8b^2 x (a + b \sec^{-1}(cx))} - \frac{3c^3 \cos(3 \sec^{-1}(cx))}{8b^2 (a + b \sec^{-1}(cx))} + \frac{c^3 \text{Ci} \left( \frac{a}{b} + \sec^{-1}(cx) \right)}{8b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.46, size = 169, normalized size = 0.74

$$\frac{-\frac{4b^2c \sqrt{1 - \frac{1}{c^2 x^2}}}{x^2 (a + b \sec^{-1}(cx))^2} + c^3 \sin\left(\frac{a}{b}\right) \text{Ci}\left(\frac{a}{b} + \sec^{-1}(cx)\right) + 9c^3 \sin\left(\frac{3a}{b}\right) \text{Ci}\left(3\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right) - c^3 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \sec^{-1}(cx)\right) - c^3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(3\left(\frac{a}{b} + \sec^{-1}(cx)\right)\right)}{8b^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*ArcSec[c\*x])^3), x]

[Out] ((-4\*b^2\*c\*Sqrt[1 - 1/(c^2\*x^2)])/(x^2\*(a + b\*ArcSec[c\*x])^2) - (12\*b)/(x^3\*(a + b\*ArcSec[c\*x])) + (8\*b\*c^2)/(a\*x + b\*x\*ArcSec[c\*x]) + c^3\*CosIntegral[a/b + ArcSec[c\*x]]\*Sin[a/b] + 9\*c^3\*CosIntegral[3\*(a/b + ArcSec[c\*x])]\*Sin[(3\*a)/b] - c^3\*Cos[a/b]\*SinIntegral[a/b + ArcSec[c\*x]] - 9\*c^3\*Cos[(3\*a)/b]\*SinIntegral[3\*(a/b + ArcSec[c\*x])])/(8\*b^3)

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^3 x^4 \operatorname{arcsec}(cx)^3 + 3 a b^2 x^4 \operatorname{arcsec}(cx)^2 + 3 a^2 b x^4 \operatorname{arcsec}(cx) + a^3 x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3\*x^4\*arcsec(c\*x)^3 + 3\*a\*b^2\*x^4\*arcsec(c\*x)^2 + 3\*a^2\*b\*x^4\*arcsec(c\*x) + a^3\*x^4), x)

**giac [B]** time = 0.16, size = 1640, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] 
$$\frac{1}{8} \cdot (36 \cdot b^2 \cdot c^2 \cdot \arccos(1/(c \cdot x))^2 \cdot \cos(a/b)^2 \cdot \cos\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 36 \cdot b^2 \cdot c^2 \cdot \arccos(1/(c \cdot x))^2 \cdot \cos(a/b)^3 \cdot \sin\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + 72 \cdot a \cdot b \cdot c^2 \cdot \arccos(1/(c \cdot x)) \cdot \cos(a/b)^2 \cdot \cos\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 72 \cdot a \cdot b \cdot c^2 \cdot \arccos(1/(c \cdot x)) \cdot \cos(a/b)^3 \cdot \sin\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 9 \cdot b^2 \cdot c^2 \cdot \arccos(1/(c \cdot x))^2 \cdot \cos\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + 36 \cdot a^2 \cdot c^2 \cdot \cos(a/b)^2 \cdot \cos\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + b^2 \cdot c^2 \cdot \arccos(1/(c \cdot x))^2 \cdot \cos\_integral(a/b + \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + 27 \cdot b^2 \cdot c^2 \cdot \arccos(1/(c \cdot x))^2 \cdot \cos(a/b) \cdot \sin\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 36 \cdot a^2 \cdot c^2 \cdot \cos(a/b)^3 \cdot \sin\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - b^2 \cdot c^2 \cdot \arccos(1/(c \cdot x))^2 \cdot \cos(a/b) \cdot \sin\_integral(a/b + \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 18 \cdot a \cdot b \cdot c^2 \cdot \arccos(1/(c \cdot x)) \cdot \cos\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + 2 \cdot a \cdot b \cdot c^2 \cdot \arccos(1/(c \cdot x)) \cdot \cos\_integral(a/b + \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + 54 \cdot a \cdot b \cdot c^2 \cdot \arccos(1/(c \cdot x)) \cdot \cos(a/b) \cdot \sin\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 2 \cdot a \cdot b \cdot c^2 \cdot \arccos(1/(c \cdot x)) \cdot \cos(a/b) \cdot \sin\_integral(a/b + \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - 9 \cdot a^2 \cdot c^2 \cdot \cos\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) \cdot \sin(a/b) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) + 27 \cdot a^2 \cdot c^2 \cdot \cos(a/b) \cdot \sin\_integral(3 \cdot a/b + 3 \cdot \arccos(1/(c \cdot x))) / (b^5 \cdot \arccos(1/(c \cdot x))^2 + 2 \cdot a \cdot b^4 \cdot \arccos(1/(c \cdot x)) + a^2 \cdot b^3) - a^2 \cdot c^2 \cdot \cos(a/b) \cdot \sin\_integral(a/b +$$

$$\begin{aligned} & \arccos(1/(c*x)) / (b^5 * \arccos(1/(c*x))^2 + 2*a*b^4 * \arccos(1/(c*x)) + a^2*b^3) \\ & + 8*b^2*c * \arccos(1/(c*x)) / ((b^5 * \arccos(1/(c*x))^2 + 2*a*b^4 * \arccos(1/(c*x))) \\ & + a^2*b^3)*x) + 8*a*b*c / ((b^5 * \arccos(1/(c*x))^2 + 2*a*b^4 * \arccos(1/(c*x)) \\ & + a^2*b^3)*x) - 4*b^2 * \sqrt{-1/(c^2*x^2) + 1} / ((b^5 * \arccos(1/(c*x))^2 + 2* \\ & a*b^4 * \arccos(1/(c*x)) + a^2*b^3)*x^2) - 12*b^2 * \arccos(1/(c*x)) / ((b^5 * \arccos \\ & (1/(c*x))^2 + 2*a*b^4 * \arccos(1/(c*x)) + a^2*b^3)*c*x^3) - 12*a*b / ((b^5 * \arccos \\ & (1/(c*x))^2 + 2*a*b^4 * \arccos(1/(c*x)) + a^2*b^3)*c*x^3) * c \end{aligned}$$

**maple** [A] time = 0.12, size = 307, normalized size = 1.35

$$c^3 \left( \frac{\sin(3 \operatorname{arcsec}(cx))}{8(a + b \operatorname{arcsec}(cx))^2 b} - \frac{3 \left( 3 \operatorname{arcsec}(cx) \operatorname{Si}\left(\frac{3a}{b} + 3 \operatorname{arcsec}(cx)\right) \cos\left(\frac{3a}{b}\right) b - 3 \operatorname{arcsec}(cx) \operatorname{Ci}\left(\frac{3a}{b} + 3 \operatorname{arcsec}(cx)\right) \right)}{8(a + b \operatorname{arcsec}(cx))^2 b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a+b\*arcsec(c\*x))^3,x)

[Out]  $c^3 * (-1/8 * \sin(3 * \operatorname{arcsec}(c*x)) / (a + b * \operatorname{arcsec}(c*x))^2 / b - 3/8 * (3 * \operatorname{arcsec}(c*x) * \operatorname{Si}(3 * a/b + 3 * \operatorname{arcsec}(c*x)) * \cos(3 * a/b) * b - 3 * \operatorname{arcsec}(c*x) * \operatorname{Ci}(3 * a/b + 3 * \operatorname{arcsec}(c*x)) * \sin(3 * a/b) * b + 3 * \operatorname{Si}(3 * a/b + 3 * \operatorname{arcsec}(c*x)) * \cos(3 * a/b) * a - 3 * \operatorname{Ci}(3 * a/b + 3 * \operatorname{arcsec}(c*x)) * \sin(3 * a/b) * a + \cos(3 * \operatorname{arcsec}(c*x)) * b) / (a + b * \operatorname{arcsec}(c*x)) / b^3 - 1/8 * ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / (a + b * \operatorname{arcsec}(c*x))^2 / b - 1/8 * (\operatorname{arcsec}(c*x) * \operatorname{Si}(a/b + \operatorname{arcsec}(c*x)) * \cos(a/b) * c * x * b - \operatorname{arcsec}(c*x) * \operatorname{Ci}(a/b + \operatorname{arcsec}(c*x)) * \sin(a/b) * c * x * b + \operatorname{Si}(a/b + \operatorname{arcsec}(c*x)) * \cos(a/b) * c * x * a - \operatorname{Ci}(a/b + \operatorname{arcsec}(c*x)) * \sin(a/b) * c * x * a) / c / x / (a + b * \operatorname{arcsec}(c*x)) / b^3)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out]  $-(24*a*b^2*\log(c)^2 - 8*(2*b^3*c^2*x^2 - 3*b^3)*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^3 + 24*a^3 - 16*(a*b^2*c^2*\log(c)^2 + a^3*c^2)*x^2 - 24*(2*a*b^2*c^2*x^2 - 3*a*b^2)*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - 2*(2*a*b^2*c^2*x^2 - 3*a*b^2)*\log(c^2*x^2)^2 - 8*(2*a*b^2*c^2*x^2 - 3*a*b^2)*\log(x)^2 + 2*(4*b^3*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - b^3*\log(c^2*x^2)^2 - 4*b^3*\log(c)^2 - 8*b^3*\log(c)*\log(x) - 4*b^3*\log(x)^2 + 8*a*b^2*\arctan(\sqrt{c*x + 1})*\sqrt{c*x - 1}) + 4*a^2*b + 4*(b^3*\log(c) + b^3*\log(x))*\log(c^2*x^2))*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 2*(12*b^3*\log(c)^2 + 36*a^2*b - 8*(b^3*c^2*\log(c)^2 + 3*a^2*b*c^2)*x^2 - (2*b^3*c^2*x^2 - 3*b^3)*\log(c^2*x^2)^2 - 4*(2*b^3*c^2*x^2 - 3*b^3)*\log(x)^2 + 4*(2*b^3*c^2*x^2*\log(c) - 3*b^3*\log(c) + (2*b^3*c^2*x^2 - 3*b^3)*\log(x))*\log(c^2*x^2) - 8*(2*b^3*c^2*x^2*\log(c) - 3*b^3*\log(c)$

```

c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*x^3*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))^4 + b^6*x^3*log(c^2*x^2)^4 + 64*b^6*x^3*log(c)*log(x
)^3 + 16*b^6*x^3*log(x)^4 + 64*a*b^5*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)
)^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x^3*log(x)^2 + 64*(b^6*log(c)^3 + a^2*b
^4*log(c))*x^3*log(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x^
3 - 8*(b^6*x^3*log(c) + b^6*x^3*log(x))*log(c^2*x^2)^3 + 8*(b^6*x^3*log(c^2
*x^2)^2 + 8*b^6*x^3*log(c)*log(x) + 4*b^6*x^3*log(x)^2 + 4*(b^6*log(c)^2 +
3*a^2*b^4)*x^3 - 4*(b^6*x^3*log(c) + b^6*x^3*log(x))*log(c^2*x^2))*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x^3*log(c)*log(x) + 3*b^6*x^3*log(
x)^2 + (3*b^6*log(c)^2 + a^2*b^4)*x^3)*log(c^2*x^2)^2 + 16*(a*b^5*x^3*log(c
^2*x^2)^2 + 8*a*b^5*x^3*log(c)*log(x) + 4*a*b^5*x^3*log(x)^2 + 4*(a*b^5*log
(c)^2 + a^3*b^3)*x^3 - 4*(a*b^5*x^3*log(c) + a*b^5*x^3*log(x))*log(c^2*x^2)
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(3*b^6*x^3*log(c)*log(x)^2 + b^6
*x^3*log(x)^3 + (3*b^6*log(c)^2 + a^2*b^4)*x^3*log(x) + (b^6*log(c)^3 + a^2
*b^4*log(c))*x^3)*log(c^2*x^2))*integrate(2*(2*a*c^2*x^2 + (2*b*c^2*x^2 - 9
*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 9*a)/(4*b^4*x^4*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^2 + b^4*x^4*log(c^2*x^2)^2 + 8*b^4*x^4*log(c)*log(x) + 4
*b^4*x^4*log(x)^2 + 8*a*b^3*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^
4*log(c)^2 + a^2*b^2)*x^4 - 4*(b^4*x^4*log(c) + b^4*x^4*log(x))*log(c^2*x^2
)), x) + 8*(2*a*b^2*c^2*x^2*log(c) - 3*a*b^2*log(c) + (2*a*b^2*c^2*x^2 - 3*
a*b^2)*log(x))*log(c^2*x^2) - 16*(2*a*b^2*c^2*x^2*log(c) - 3*a*b^2*log(c))*
log(x))/(16*b^6*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x^3*log(c^2
*x^2)^4 + 64*b^6*x^3*log(c)*log(x)^3 + 16*b^6*x^3*log(x)^4 + 64*a*b^5*x^3*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x^3*lo
g(x)^2 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^3*log(x) + 16*(b^6*log(c)^4 +
2*a^2*b^4*log(c)^2 + a^4*b^2)*x^3 - 8*(b^6*x^3*log(c) + b^6*x^3*log(x))*lo
g(c^2*x^2)^3 + 8*(b^6*x^3*log(c^2*x^2)^2 + 8*b^6*x^3*log(c)*log(x) + 4*b^6*
x^3*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x^3 - 4*(b^6*x^3*log(c) + b^6*x
^3*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x
^3*log(c)*log(x) + 3*b^6*x^3*log(x)^2 + (3*b^6*log(c)^2 + a^2*b^4)*x^3)*log
(c^2*x^2)^2 + 16*(a*b^5*x^3*log(c^2*x^2)^2 + 8*a*b^5*x^3*log(c)*log(x) + 4*
a*b^5*x^3*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x^3 - 4*(a*b^5*x^3*log(c)
+ a*b^5*x^3*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32
*(3*b^6*x^3*log(c)*log(x)^2 + b^6*x^3*log(x)^3 + (3*b^6*log(c)^2 + a^2*b^4)
*x^3*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x^3)*log(c^2*x^2))

```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4\*(a + b\*acos(1/(c\*x)))^3), x)

[Out] `int(1/(x^4*(a + b*acos(1/(c*x))))^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \operatorname{asec}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(a+b*asec(c*x))**3,x)`

[Out] `Integral(1/(x**4*(a + b*asec(c*x))**3), x)`

$$3.51 \quad \int (dx)^m \left( a + b \sec^{-1}(cx) \right)^3 dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left((dx)^m (a + b \sec^{-1}(cx))^3, x\right)$$

[Out] Unintegrable((d\*x)^m\*(a+b\*arcsec(c\*x))^3,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (dx)^m \left( a + b \sec^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcSec[c\*x])^3,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcSec[c\*x])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \sec^{-1}(cx) \right)^3 dx = \int (dx)^m \left( a + b \sec^{-1}(cx) \right)^3 dx$$

**Mathematica [A]** time = 6.79, size = 0, normalized size = 0.00

$$\int (dx)^m \left( a + b \sec^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcSec[c\*x])^3,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcSec[c\*x])^3, x]

**fricas [A]** time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \operatorname{arcsec}(cx)^3 + 3ab^2 \operatorname{arcsec}(cx)^2 + 3a^2b \operatorname{arcsec}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x))^3,x, algorithm="fricas")

[Out] integral((b^3\*arcsec(c\*x)^3 + 3\*a\*b^2\*arcsec(c\*x)^2 + 3\*a^2\*b\*arcsec(c\*x) + a^3)\*(d\*x)^m, x)



**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^3\*(d\*x)^m, x)

**maple** [A] time = 3.31, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arcsec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arcsec(c\*x))^3,x)

[Out] int((d\*x)^m\*(a+b\*arcsec(c\*x))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x))^3,x, algorithm="maxima")

[Out] (d\*x)^(m + 1)\*a^3/(d\*(m + 1)) + 1/4\*(4\*b^3\*d^m\*x\*x^m\*arctan(sqrt(c\*x + 1))\*sqrt(c\*x - 1))^3 - 3\*b^3\*d^m\*x\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*log(c^2\*x^2)^2 - 4\*(m + 1)\*integrate(3/4\*(4\*(a\*b^2\*d^m\*m + a\*b^2\*d^m - (a\*b^2\*c^2\*d^m\*m + a\*b^2\*c^2\*d^m)\*x^2)\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 - (a\*b^2\*d^m\*m + a\*b^2\*d^m - (a\*b^2\*c^2\*d^m\*m + a\*b^2\*c^2\*d^m)\*x^2)\*x^m\*log(c^2\*x^2)^2 - 4\*(a\*b^2\*d^m\*m + a\*b^2\*d^m - (a\*b^2\*c^2\*d^m\*m + a\*b^2\*c^2\*d^m)\*x^2)\*x^m\*log(x)^2 - 8\*(a\*b^2\*d^m\*m\*log(c) + a\*b^2\*d^m\*log(c) - (a\*b^2\*c^2\*d^m\*m\*log(c) + a\*b^2\*c^2\*d^m\*log(c))\*x^2)\*x^m\*log(x) + (4\*b^3\*d^m\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 - b^3\*d^m\*x^m\*log(c^2\*x^2)^2)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - 4\*(a\*b^2\*d^m\*m\*log(c)^2 + a\*b^2\*d^m\*log(c)^2 - (a\*b^2\*c^2\*d^m\*m\*log(c)^2 + a\*b^2\*c^2\*d^m\*log(c)^2)\*x^2)\*x^m - 4\*((b^3\*d^m\*m + b^3\*d^m - (b^3\*c^2\*d^m\*m + b^3\*c^2\*d^m)\*x^2)\*x^m\*log(x)^2 + 2\*(b^3\*d^m\*m\*log(c) + b^3\*d^m\*log(c) - (b^3\*c^2\*d^m\*m\*log(c) + b^3\*c^2\*d^m\*log(c))\*x^2)\*x^m\*log(x) + ((b^3\*log(c)^2 - a^2\*b)\*d^m\*m - ((b^3\*c^2\*log(c)^2 - a^2\*b\*c^2)\*d^m\*m + (b^3\*c^2\*log(c)^2 - a^2\*b\*c^2)\*d^m)\*x^2 + (b^3\*log(c)^2 - a^2\*b)\*d^m)\*x^m - ((b^3\*d^m\*m + b^3\*d^m - (b^3\*c^2\*d^m\*m + b^3\*c^2\*d^m)\*x^2)\*x^m\*log(x) + (b^3\*d^m\*m\*log(c) - (b^3\*c^2\*d^m\*m\*log(c) + (b^3\*c^2\*log(c) + b^3\*c^2)\*d^m)\*x

$$\begin{aligned} &^2 + (b^3 \log(c) + b^3) d^m x^m \log(c^2 x^2) \arctan(\sqrt{c x + 1} \sqrt{c x - 1}) \\ &+ 4((a b^2 d^m m + a b^2 d^m - (a b^2 c^2 d^m m + a b^2 c^2 d^m) x^2) x^m \log(x) \\ &+ (a b^2 d^m m \log(c) + a b^2 d^m \log(c) - (a b^2 c^2 d^m m \log(c) + a b^2 c^2 d^m \log(c)) x^2) x^m) \log(c^2 x^2) / ((c^2 m + c^2) x^2 - m - 1), x) / (m + 1) \end{aligned}$$

**mupad [A]** time = 0.00, size = -1, normalized size = -0.05

$$\int (dx)^m \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*acos(1/(c\*x)))^3,x)

[Out] int((d\*x)^m\*(a + b\*acos(1/(c\*x)))^3, x)

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{asec}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*asec(c\*x))\*\*3,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*asec(c\*x))\*\*3, x)

$$3.52 \quad \int (dx)^m \left( a + b \sec^{-1}(cx) \right)^2 dx$$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m (a + b \sec^{-1}(cx))^2, x\right)$$

[Out] Unintegrable((d\*x)^m\*(a+b\*arcsec(c\*x))^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (dx)^m \left( a + b \sec^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcSec[c\*x])^2,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcSec[c\*x])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \sec^{-1}(cx) \right)^2 dx = \int (dx)^m \left( a + b \sec^{-1}(cx) \right)^2 dx$$

Mathematica [A] time = 4.51, size = 0, normalized size = 0.00

$$\int (dx)^m \left( a + b \sec^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcSec[c\*x])^2,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcSec[c\*x])^2, x]

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \text{arcsec}(cx)^2 + 2ab \text{arcsec}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2)\*(d\*x)^m, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)^2\*(d\*x)^m, x)

**maple** [A] time = 3.12, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arcsec}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arcsec(c\*x))^2,x)

[Out] int((d\*x)^m\*(a+b\*arcsec(c\*x))^2,x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (dx)^m \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*acos(1/(c\*x)))^2,x)

[Out] int((d\*x)^m\*(a + b\*acos(1/(c\*x)))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{asec}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*asec(c\*x))\*\*2,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*asec(c\*x))\*\*2, x)

### 3.53 $\int (dx)^m (a + b \sec^{-1}(cx)) dx$

Optimal. Leaf size=67

$$\frac{(dx)^{m+1} (a + b \sec^{-1}(cx))}{d(m+1)} - \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; \frac{1}{c^2 x^2}\right)}{cm(m+1)}$$

[Out]  $(d*x)^{(1+m)*(a+b*\text{arcsec}(c*x))/d/(1+m)-b*(d*x)^m*\text{hypergeom}([1/2, -1/2*m], [1-1/2*m], 1/c^2/x^2)/c/m/(1+m)$

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5220, 339, 364}

$$\frac{(dx)^{m+1} (a + b \sec^{-1}(cx))}{d(m+1)} - \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; \frac{1}{c^2 x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^m*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $((d*x)^{(1+m)*(a + b*\text{ArcSec}[c*x])}/(d*(1+m)) - (b*(d*x)^m*\text{Hypergeometric}2F1[1/2, -m/2, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1+m))$

#### Rule 339

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_)})^{(p_*)}, x\_Symbol] \rightarrow -\text{Dist}[(c*x)^{(m+1)}*(1/x)^{(m+1)}/c, \text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x], x] /;$  FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]

#### Rule 364

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric}2F1[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])/((c*(m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rule 5220

$\text{Int}[(a_*) + \text{ArcSec}[(c_*)*(x_*)]*(b_*)]*((d_*)*(x_*)^{(m_*)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcSec}[c*x])]/(d*(m+1)), x] - \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{(m-1)}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int (dx)^m (a + b \sec^{-1}(cx)) dx &= \frac{(dx)^{1+m} (a + b \sec^{-1}(cx))}{d(1+m)} - \frac{(bd) \int \frac{(dx)^{-1+m}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{c(1+m)} \\
&= \frac{(dx)^{1+m} (a + b \sec^{-1}(cx))}{d(1+m)} + \frac{\left(b \left(\frac{1}{x}\right)^m (dx)^m\right) \text{Subst} \left( \int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{c(1+m)} \\
&= \frac{(dx)^{1+m} (a + b \sec^{-1}(cx))}{d(1+m)} - \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; \frac{1}{c^2x^2}\right)}{cm(1+m)}
\end{aligned}$$

**Mathematica** [A] time = 0.26, size = 82, normalized size = 1.22

$$\frac{x(dx)^m \left( (m+1) (a + b \sec^{-1}(cx)) + \frac{bcx \sqrt{1-\frac{1}{c^2x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; c^2x^2\right)}{\sqrt{1-c^2x^2}} \right)}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcSec[c\*x]),x]

[Out] (x\*(d\*x)^m\*((1+m)\*(a + b\*ArcSec[c\*x]) + (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/Sqrt[1 - c^2\*x^2]))/(1+m)^2

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}((b \operatorname{arcsec}(cx) + a)(dx)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)\*(d\*x)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsec}(cx) + a)(dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*(d\*x)^m, x)

maple [F] time = 4.37, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arcsec(c\*x)),x)

[Out] int((d\*x)^m\*(a+b\*arcsec(c\*x)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left( d^m x x^m \arctan(\sqrt{cx+1} \sqrt{cx-1}) + (c^2 d^m m + c^2 d^m) \int \frac{\sqrt{cx+1} \sqrt{cx-1} x^m}{c^2 m - (c^4 m + c^4) x^2 + c^2} dx \right) b}{m+1} + \frac{(dx)^{m+1} a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] (d^m\*x\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - (c^2\*d^m\*m + c^2\*d^m)\*integrate(-sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*x^m/(c^2\*m - (c^4\*m + c^4)\*x^2 + c^2), x)))\*b/(m + 1) + (d\*x)^(m + 1)\*a/(d\*(m + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^m \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*acos(1/(c\*x))),x)

[Out] int((d\*x)^m\*(a + b\*acos(1/(c\*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{asec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*asec(c\*x)),x)

[Out] Integral((d\*x)\*\*m\*(a + b\*asec(c\*x)), x)

$$3.54 \quad \int \frac{(dx)^m}{a+b \sec^{-1}(cx)} dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b \sec^{-1}(cx)}, x\right)$$

[Out] Unintegrable((d\*x)^m/(a+b\*arcsec(c\*x)), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b \sec^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \sec^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b \sec^{-1}(cx)} dx$$

**Mathematica [A]** time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b \sec^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcSec[c\*x]), x]

**fricas [A]** time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b \operatorname{arcsec}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x)^m/(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arcsec(c\*x) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \operatorname{arcsec}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arcsec(c\*x) + a), x)

**maple** [A] time = 3.83, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{arcsec}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arcsec(c\*x)),x)

[Out] int((d\*x)^m/(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \operatorname{arcsec}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arcsec(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^m}{a + b \operatorname{acos}\left(\frac{1}{cx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a + b\*acos(1/(c\*x))),x)

[Out] int((d\*x)^m/(a + b\*acos(1/(c\*x))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{asec}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*asec(c*x)),x)
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```
[Out] Integral((d*x)**m/(a + b*asec(c*x)), x)
```

$$3.55 \quad \int \frac{(dx)^m}{(a+b \sec^{-1}(cx))^2} dx$$

Optimal. Leaf size=19

$$\text{Int} \left( \frac{(dx)^m}{(a + b \sec^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable((d\*x)^m/(a+b\*arcsec(c\*x))^2, x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(dx)^m}{(a + b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcSec[c\*x])^2, x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcSec[c\*x])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{(a + b \sec^{-1}(cx))^2} dx = \int \frac{(dx)^m}{(a + b \sec^{-1}(cx))^2} dx$$

Mathematica [A] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \sec^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcSec[c\*x])^2, x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcSec[c\*x])^2, x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx)^m}{b^2 \text{arcsec}(cx)^2 + 2ab \text{arcsec}(cx) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arcsec(c\*x))^2,x, algorithm="fricas")

[Out] integral((d\*x)^m/(b^2\*arcsec(c\*x)^2 + 2\*a\*b\*arcsec(c\*x) + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b \operatorname{arcsec}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arcsec(c\*x))^2,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arcsec(c\*x) + a)^2, x)

**maple** [A] time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \operatorname{arcsec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arcsec(c\*x))^2,x)

[Out] int((d\*x)^m/(a+b\*arcsec(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$4(bd^mxx^m \arctan(\sqrt{cx+1}\sqrt{cx-1}) + ad^mxx^m)\sqrt{cx+1}\sqrt{cx-1} - 4(4b^3 \arctan(\sqrt{cx+1}\sqrt{cx-1})^2 + b^3 \log$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arcsec(c\*x))^2,x, algorithm="maxima")

[Out] -(4\*(b\*d^m\*x\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + a\*d^m\*x\*x^m)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - (4\*b^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))^2 + b^3\*log(c^2\*x^2)^2 + 4\*b^3\*log(c)^2 + 8\*b^3\*log(c)\*log(x) + 4\*b^3\*log(x)^2 + 8\*a\*b^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*a^2\*b - 4\*(b^3\*log(c) + b^3\*log(x))\*log(c^2\*x^2))\*integrate(4\*((b\*d^m\*m - (b\*c^2\*d^m\*m + 2\*b\*c^2\*d^m)\*x^2 + b\*d^m)\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + (a\*d^m\*m - (a\*c^2\*d^m\*m + 2\*a\*c^2\*d^m)\*x^2 + a\*d^m)\*x^m)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/(4\*b^3\*log(c)^2 + 4\*a^2\*b - 4\*(b^3\*c^2\*log(c)^2 + a^2\*b\*c^2)\*x^2 - 4\*(b^3\*c^2\*x^2 - b^3)

```
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2
- 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2
- b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)),
x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b
^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^m}{\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*acos(1/(c*x)))^2, x)
```

```
[Out] int((d*x)^m/(a + b*acos(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \operatorname{asec}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*asec(c*x))**2, x)
```

```
[Out] Integral((d*x)**m/(a + b*asec(c*x))**2, x)
```

### 3.56 $\int (d + ex)^3 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=167

$$\frac{(d + ex)^4 (a + b \sec^{-1}(cx))}{4e} - \frac{bde^2 x^2 \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} - \frac{be^3 x^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{12c} - \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}} (9c^2 d^2 + e^2)}{6c^3} - \frac{bd (2c^2 d^2 + e^2) \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{2c^3}$$

[Out]  $\frac{1}{4} b d^4 \operatorname{arccsc}(c x) / e + \frac{1}{4} (e x + d)^4 (a + b \operatorname{arcsec}(c x)) / e - \frac{1}{2} b d (2 c^2 d^2 + e^2) \operatorname{arctanh}\left(\sqrt{1 - 1/c^2/x^2}\right) / c^3 - \frac{1}{6} b e (9 c^2 d^2 + e^2) x \sqrt{1 - 1/c^2/x^2} / c^3 - \frac{1}{2} b d e^2 x^2 \sqrt{1 - 1/c^2/x^2} / c - \frac{1}{12} b e^3 x^3 \sqrt{1 - 1/c^2/x^2} / c$

**Rubi [A]** time = 0.40, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {5226, 1568, 1475, 1807, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^4 (a + b \sec^{-1}(cx))}{4e} - \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}} (9c^2 d^2 + e^2)}{6c^3} - \frac{bd (2c^2 d^2 + e^2) \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{2c^3} - \frac{bde^2 x^2 \sqrt{1 - \frac{1}{c^2 x^2}}}{2c}$$

Antiderivative was successfully verified.

[In] `Int[(d + e*x)^3*(a + b*ArcSec[c*x]),x]`

[Out]  $-\frac{b e (9 c^2 d^2 + e^2) \sqrt{1 - 1/(c^2 x^2)} x}{6 c^3} - \frac{b d e^2 \sqrt{1 - 1/(c^2 x^2)} x^2}{2 c} - \frac{b e^3 \sqrt{1 - 1/(c^2 x^2)} x^3}{12 c} + \frac{b d^4 \operatorname{ArcCsc}[c x]}{4 e} + \frac{(d + e x)^4 (a + b \operatorname{ArcSec}[c x])}{4 e} - \frac{b d (2 c^2 d^2 + e^2) \operatorname{ArcTanh}\left[\sqrt{1 - 1/(c^2 x^2)}\right]}{2 c^3}$

#### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

#### Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p, x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1475

Int[(x\_)^(m\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.))^p, x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(d + e\*x)^q\*(a + c\*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2\*n] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1568

Int[(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^(mn\_.))^q, x\_Symbol] := Int[x^(m + mn\*q)\*(e + d/x^mn)^q\*(a + c\*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2\*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])

### Rule 1807

Int[(Pq\_)\*((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, c\*x, x], R = PolynomialRemainder[Pq, c\*x, x]}, Simp[(R\*(c\*x)^(m + 1)\*(a + b\*x^2)^(p + 1))/(a\*c\*(m + 1)), x] + Dist[1/(a\*c\*(m + 1)), Int[(c\*x)^(m + 1)\*(a + b\*x^2)^p\*ExpandToSum[a\*c\*(m + 1)\*Q - b\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ[m, -1] && (IntegerQ[2\*p] || NeQ[Expon[Pq, x], 1])

### Rule 5226

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(e\*(m + 1)), x] - Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /;

`FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]`

Rubi steps



$$\begin{aligned}
\int (d+ex)^3 (a+b\sec^{-1}(cx)) dx &= \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} - \frac{b \int \frac{(d+ex)^4 dx}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{4ce} \\
&= \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} - \frac{b \int \frac{\left(e+\frac{d}{x}\right)^4 x^2 dx}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{4ce} \\
&= \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} + \frac{b \operatorname{Subst} \left( \int \frac{(e+dx)^4}{x^4 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} \\
&= -\frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} - \frac{b \operatorname{Subst} \left( \int \frac{-12de^3-2e^2 \left(9d^2+\frac{e^2}{c^2}\right)}{x^3 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{12c} \\
&= -\frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} - \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} + \frac{b \operatorname{Subst} \left( \int \frac{12de^3+2e^2 \left(9d^2+\frac{e^2}{c^2}\right)}{x^3 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{12c} \\
&= -\frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} - \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} - \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} \\
&= -\frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} - \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} - \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b\sec^{-1}(cx))}{4e} \\
&= -\frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} - \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} - \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{bd^4 \operatorname{csch}^{-1}\left(\frac{d+ex}{c}\right)}{4c} \\
&= -\frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} - \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} - \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{bd^4 \operatorname{csch}^{-1}\left(\frac{d+ex}{c}\right)}{4c} \\
&= -\frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} - \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} - \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{bd^4 \operatorname{csch}^{-1}\left(\frac{d+ex}{c}\right)}{4c}
\end{aligned}$$







$$\begin{aligned}
& + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2 - 12*b* \\
& c*d*\sqrt{-1/(c^2*x^2) + 1}*e^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1) \\
& )^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3 \\
& /((1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) \\
& - 108*b*c^2*d^2*(1/(c^2*x^2) - 1)^2*\sqrt{-1/(c^2*x^2) + 1}*e/((c^5 + 4*c^5* \\
& (1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1) \\
& ^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1 \\
& /((c*x) + 1)^8)*(1/(c*x) + 1)^5) - 12*a*c^3*d^3*(1/(c^2*x^2) - 1)^4/((c^5 + \\
& 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) \\
& ) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1) \\
& )^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 3*b*arccos(1/(c*x))*e^3/(c^5 + 4*c^ \\
& 5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + \\
& 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/ \\
& (1/(c*x) + 1)^8) - 24*a*c*d*(1/(c^2*x^2) - 1)*e^2/((c^5 + 4*c^5*(1/(c^2*x^2) \\
& ) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5* \\
& (1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& ^8)*(1/(c*x) + 1)^2) + 18*b*c^2*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))*e/( \\
& (c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/ \\
& (1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x \\
& ^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 36*b*c*d*(1/(c^2*x^2) - 1)^2 \\
& *e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x \\
& ^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^ \\
& 5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + \\
& 1)^8)*(1/(c*x) + 1)^4) + 36*b*c*d*(1/(c^2*x^2) - 1)^2*e^2*\log(\text{abs}(\sqrt{-1/( \\
& c^2*x^2) + 1} - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1) \\
& ^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/ \\
& (1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) \\
& + 12*b*c*d*(-1/(c^2*x^2) + 1)^(3/2)*e^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1 \\
& /((c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x \\
& ^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c \\
& *x) + 1)^3) - 36*b*c^2*d^2*(1/(c^2*x^2) - 1)^3*\sqrt{-1/(c^2*x^2) + 1}*e/((c \\
& ^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1 \\
& /((c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) \\
& ) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) + 3*a*e^3/(c^5 + 4*c^5*(1/(c^2*x \\
& ^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^ \\
& 5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + \\
& 1)^8) - 12*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))*e^3/((c^5 + 4*c^5*(1/(c^2*x^ \\
& 2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5 \\
& *(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& )^8)*(1/(c*x) + 1)^2) + 24*b*c*d*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))*e^2/(( \\
& c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/( \\
& 1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^ \\
& 2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 18*a*c^2*d^2*(1/(c^2*x^2) - 1) \\
& )^4*e/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) \\
& - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1
\end{aligned}$$

$$\begin{aligned}
& / (c^2 x^2 - 1)^4 / (1/(c x) + 1)^8 * (1/(c x) + 1)^8 - 24 * b * c * d * (1/(c^2 x^2) \\
& - 1)^3 * e^2 * \log(\text{abs}(\text{sqrt}(-1/(c^2 x^2) + 1) + 1/(c x) + 1)) / ((c^5 + 4 * c^5 * (1 / \\
& / (c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 \\
& + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c \\
& c x) + 1)^8) * (1/(c x) + 1)^6 + 24 * b * c * d * (1/(c^2 x^2) - 1)^3 * e^2 * \log(\text{abs}(\text{sq} \\
& \text{rt}(-1/(c^2 x^2) + 1) - 1/(c x) - 1)) / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c * \\
& x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) \\
& - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) \\
& + 1)^6) - 6 * b * \text{sqrt}(-1/(c^2 x^2) + 1) * e^3 / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1 / \\
& / (c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) \\
& ^2) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c \\
& * x) + 1)) + 12 * b * c * d * (1/(c^2 x^2) - 1)^2 * \text{sqrt}(-1/(c^2 x^2) + 1) * e^2 / ((c^5 + \\
& 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x \\
& x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - \\
& 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^5) - 12 * a * (1/(c^2 x^2) - 1) * e^3 / ((c^5 + \\
& 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x \\
& x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - \\
& 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^2) + 18 * b * (1/(c^2 x^2) - 1)^2 * \arccos(1 / \\
& (c x)) * e^3 / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1/(c^2 * \\
& x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^6 + c \\
& ^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^4) + 24 * a * c * d * (1/(c^2 \\
& * x^2) - 1)^3 * e^2 / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1 / \\
& / (c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1) \\
& ^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^6) - 12 * b * c * d * ( \\
& 1/(c^2 x^2) - 1)^4 * \arccos(1/(c x)) * e^2 / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c \\
& c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2 \\
& ) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x \\
& ) + 1)^8) - 6 * b * c * d * (1/(c^2 x^2) - 1)^4 * e^2 * \log(\text{abs}(\text{sqrt}(-1/(c^2 x^2) + 1) \\
& + 1/(c x) + 1)) / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1 / \\
& / (c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^ \\
& 6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^8) + 6 * b * c * d * (1 / \\
& (c^2 x^2) - 1)^4 * e^2 * \log(\text{abs}(\text{sqrt}(-1/(c^2 x^2) + 1) - 1/(c x) - 1)) / ((c^5 + \\
& 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x \\
& x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - \\
& 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^8) + 10 * b * (-1/(c^2 x^2) + 1)^(3/2) * e^3 / \\
& ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 \\
& / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / (1/(c x) + 1)^6 + c^5 * (1/(c^2 * \\
& x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^3) + 12 * b * c * d * (1/(c^2 x^2) - 1)^ \\
& 3 * \text{sqrt}(-1/(c^2 x^2) + 1) * e^2 / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1)^ \\
& 2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 / ( \\
& 1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^7) \\
& + 18 * a * (1/(c^2 x^2) - 1)^2 * e^3 / ((c^5 + 4 * c^5 * (1/(c^2 x^2) - 1) / (1/(c x) + 1) \\
& )^2 + 6 * c^5 * (1/(c^2 x^2) - 1)^2 / (1/(c x) + 1)^4 + 4 * c^5 * (1/(c^2 x^2) - 1)^3 \\
& / (1/(c x) + 1)^6 + c^5 * (1/(c^2 x^2) - 1)^4 / (1/(c x) + 1)^8) * (1/(c x) + 1)^4) \\
& ) - 12 * b * (1/(c^2 x^2) - 1)^3 * \arccos(1/(c x)) * e^3 / ((c^5 + 4 * c^5 * (1/(c^2 x^2)
\end{aligned}$$

$$\begin{aligned}
& - 1)/((1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*( \\
& 1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) \\
& *(1/(c*x) + 1)^6) - 12*a*c*d*(1/(c^2*x^2) - 1)^4*e^2/((c^5 + 4*c^5*(1/(c^2 \\
& *x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4 \\
& *c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) \\
& + 1)^8)*(1/(c*x) + 1)^8) - 10*b*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) \\
& *e^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - \\
& 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/ \\
& (c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) - 12*a*(1/(c^2*x^2) - 1) \\
& ^3*e^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) \\
& - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*( \\
& 1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 3*b*(1/(c^2*x^2) - 1) \\
& )^4*arccos(1/(c*x))*e^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6 \\
& *c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c* \\
& x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 6*b \\
& *(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)*e^3/((c^5 + 4*c^5*(1/(c^2*x^2) \\
& - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/ \\
& (c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 \\
& )*(1/(c*x) + 1)^7) + 3*a*(1/(c^2*x^2) - 1)^4*e^3/((c^5 + 4*c^5*(1/(c^2*x^2) \\
& - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*( \\
& 1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^ \\
& 8)*(1/(c*x) + 1)^8))*c
\end{aligned}$$

**maple [B]** time = 0.06, size = 486, normalized size = 2.91

$$\frac{ae^3x^4}{4} + ae^2x^3d + \frac{3aex^2d^2}{2} + axd^3 + \frac{ad^4}{4e} + \frac{be^3\operatorname{arcsec}(cx)x^4}{4} + be^2\operatorname{arcsec}(cx)x^3d + \frac{3be\operatorname{arcsec}(cx)x^2d^2}{2} + b\operatorname{arcsec}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(a+b\*arcsec(c\*x)),x)

[Out]  $\frac{1}{4}ae^3x^4 + ae^2x^3d + \frac{3}{2}aex^2d^2 + axd^3 + \frac{1}{4}ad^4 + \frac{1}{4}be^3\operatorname{arcsec}(cx)x^4 + be^2\operatorname{arcsec}(cx)x^3d + \frac{3}{2}be\operatorname{arcsec}(cx)x^2d^2 + b\operatorname{arcsec}(cx)x^2d^2 + b\operatorname{arcsec}(cx)x^2d^2 + \frac{1}{4}b/e\operatorname{arcsec}(cx)d^4 + \frac{1}{4}c/b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d^4*\arctan(1/(c^2*x^2-1)^{(1/2)}) - 1/c^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d^3*\ln(c*x+(c^2*x^2-1)^{(1/2)}) - 1/12/c*b*e^3/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^3 - 1/12/c^3*b*e^3/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x - 1/2/c*b*e^2/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/d*x^2 + 1/2/c^3*b*e^2/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/d - 3/2/c*b*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d^2 + 3/2/c^3*b*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d^2 - 1/2/c^4*b*e^2*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d*\ln(c*x+(c^2*x^2-1)^{(1/2)}) + 1/6/c^5*b*e^3/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x$

**maxima** [A] time = 0.33, size = 272, normalized size = 1.63

$$\frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2 \operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c}\right)bd^2e + \frac{1}{4}\left(4x^3 \operatorname{arcsec}(cx) - \frac{2\sqrt{-\frac{1}{c^2x^2} + 1}}{c^2\left(\frac{1}{2x^2} - 1\right) + c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1}\right)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/4\*a\*e^3\*x^4 + a\*d\*e^2\*x^3 + 3/2\*a\*d^2\*e\*x^2 + 3/2\*(x^2\*arcsec(c\*x) - x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*b\*d^2\*e + 1/4\*(4\*x^3\*arcsec(c\*x) - (2\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) - 1) + c^2) + log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b\*d\*e^2 + 1/12\*(3\*x^4\*arcsec(c\*x) - (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*e^3 + a\*d^3\*x + 1/2\*(2\*c\*x\*arcsec(c\*x) - log(sqrt(-1/(c^2\*x^2) + 1) + 1) + log(-sqrt(-1/(c^2\*x^2) + 1) + 1))\*b\*d^3/c

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))\*(d + e\*x)^3,x)

[Out] int((a + b\*acos(1/(c\*x)))\*(d + e\*x)^3, x)

**sympy** [A] time = 7.88, size = 362, normalized size = 2.17

$$ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x \operatorname{asec}(cx) + \frac{3bd^2ex^2 \operatorname{asec}(cx)}{2} + bde^2x^3 \operatorname{asec}(cx) + \frac{be^3x^4 \operatorname{asec}(cx)}{4} - \frac{bd^3 \left\{ \begin{array}{l} \operatorname{acos} \\ -i \operatorname{asin} \end{array} \right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(a+b\*asec(c\*x)),x)

[Out] a\*d\*\*3\*x + 3\*a\*d\*\*2\*e\*x\*\*2/2 + a\*d\*e\*\*2\*x\*\*3 + a\*e\*\*3\*x\*\*4/4 + b\*d\*\*3\*x\*asec(c\*x) + 3\*b\*d\*\*2\*e\*x\*\*2\*asec(c\*x)/2 + b\*d\*e\*\*2\*x\*\*3\*asec(c\*x) + b\*e\*\*3\*x\*\*4



$$\begin{aligned}
& 4*\operatorname{asec}(c*x)/4 - b*d**3*\operatorname{Piecewise}(\operatorname{acosh}(c*x), \operatorname{Abs}(c**2*x**2) > 1), (-I*\operatorname{asin}(c*x), \operatorname{True}))/c - 3*b*d**2*e*\operatorname{Piecewise}(\operatorname{sqrt}(c**2*x**2 - 1)/c, \operatorname{Abs}(c**2*x**2) > 1), (I*\operatorname{sqrt}(-c**2*x**2 + 1)/c, \operatorname{True}))/2*c - b*d*e**2*\operatorname{Piecewise}(x*\operatorname{sqrt}(c**2*x**2 - 1)/(2*c) + \operatorname{acosh}(c*x)/(2*c**2), \operatorname{Abs}(c**2*x**2) > 1), (-I*c*x**3/(2*\operatorname{sqrt}(-c**2*x**2 + 1)) + I*x/(2*c*\operatorname{sqrt}(-c**2*x**2 + 1)) - I*\operatorname{asin}(c*x)/(2*c**2), \operatorname{True}))/c - b*e**3*\operatorname{Piecewise}(x**2*\operatorname{sqrt}(c**2*x**2 - 1)/(3*c) + 2*\operatorname{sqrt}(c**2*x**2 - 1)/(3*c**3), \operatorname{Abs}(c**2*x**2) > 1), (I*x**2*\operatorname{sqrt}(-c**2*x**2 + 1)/(3*c) + 2*I*\operatorname{sqrt}(-c**2*x**2 + 1)/(3*c**3), \operatorname{True}))/4*c
\end{aligned}$$

### 3.57 $\int (d + ex)^2 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=124

$$\frac{(d + ex)^3 (a + b \sec^{-1}(cx))}{3e} - \frac{bdex\sqrt{1 - \frac{1}{c^2x^2}}}{c} - \frac{be^2x^2\sqrt{1 - \frac{1}{c^2x^2}}}{6c} - \frac{b(6c^2d^2 + e^2)\tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3} + \frac{bd^3 \csc^{-1}(cx)}{3e}$$

[Out]  $\frac{1}{3}b*d^3*\text{arccsc}(c*x)/e + \frac{1}{3}*(e*x+d)^3*(a+b*\text{arcsec}(c*x))/e - \frac{1}{6}b*(6*c^2*d^2 + e^2)*\text{arctanh}((1-1/c^2/x^2)^{(1/2)})/c^3 - b*d*e*x*(1-1/c^2/x^2)^{(1/2)}/c - \frac{1}{6}b*e^2*x^2*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.27, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {5226, 1568, 1475, 1807, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^3 (a + b \sec^{-1}(cx))}{3e} - \frac{b(6c^2d^2 + e^2)\tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3} - \frac{bdex\sqrt{1 - \frac{1}{c^2x^2}}}{c} - \frac{be^2x^2\sqrt{1 - \frac{1}{c^2x^2}}}{6c} + \frac{bd^3 \csc^{-1}(cx)}{3e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^2*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-\frac{(b*d*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/c - (b*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^2)/(6*c) + (b*d^3*\text{ArcCsc}[c*x])/(3*e) + ((d + e*x)^3*(a + b*\text{ArcSec}[c*x]))/(3*e) - (b*(6*c^2*d^2 + e^2)*\text{ArcTanh}[\text{Sqrt}[1 - 1/(c^2*x^2)]])/(6*c^3)}$

#### Rule 63

$\text{Int}[(a_. + (b_.)*(x_)^m)*((c_.) + (d_.)*(x_)^n), x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a_. + (b_.)*(x_)^2), x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^p
_, x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^p_.*((d_) + (e_.)*(x_)^(n_.))^q
_, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^q_.*((a_) + (c_.)*(x_)^(n2_.))^
p_, x_Symbol] := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] || !IntegerQ[p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^p_, x_Symbol] := With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5226

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2 (a+b \sec^{-1}(cx)) dx &= \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} - \frac{b \int \frac{(d+ex)^3 dx}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{3ce} \\
&= \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} - \frac{b \int \frac{\left(\frac{e+d}{x}\right)^3 x dx}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{3ce} \\
&= \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} + \frac{b \operatorname{Subst} \left( \int \frac{(e+dx)^3 dx, x, \frac{1}{x}}{x^3 \sqrt{1-\frac{x^2}{c^2}}} \right)}{3ce} \\
&= -\frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} - \frac{b \operatorname{Subst} \left( \int \frac{-6de^2 - e \left(6d^2 + \frac{e^2}{2}\right) x - 2}{x^2 \sqrt{1-\frac{x^2}{c^2}}} dx \right)}{6ce} \\
&= -\frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} + \frac{b \operatorname{Subst} \left( \int \frac{bd^3}{x^2} dx \right)}{6ce} \\
&= -\frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} + \frac{(bd^3) \operatorname{Subst} \left( \int \frac{1}{x^2} dx \right)}{6ce} \\
&= -\frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{bd^3 \csc^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} \\
&= -\frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{bd^3 \csc^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e} \\
&= -\frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{bd^3 \csc^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b \sec^{-1}(cx))}{3e}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 124, normalized size = 1.00

$$\frac{c^2x \left( 2ac(3d^2 + 3dex + e^2x^2) - be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex) \right) + 2bc^3x \sec^{-1}(cx)(3d^2 + 3dex + e^2x^2) - b(6c^2d^2 + e^2) \log\left(\frac{d+ex}{x}\right)}{6c^3}$$



$$\begin{aligned}
& (c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*b*c^3*d*x^2*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x)) * e/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 6*b*c^2*d*x*\sqrt{-1/(c^2*x^2) + 1} * e/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*b*c^2*d^2*\arccos(1/(c*x)))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^3*d*x^2*(1/(c^2*x^2) - 1)^4 * e/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 6*b*c^2*d^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1)) / (c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*b*c^2*d^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1)) / (c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 18*b*c^2*d*x*(-1/(c^2*x^2) + 1)^(3/2) * e/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 6*a*c^2*d^2/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*b*c^2*d^2*(1/(c^2*x^2) - 1)*\arccos(1/(c*x)) / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 6*b*c*d*\arccos(1/(c*x)) * e / (c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 18*b*c^2*d^2*(1/(c^2*x^2) - 1)*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1)) / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 18*b*c^2*d^2*(1/(c^2*x^2) - 1)*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1)) / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 18*b*c^2*d*x*(1/(c^2*x^2) - 1)^2*\sqrt{-1/(c^2*x^2) + 1} * e/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*a*c^2*d^2*(1/(c^2*x^2) - 1) / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 6*b*c^2*d^2*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x)) / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*a*c*d * e / (c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 18*b*c*d*(1/(c^2*x^2) - 1)*\arccos(1/(c*x)) * e / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 18*b*c^2*d^2*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1)) / ((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) -
\end{aligned}$$



```

)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6*(1/(c*x) + 1)^2) - 3*b*(1/(c^2*x^2) - 1)*e^2*log(abs(sqrt(-
1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 2*b*sqrt(-1/(c^2*x^2) + 1)*e^2/((c^4 +
3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)) + 6*a*(
1/(c^2*x^2) - 1)*e^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^
4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6)*(1/(c*x) + 1)^2) - 6*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))*e^2/((c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*
a*c*d*(1/(c^2*x^2) - 1)^3*e/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6)*(1/(c*x) + 1)^6) + 3*b*(1/(c^2*x^2) - 1)^2*e^2*log(abs(sqrt(-1
/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 3*b*(1/(c^2*x^2) - 1)^2*e^2*log(abs(sqr
t(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*a*(1/(c^2*x^2) - 1)^2*e^2/((c^4 +
3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 2*b
*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))*e^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + b*(1/(c^2*x^2) - 1)^3*e^2*log(
abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - b*(1/(c^2*x^2) - 1)^3*e^2*lo
g(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - 2*b*(1/(c^2*x^2) - 1)^2*sq
rt(-1/(c^2*x^2) + 1)*e^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x)
) + 1)^6)*(1/(c*x) + 1)^5) + 2*a*(1/(c^2*x^2) - 1)^3*e^2/((c^4 + 3*c^4*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6))*c

```

**maple [B]** time = 0.05, size = 362, normalized size = 2.92

$$\frac{a e^2 x^3}{3} + a e d x^2 + a x d^2 + \frac{a d^3}{3e} + \frac{b e^2 \operatorname{arcsec}(cx) x^3}{3} + b e \operatorname{arcsec}(cx) x^2 d + b \operatorname{arcsec}(cx) x d^2 + \frac{b \operatorname{arcsec}(cx) d^3}{3e} + \frac{b \sqrt{c^2 x^2 - 1}}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.





Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(a+b\*asec(c\*x)),x)

[Out] a\*d\*\*2\*x + a\*d\*e\*x\*\*2 + a\*e\*\*2\*x\*\*3/3 + b\*d\*\*2\*x\*asec(c\*x) + b\*d\*e\*x\*\*2\*asec(c\*x) + b\*e\*\*2\*x\*\*3\*asec(c\*x)/3 - b\*d\*\*2\*Piecewise((acosh(c\*x), Abs(c\*\*2\*x\*\*2) > 1), (-I\*asin(c\*x), True))/c - b\*d\*e\*Piecewise((sqrt(c\*\*2\*x\*\*2 - 1)/c, Abs(c\*\*2\*x\*\*2) > 1), (I\*sqrt(-c\*\*2\*x\*\*2 + 1)/c, True))/c - b\*e\*\*2\*Piecewise((x\*sqrt(c\*\*2\*x\*\*2 - 1)/(2\*c) + acosh(c\*x)/(2\*c\*\*2), Abs(c\*\*2\*x\*\*2) > 1), (-I\*c\*x\*\*3/(2\*sqrt(-c\*\*2\*x\*\*2 + 1)) + I\*x/(2\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)) - I\*asin(c\*x)/(2\*c\*\*2), True))/(3\*c)

### 3.58 $\int (d + ex) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=84

$$\frac{(d + ex)^2 (a + b \sec^{-1}(cx))}{2e} - \frac{bd \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} - \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} + \frac{bd^2 \csc^{-1}(cx)}{2e}$$

[Out]  $\frac{1}{2} b d^2 \operatorname{arccsc}(c x) / e + \frac{1}{2} (e x + d)^2 (a + b \operatorname{arcsec}(c x)) / e - b d \operatorname{arctanh}\left(\frac{1 - 1/c^2/x^2}{1 + 1/c^2/x^2}\right) / c - \frac{1}{2} b e x \sqrt{1 - 1/c^2/x^2} / c$

**Rubi [A]** time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {5226, 1568, 1396, 1807, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^2 (a + b \sec^{-1}(cx))}{2e} - \frac{bd \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} - \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} + \frac{bd^2 \csc^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + b\*ArcSec[c\*x]),x]

[Out]  $-(b * e * \sqrt{1 - 1/(c^2 * x^2)}) * x / (2 * c) + (b * d^2 * \operatorname{ArcCsc}[c * x]) / (2 * e) + ((d + e * x)^2 * (a + b * \operatorname{ArcSec}[c * x])) / (2 * e) - (b * d * \operatorname{ArcTanh}[\sqrt{1 - 1/(c^2 * x^2)}]) / c$

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^(2))^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 216

Int[1/Sqrt[(a\_.) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 1396

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol
] := -Subst[Int[((d + e/x^n)^q*(a + c/x^(2*n))^p)/x^2, x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

#### Rule 1568

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^(mn_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(
p_), x_Symbol] := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] || !IntegerQ[p])
```

#### Rule 1807

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

#### Rule 5226

```
Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((d_) + (e_)*(x_))^(m_), x_Symbol
] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int (d+ex)(a+b\sec^{-1}(cx)) dx &= \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} - \frac{b \int \frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{2ce} \\
&= \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} - \frac{b \int \frac{\left(\frac{e+dx}{x}\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{2ce} \\
&= \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} + \frac{b \operatorname{Subst}\left(\int \frac{(e+dx)^2}{x^2\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2ce} \\
&= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} - \frac{b \operatorname{Subst}\left(\int \frac{-2de-d^2x}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2ce} \\
&= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} + \frac{(bd) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{bd^2 \csc^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} + \frac{(bd) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{bd^2 \csc^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} - (bcd) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right) \\
&= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{bd^2 \csc^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} - \frac{bd \tanh^{-1}\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 114, normalized size = 1.36

$$adx + \frac{1}{2}aex^2 - \frac{bdx\sqrt{1-\frac{1}{c^2x^2}} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} - \frac{bex\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c} + bdx \sec^{-1}(cx) + \frac{1}{2}bex^2 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + b\*ArcSec[c\*x]), x]

[Out]  $a*d*x + (a*e*x^2)/2 - (b*e*x*\text{Sqrt}[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*$   
 $\text{ArcSec}[c*x] + (b*e*x^2*\text{ArcSec}[c*x])/2 - (b*d*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{ArcTan}$   
 $h[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/\text{Sqrt}[-1 + c^2*x^2]$

**fricas** [A] time = 0.45, size = 130, normalized size = 1.55

$$\frac{ac^2ex^2 + 2ac^2dx + 2bcd \log\left(-cx + \sqrt{c^2x^2 - 1}\right) - \sqrt{c^2x^2 - 1}be + (bc^2ex^2 + 2bc^2dx - 2bc^2d - bc^2e) \text{arcsec}(cx) + \dots}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")`

[Out]  $1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x + 2*b*c*d*\log(-c*x + \text{sqrt}(c^2*x^2 - 1)) - \text{sqrt}(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*\text{arcsec}(c*x) + 2*(2*b*c^2*d + b*c^2*e)*\text{arctan}(-c*x + \text{sqrt}(c^2*x^2 - 1)))/c^2$

**giac** [B] time = 0.68, size = 1555, normalized size = 18.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="giac")`

[Out]  $1/2*(2*b*c*d*\arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 2*b*c*d*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 2*b*c*d*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 2*a*c*d/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + b*\arccos(1/(c*x))*e/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 4*b*c*d*(1/(c^2*x^2) - 1)*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2 + 4*b*c*d*(1/(c^2*x^2) - 1)*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2 - 2*b*c*d*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*c*d*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4 + a*e/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 2*b*(1/(c^2*x^2) - 1)*\arccos(1/(c*x))*e/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*c*d*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4$

$$\begin{aligned}
& + 2*b*c*d*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)) / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^4) - 2*b*\text{sqrt}(-1/(c^2*x^2) + 1)*e / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^4) * (1/(c*x) + 1) - 2*a*c*d*(1/(c^2*x^2) - 1)^2 / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^4) - 2*a*(1/(c^2*x^2) - 1)*e / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^4) * (1/(c*x) + 1)^2 + b*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))*e / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^4) + 2*b*(-1/(c^2*x^2) + 1)^(3/2)*e / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^3) + a*(1/(c^2*x^2) - 1)^2*e / ((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2 / (1/(c*x) + 1)^4) * (1/(c*x) + 1)^4) * c
\end{aligned}$$

**maple [A]** time = 0.05, size = 141, normalized size = 1.68

$$\frac{ax^2e}{2} + adx + \frac{b \operatorname{arcsec}(cx)x^2e}{2} + b \operatorname{arcsec}(cx)xd - \frac{b\sqrt{c^2x^2-1} d \ln\left(cx + \sqrt{c^2x^2-1}\right)}{c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}x} - \frac{bx e}{2c\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{be}{2c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(a+b\*arcsec(c\*x)),x)

[Out] 1/2\*a\*x^2\*e+a\*d\*x+1/2\*b\*arcsec(c\*x)\*x^2\*e+b\*arcsec(c\*x)\*x\*d-1/c^2\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*(c^2\*x^2-1)^(1/2)\*d\*ln(c\*x+(c^2\*x^2-1)^(1/2))-1/2/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x\*e+1/2/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*e

**maxima [A]** time = 0.32, size = 93, normalized size = 1.11

$$\frac{1}{2} aex^2 + \frac{1}{2} \left( x^2 \operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c} \right) be + adx + \frac{\left( 2cx \operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1}\right) \right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/2\*a\*e\*x^2 + 1/2\*(x^2\*arcsec(c\*x) - x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*b\*e + a\*d\*x + 1/2\*(2\*c\*x\*arcsec(c\*x) - log(sqrt(-1/(c^2\*x^2) + 1) + 1) + log(-sqrt(-1/(c^2\*x^2) + 1)))\*b\*d/c

**mupad [B]** time = 0.90, size = 77, normalized size = 0.92

$$\frac{ax(2d+ex)}{2} - \frac{bd \operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{c} + bdx \operatorname{acos}\left(\frac{1}{cx}\right) - \frac{bex\left(\sqrt{1-\frac{1}{c^2x^2}} - cx \operatorname{acos}\left(\frac{1}{cx}\right)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*acos(1/(c*x)))*(d + e*x),x)`

[Out] `(a*x*(2*d + e*x))/2 - (b*d*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*d*x*acos(1/(c*x)) - (b*e*x*((1 - 1/(c^2*x^2))^(1/2) - c*x*acos(1/(c*x))))/(2*c)`

**sympy [A]** time = 4.42, size = 104, normalized size = 1.24

$$adx + \frac{aex^2}{2} + bdx \operatorname{asec}(cx) + \frac{bex^2 \operatorname{asec}(cx)}{2} - \frac{bd \left( \begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases} \right)}{c} - \frac{be \left( \begin{cases} \frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2| > 1 \\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*asec(c*x)),x)`

[Out] `a*d*x + a*e*x**2/2 + b*d*x*asec(c*x) + b*e*x**2*asec(c*x)/2 - b*d*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/2*c`



### 3.59 $\int (a + b \sec^{-1}(cx)) dx$

Optimal. Leaf size=32

$$ax - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \sec^{-1}(cx)$$

[Out] a\*x+b\*x\*arcsec(c\*x)-b\*arctanh((1-1/c^2/x^2)^(1/2))/c

**Rubi** [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5214, 266, 63, 208}

$$ax - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcSec[c\*x], x]

[Out] a\*x + b\*x\*ArcSec[c\*x] - (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 5214

`Int[ArcSec[(c_.)*(x_)], x_Symbol] := Simp[x*ArcSec[c*x], x] - Dist[1/c, Int[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]`

### Rubi steps

$$\begin{aligned}
 \int (a + b \sec^{-1}(cx)) dx &= ax + b \int \sec^{-1}(cx) dx \\
 &= ax + bx \sec^{-1}(cx) - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{c} \\
 &= ax + bx \sec^{-1}(cx) + \frac{b \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{2c} \\
 &= ax + bx \sec^{-1}(cx) - (bc) \operatorname{Subst}\left(\int \frac{1}{c^2 - c^2 x^2} dx, x, \sqrt{1 - \frac{1}{c^2 x^2}}\right) \\
 &= ax + bx \sec^{-1}(cx) - \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 59, normalized size = 1.84

$$ax - \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2 x^2 - 1}}\right)}{\sqrt{c^2 x^2 - 1}} + bx \sec^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcSec[c\*x], x]

[Out] a\*x + b\*x\*ArcSec[c\*x] - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/Sqrt[-1 + c^2\*x^2]

**fricas [B]** time = 0.43, size = 63, normalized size = 1.97

$$\frac{acx + 2bc \arctan\left(-cx + \sqrt{c^2 x^2 - 1}\right) + (bcx - bc) \operatorname{arcsec}(cx) + b \log\left(-cx + \sqrt{c^2 x^2 - 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arcsec(c\*x),x, algorithm="fricas")

[Out]  $(a*c*x + 2*b*c*\arctan(-c*x + \sqrt{c^2*x^2 - 1})) + (b*c*x - b*c)*\operatorname{arcsec}(c*x) + b*\log(-c*x + \sqrt{c^2*x^2 - 1}))/c$

**giac** [B] time = 0.15, size = 63, normalized size = 1.97

$$\frac{1}{2}bc \left( \frac{2x \arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)}{c^2} \right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsec(c*x),x, algorithm="giac")`

[Out]  $1/2*b*c*(2*x*\arccos(1/(c*x)))/c - (\log(\sqrt{-1/(c^2*x^2) + 1} + 1) + 1) - \log(-\sqrt{-1/(c^2*x^2) + 1} + 1))/c^2 + a*x$

**maple** [A] time = 0.04, size = 38, normalized size = 1.19

$$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arcsec(c*x),x)`

[Out]  $a*x + b*x*\operatorname{arcsec}(c*x) - b/c*\ln(c*x + c*x*(1 - 1/c^2/x^2)^{(1/2)})$

**maxima** [A] time = 0.32, size = 53, normalized size = 1.66

$$ax + \frac{\left(2cx \operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsec(c*x),x, algorithm="maxima")`

[Out]  $a*x + 1/2*(2*c*x*\operatorname{arcsec}(c*x) - \log(\sqrt{-1/(c^2*x^2) + 1} + 1) + \log(-\sqrt{-1/(c^2*x^2) + 1} + 1))*b/c$

**mupad** [B] time = 0.85, size = 34, normalized size = 1.06

$$ax + bx \operatorname{acos}\left(\frac{1}{cx}\right) - \frac{b \operatorname{atanh}\left(\frac{1}{\sqrt{1 - \frac{1}{c^2x^2}}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + b*acos(1/(c*x)), x)`

[Out] `a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c`

sympy [A] time = 2.27, size = 32, normalized size = 1.00

$$ax + b \left( x \operatorname{asec}(cx) - \frac{\begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*asec(c*x), x)`

[Out] `a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c)`



$e^2))E^{(I*ArcSec[c*x])}/(c*d)]/(x^2*sqrt[1 - 1/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])*E^{(I*ArcSec[c*x])})/(c*d)]/(x^2*sqrt[1 - 1/(c^2*x^2)]), x], x] + Dist[b/(c*e), Int[Log[1 + E^{(2*I*ArcSec[c*x])})/(x^2*sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[((a + b*ArcSec[c*x])*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])*E^{(I*ArcSec[c*x])})/(c*d)])]/e, x] - Simp[((a + b*ArcSec[c*x])*Log[1 + E^{(2*I*ArcSec[c*x])})]/e, x] /; FreeQ[{a, b, c, d, e}, x]$

### Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex} dx = \frac{(a + b \sec^{-1}(cx)) \log\left(1 + \frac{(e - \sqrt{-c^2 d^2 + e^2}) e^{i \sec^{-1}(cx)}}{cd}\right)}{e} + \frac{(a + b \sec^{-1}(cx)) \log\left(1 + \frac{(e + \sqrt{-c^2 d^2 + e^2}) e^{i \sec^{-1}(cx)}}{cd}\right)}{e}$$

$$= \frac{(a + b \sec^{-1}(cx)) \log\left(1 + \frac{(e - \sqrt{-c^2 d^2 + e^2}) e^{i \sec^{-1}(cx)}}{cd}\right)}{e} + \frac{(a + b \sec^{-1}(cx)) \log\left(1 + \frac{(e + \sqrt{-c^2 d^2 + e^2}) e^{i \sec^{-1}(cx)}}{cd}\right)}{e}$$

**Mathematica [A]** time = 0.62, size = 333, normalized size = 1.35

$$\frac{a \log(d + ex)}{e} + \frac{b \left( -i \left( \text{Li}_2 \left( \frac{(\sqrt{e^2 - c^2 d^2} - e) e^{i \sec^{-1}(cx)}}{cd} \right) + \text{Li}_2 \left( -\frac{(e + \sqrt{e^2 - c^2 d^2}) e^{i \sec^{-1}(cx)}}{cd} \right) \right) + \log \left( 1 + \frac{(e - \sqrt{e^2 - c^2 d^2}) e^{i \sec^{-1}(cx)}}{cd} \right) \right)}{2e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x), x]

[Out] (a\*Log[d + e\*x])/e + (b\*((4\*I)\*ArcSin[Sqrt[1 + e/(c\*d)]/Sqrt[2]]\*ArcTan[(-(c\*d) + e)\*Tan[ArcSec[c\*x]/2]]/Sqrt[-(c^2\*d^2) + e^2]] + (ArcSec[c\*x] + 2\*ArcSin[Sqrt[1 + e/(c\*d)]/Sqrt[2]])\*Log[1 + ((e - Sqrt[-(c^2\*d^2) + e^2])\*E^{(I\*ArcSec[c\*x])})/(c\*d)] + (ArcSec[c\*x] - 2\*ArcSin[Sqrt[1 + e/(c\*d)]/Sqrt[2]])\*Log[1 + ((e + Sqrt[-(c^2\*d^2) + e^2])\*E^{(I\*ArcSec[c\*x])})/(c\*d)] - ArcSec[c\*x]\*Log[1 + E^{((2\*I)\*ArcSec[c\*x])}] - I\*(PolyLog[2, ((-e + Sqrt[-(c^2\*d^2) + e^2])\*E^{(I\*ArcSec[c\*x])})/(c\*d)] + PolyLog[2, -(((e + Sqrt[-(c^2\*d^2) + e^2])\*E^{(I\*ArcSec[c\*x])})/(c\*d))]) + (I/2)\*PolyLog[2, -E^{((2\*I)\*ArcSec[c\*x])}])/e

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b \operatorname{arcsec}(cx) + a}{ex + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e\*x + d), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym  
m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [A] time = 0.82, size = 456, normalized size = 1.85

$$\frac{a \ln(cex + dc)}{e} - \frac{b \operatorname{arcsec}(cx) \ln\left(1 + i\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)\right)}{e} - \frac{b \operatorname{arcsec}(cx) \ln\left(1 - i\left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)\right)}{e} + \frac{ib \operatorname{dilog}\left(\dots\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x+d),x)

[Out] a\*ln(c\*e\*x+c\*d)/e-b/e\*arcsec(c\*x)\*ln(1+I\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2)))-b/e\*arcsec(c\*x)\*ln(1-I\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2)))+I\*b/e\*dilog(1+I\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2)))+I\*b/e\*dilog(1-I\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2)))+b/e\*a\*rcsec(c\*x)\*ln((-d\*c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))+(-c^2\*d^2+e^2)^(1/2)-e)/(-e+(-c^2\*d^2+e^2)^(1/2)))+b/e\*arcsec(c\*x)\*ln((d\*c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))+(-c^2\*d^2+e^2)^(1/2)+e)/(e+(-c^2\*d^2+e^2)^(1/2)))-I\*b/e\*dilog((-d\*c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))+(-c^2\*d^2+e^2)^(1/2)-e)/(-e+(-c^2\*d^2+e^2)^(1/2)))-I\*b/e\*dilog((d\*c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))+(-c^2\*d^2+e^2)^(1/2)+e)/(e+(-c^2\*d^2+e^2)^(1/2)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{ex+d}\right)}{ex+d} dx + \frac{a \log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d),x, algorithm="maxima")

[Out] b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x + d), x) + a\*log(e\*x + d)/e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x),x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x+d),x)

[Out] Integral((a + b\*asec(c\*x))/(d + e\*x), x)



$$3.61 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex)^2} dx$$

Optimal. Leaf size=104

$$-\frac{a+b \sec^{-1}(cx)}{e(d+ex)} - \frac{b \tanh^{-1}\left(\frac{c^2 d + \frac{e}{x}}{c \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2}}\right)}{d \sqrt{c^2 d^2 - e^2}} - \frac{b \csc^{-1}(cx)}{de}$$

[Out]  $-b \operatorname{arccsc}(c x) / d / e + (-a - b \operatorname{arcsec}(c x)) / e / (e x + d) - b \operatorname{arctanh}\left(\frac{c^2 d + e / x}{c \sqrt{1 - 1 / c^2 x^2}}\right) / d / \left(c^2 d^2 - e^2\right)^{1 / 2} / \left(1 - 1 / c^2 x^2\right)^{1 / 2} / d / \left(c^2 d^2 - e^2\right)^{1 / 2}$

**Rubi [A]** time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5226, 1568, 1475, 844, 216, 725, 206}

$$-\frac{a+b \sec^{-1}(cx)}{e(d+ex)} - \frac{b \tanh^{-1}\left(\frac{c^2 d + \frac{e}{x}}{c \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2}}\right)}{d \sqrt{c^2 d^2 - e^2}} - \frac{b \csc^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x)^2, x]

[Out]  $-((b \operatorname{ArcCsc}[c x]) / (d e)) - (a + b \operatorname{ArcSec}[c x]) / (e (d + e x)) - (b \operatorname{ArcTanh}[(c^2 d + e / x) / (c \operatorname{Sqrt}[c^2 d^2 - e^2] \operatorname{Sqrt}[1 - 1 / (c^2 x^2)])]) / (d \operatorname{Sqrt}[c^2 d^2 - e^2])$

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ

[{a, c, d, e}, x]

### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1475

Int[(x\_)^(m\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(d + e\*x)^q\*(a + c\*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2\*n] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1568

Int[(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^(mn\_.))^(q\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.))^(p\_.), x\_Symbol] :> Int[x^(m + mn\*q)\*(e + d/x^mn)^q\*(a + c\*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2\*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])

### Rule 5226

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + (e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(e\*(m + 1)), x] - Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex)^2} dx &= -\frac{a + b \sec^{-1}(cx)}{e(d + ex)} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 (d + ex)} dx}{ce} \\
&= -\frac{a + b \sec^{-1}(cx)}{e(d + ex)} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} \left(e + \frac{d}{x}\right) x^3} dx}{ce} \\
&= -\frac{a + b \sec^{-1}(cx)}{e(d + ex)} - \frac{b \operatorname{Subst} \left( \int \frac{x}{(e + dx) \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce} \\
&= -\frac{a + b \sec^{-1}(cx)}{e(d + ex)} + \frac{b \operatorname{Subst} \left( \int \frac{1}{(e + dx) \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd} - \frac{b \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cde} \\
&= -\frac{b \csc^{-1}(cx)}{de} - \frac{a + b \sec^{-1}(cx)}{e(d + ex)} - \frac{b \operatorname{Subst} \left( \int \frac{1}{d^2 - \frac{e^2}{c^2} - x^2} dx, x, \frac{d + \frac{e}{c^2 x}}{\sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{cd} \\
&= -\frac{b \csc^{-1}(cx)}{de} - \frac{a + b \sec^{-1}(cx)}{e(d + ex)} - \frac{b \tanh^{-1} \left( \frac{c^2 d + \frac{e}{x}}{c \sqrt{c^2 d^2 - e^2} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{d \sqrt{c^2 d^2 - e^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 142, normalized size = 1.37

$$-\frac{a}{e(d + ex)} + \frac{b \log \left( cx \left( cd - \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2} \right) + e \right)}{d \sqrt{c^2 d^2 - e^2}} - \frac{b \log(d + ex)}{d \sqrt{c^2 d^2 - e^2}} - \frac{b \sin^{-1} \left( \frac{1}{cx} \right)}{de} - \frac{b \sec^{-1}(cx)}{e(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x)^2,x]

[Out] -(a/(e\*(d + e\*x))) - (b\*ArcSec[c\*x])/(e\*(d + e\*x)) - (b\*ArcSin[1/(c\*x)])/(d\*e) - (b\*Log[d + e\*x])/(d\*Sqrt[c^2\*d^2 - e^2]) + (b\*Log[e + c\*(c\*d - Sqrt[c^2\*d^2 - e^2]\*Sqrt[1 - 1/(c^2\*x^2)])\*x])/(d\*Sqrt[c^2\*d^2 - e^2])

**fricas** [B] time = 0.49, size = 477, normalized size = 4.59

$$\frac{ac^2d^3 - ade^2 - \sqrt{c^2d^2 - e^2} (be^2x + bde) \log\left(\frac{c^3d^2x + cde - \sqrt{c^2d^2 - e^2} (c^2dx + e) + (c^2d^2 - \sqrt{c^2d^2 - e^2} cd - e^2) \sqrt{c^2x^2 - 1}}{ex + d}\right) + (bc^2d^3 - bde^2)}{c^2d^4e - d^2e^3 + (c^2d^3e^2 - de^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^2,x, algorithm="fricas")

[Out]  $[-(a*c^2*d^3 - a*d*e^2 - \sqrt{c^2*d^2 - e^2}*(b*e^2*x + b*d*e))*\log((c^3*d^2*x + c*d*e - \sqrt{c^2*d^2 - e^2}*(c^2*d*x + e) + (c^2*d^2 - \sqrt{c^2*d^2 - e^2})*c*d - e^2)*\sqrt{c^2*x^2 - 1})/(e*x + d) + (b*c^2*d^3 - b*d*e^2)*\operatorname{arcsec}(c*x) - 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*\arctan(-c*x + \sqrt{c^2*x^2 - 1})/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a*c^2*d^3 - a*d*e^2 - 2*\sqrt{-c^2*d^2 + e^2}*(b*e^2*x + b*d*e)*\arctan(-(\sqrt{-c^2*d^2 + e^2})*\sqrt{c^2*x^2 - 1}*e - \sqrt{-c^2*d^2 + e^2}*(c*e*x + c*d))/(c^2*d^2 - e^2) + (b*c^2*d^3 - b*d*e^2)*\operatorname{arcsec}(c*x) - 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*\arctan(-c*x + \sqrt{c^2*x^2 - 1})/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [B] time = 0.06, size = 214, normalized size = 2.06

$$\frac{\frac{ca}{(cex + dc)e} - \frac{cb \operatorname{arcsec}(cx)}{(cex + dc)e} - \frac{b\sqrt{c^2x^2 - 1} \arctan\left(\frac{1}{\sqrt{c^2x^2 - 1}}\right)}{ce\sqrt{\frac{c^2x^2 - 1}{c^2x^2}} xd} + \frac{b\sqrt{c^2x^2 - 1} \ln\left(\frac{2\sqrt{c^2x^2 - 1} \sqrt{\frac{c^2d^2 - e^2}{e^2}} e - 2c^2dx - 2e}{cex + dc}\right)}{ce\sqrt{\frac{c^2x^2 - 1}{c^2x^2}} xd\sqrt{\frac{c^2d^2 - e^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/(e*x+d)^2,x)`

[Out] 
$$-c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arcsec(c*x)-1/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d*arctan(1/(c^2*x^2-1)^{(1/2)})+1/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/((c^2*d^2-e^2)/e^2)^{(1/2)}*\ln(2*(c^2*x^2-1)^{(1/2)}*((c^2*d^2-e^2)/e^2)^{(1/2)}*e^{-c^2*d*x-e}/(c*e*x+c*d))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \frac{\left( 2(c^2e^2x+c^2de)c \left( \frac{\arctan\left(\frac{(\sqrt{cx+1}-\sqrt{cx-1})^2e+2cd}{2\sqrt{-c^2d^2+e^2}}\right)e^{-\arctan\left(\frac{1}{2}(\sqrt{cx+1}-\sqrt{cx-1})^2\right)}}{\sqrt{-c^2d^2+e^2}cd} \right)}{c^2e} - \arctan(\sqrt{cx+1}\sqrt{cx-1}) \right)}{e^2x+de} - \frac{a}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="maxima")`

[Out] 
$$((c^2*e^2*x + c^2*d*e)*integrate(x*e^{(1/2*\log(c*x + 1) + 1/2*\log(c*x - 1))}/(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e)*e^{(\log(c*x + 1) + \log(c*x - 1))}), x) - \arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1}))*b/(e^2*x + d*e) - a/(e^2*x + d*e)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*acos(1/(c*x)))/(d + e*x)^2,x)`

[Out] `int((a + b*acos(1/(c*x)))/(d + e*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/(e*x+d)**2,x)`

[Out] `Integral((a + b*asec(c*x))/(d + e*x)**2, x)`

$$3.62 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex)^3} dx$$

Optimal. Leaf size=172

$$\frac{a+b \sec^{-1}(cx)}{2e(d+ex)^2} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d(c^2d^2-e^2)\left(\frac{d}{x}+e\right)} - \frac{b(2c^2d^2-e^2) \tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2(c^2d^2-e^2)^{3/2}} - \frac{b \csc^{-1}(cx)}{2d^2e}$$

[Out]  $-1/2*b*\text{arccsc}(c*x)/d^2/e+1/2*(-a-b*\text{arcsec}(c*x))/e/(e*x+d)^2-1/2*b*(2*c^2*d^2-e^2)*\text{arctanh}((c^2*d+e/x)/c/(c^2*d^2-e^2)^{(1/2)/(1-1/c^2/x^2)^{(1/2)})}/d^2/(c^2*d^2-e^2)^{(3/2)}+1/2*b*c*e*(1-1/c^2/x^2)^{(1/2)}/d/(c^2*d^2-e^2)/(e+d/x)$

**Rubi [A]** time = 0.29, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5226, 1568, 1475, 1651, 844, 216, 725, 206}

$$\frac{a+b \sec^{-1}(cx)}{2e(d+ex)^2} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d(c^2d^2-e^2)\left(\frac{d}{x}+e\right)} - \frac{b(2c^2d^2-e^2) \tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2(c^2d^2-e^2)^{3/2}} - \frac{b \csc^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x)^3,x]

[Out]  $(b*c*e*\text{Sqrt}[1-1/(c^2*x^2)])/(2*d*(c^2*d^2-e^2)*(e+d/x)) - (b*\text{ArcCsc}[c*x])/(2*d^2*e) - (a+b*\text{ArcSec}[c*x])/(2*e*(d+e*x)^2) - (b*(2*c^2*d^2-e^2)*\text{ArcTanh}[(c^2*d+e/x)/(c*\text{Sqrt}[c^2*d^2-e^2]*\text{Sqrt}[1-1/(c^2*x^2)])])/(2*d^2*(c^2*d^2-e^2)^{(3/2)})$

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] || !IntegerQ[p])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5226

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex)^3} dx &= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 (d + ex)^2} dx}{2ce} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} \left(e + \frac{d}{x}\right)^2 x^4} dx}{2ce} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} - \frac{b \operatorname{Subst}\left(\int \frac{x^2}{(e + dx)^2 \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} - \frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc) \operatorname{Subst}\left(\int \frac{e - \left(d - \frac{e^2}{c^2 d}\right)x}{(e + dx) \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2e(c^2 d^2 - e^2)} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} - \frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} - \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2cd^2 e} + \frac{(bc \left(2 - \frac{e^2}{c^2 d^2}\right)) \operatorname{Subst}\left(\int \frac{1}{d^2 - \frac{e^2}{c^2} - x} dx, x, \frac{1}{x}\right)}{2(c^2 d^2 - e^2)} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} - \frac{b \csc^{-1}(cx)}{2d^2 e} - \frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} - \frac{(bc \left(2 - \frac{e^2}{c^2 d^2}\right)) \operatorname{Subst}\left(\int \frac{1}{d^2 - \frac{e^2}{c^2} - x} dx, x, \frac{1}{x}\right)}{2(c^2 d^2 - e^2)} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} - \frac{b \csc^{-1}(cx)}{2d^2 e} - \frac{a + b \sec^{-1}(cx)}{2e(d + ex)^2} - \frac{b(2c^2 d^2 - e^2) \tanh^{-1}\left(\frac{c^2 d - \frac{e^2}{c}}{c \sqrt{c^2 d^2 - e^2}}\right)}{2d^2 (c^2 d^2 - e^2)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.49, size = 247, normalized size = 1.44

$$\frac{1}{2} \left( -\frac{a}{e(d + ex)^2} + \frac{bcex \sqrt{1 - \frac{1}{c^2 x^2}}}{d(c^2 d^2 - e^2)(d + ex)} + \frac{b(2c^2 d^2 - e^2) \log\left(cx \left(cd - \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2}\right) + e\right)}{d^2 (cd - e)(cd + e) \sqrt{c^2 d^2 - e^2}} + \frac{b(e^2 - 2c^2 d^2)}{d^2 (cd - e)(cd + e)} \right)$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x)^3,x]

[Out] 
$$\begin{aligned} & -(a/(e*(d + e*x)^2)) + (b*c*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)* \\ & (d + e*x)) - (b*\text{ArcSec}[c*x])/(e*(d + e*x)^2) - (b*\text{ArcSin}[1/(c*x)])/(d^2*e) \\ & + (b*(-2*c^2*d^2 + e^2)*\text{Log}[d + e*x])/(d^2*(c*d - e)*(c*d + e)*\text{Sqrt}[c^2*d^2 \\ & - e^2]) + (b*(2*c^2*d^2 - e^2)*\text{Log}[e + c*(c*d - \text{Sqrt}[c^2*d^2 - e^2])* \\ & \text{Sqrt}[1 - 1/(c^2*x^2)]]*x)/(d^2*(c*d - e)*(c*d + e)*\text{Sqrt}[c^2*d^2 - e^2])/2 \end{aligned}$$

**fricas** [B] time = 0.99, size = 1117, normalized size = 6.49

$$\frac{ac^4d^6 - bc^3d^5e - 2ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 - (bc^3d^3e^3 - bcde^5)x^2 - (2bc^2d^4e - bd^2e^3 + (2bc^2d^2e^3 - be^5)x^2 - \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4 \\ & - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d \\ & ^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*\text{sqrt}(c^2*d^2 - e^2)* \\ & \text{log}((c^3*d^2*x + c*d*e - \text{sqrt}(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - \text{sqrt} \\ & (c^2*d^2 - e^2)*c*d - e^2)*\text{sqrt}(c^2*x^2 - 1)))/(e*x + d)) - 2*(b*c^3*d^4*e^ \\ & 2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*\text{arcsec}(c*x) \\ & - 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2 \\ & *e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*\text{arctan}(- \\ & c*x + \text{sqrt}(c^2*x^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b* \\ & d*e^5)*x)*\text{sqrt}(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^ \\ & 6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3 \\ & *e^6)*x), -1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a \\ & *d^2*e^4 - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - 2*(2*b*c^2*d^4*e - b*d^2*e^3 + \\ & (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*\text{sqrt}(-c^2 \\ & *d^2 + e^2)*\text{arctan}(-(\text{sqrt}(-c^2*d^2 + e^2))*\text{sqrt}(c^2*x^2 - 1)*e - \text{sqrt}(-c^2*d \\ & ^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) - 2*(b*c^3*d^4*e^2 - b*c*d^2*e^4) \\ & *x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*\text{arcsec}(c*x) - 2*(b*c^4*d^6 - \\ & 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^ \\ & 2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*\text{arctan}(-c*x + \text{sqrt}(c^2*x \\ & ^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*d*e^5)*x)*\text{sqrt}(c \\ & ^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^ \\ & 4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)] \end{aligned}$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^3,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [B] time = 0.07, size = 1005, normalized size = 5.84

$$\frac{\frac{c^2 a}{2(cex + dc)^2 e} - \frac{c^2 b \operatorname{arcsec}(cx)}{2(cex + dc)^2 e} - \frac{c^2 b \sqrt{c^2 x^2 - 1} \arctan\left(\frac{1}{\sqrt{c^2 x^2 - 1}}\right)}{2\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} (c^2 d^2 - e^2)(cex + dc)} - \frac{c^2 b \sqrt{c^2 x^2 - 1} d \arctan\left(\frac{1}{\sqrt{c^2 x^2 - 1}}\right)}{2e\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x (c^2 d^2 - e^2)(cex + dc)} + \frac{c^2 b \sqrt{c^2 x^2 - 1}}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x+d)^3,x)

[Out] 
$$\begin{aligned} & -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arcsec(c*x)-1/2*c^2*b* \\ & (c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*arc \\ & tan(1/(c^2*x^2-1)^{(1/2)})-1/2*c^2*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2 \\ & )^{(1/2)}/x*d/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^{(1/2)})+c^2*b*(c^ \\ & 2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d \\ & ^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2-1)^{(1/2)}*((c^2*d^2-e^2)/e^2)^{(1/2)}*e-c^2 \\ & *d*x-e)/(c*e*x+c*d))+c^2*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/ \\ & x*d/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2-1)^{( \\ & 1/2)}*((c^2*d^2-e^2)/e^2)^{(1/2)}*e-c^2*d*x-e)/(c*e*x+c*d))+1/2*c^2*b*e/((c^2* \\ & x^2-1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d^2-e^2)/(c*e*x+c*d)-1/2*b*e/((c^2*x^2-1)/c^ \\ & 2/x^2)^{(1/2)}/x/d/(c^2*d^2-e^2)/(c*e*x+c*d)+1/2*b*e^2*(c^2*x^2-1)^{(1/2)}/((c^ \\ & 2*x^2-1)/c^2/x^2)^{(1/2)}/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^ \\ & (1/2))+1/2*b*e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d^2-e \\ & ^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^{(1/2)})-1/2*b*e^2*(c^2*x^2-1)^{(1/2)}/((c \\ & ^2*x^2-1)/c^2/x^2)^{(1/2)}/d^2/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x \\ & +c*d)*ln(2*((c^2*x^2-1)^{(1/2)}*((c^2*d^2-e^2)/e^2)^{(1/2)}*e-c^2*d*x-e)/(c*e*x \\ & +c*d))-1/2*b*e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/((c^2*d^2- \\ & e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2-1)^{(1/2)}*((c^2*d^2 \\ & -e^2)/e^2)^{(1/2)}*e-c^2*d*x-e)/(c*e*x+c*d)) \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left( (c^2 e^3 x^2 + 2 c^2 d e^2 x + c^2 d^2 e) \int \frac{x e^{\left( \frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1) \right)}}{c^2 e^3 x^4 + 2 c^2 d e^2 x^3 - 2 d e^2 x + (c^2 e^3 x^4 + 2 c^2 d e^2 x^3 - 2 d e^2 x - d^2 e + (c^2 d^2 e - e^3) x^2)(cx+1)(cx-1) - d^2 e + (c^2 d^2 e - e^3) x^2} dx \right)}{2 (e^3 x^2 + 2 d e^2 x + d^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^3,x, algorithm="maxima")

[Out] 1/2\*(2\*(c^2\*e^3\*x^2 + 2\*c^2\*d\*e^2\*x + c^2\*d^2\*e)\*integrate(1/2\*x\*e^(1/2\*log(c\*x + 1) + 1/2\*log(c\*x - 1))/(c^2\*e^3\*x^4 + 2\*c^2\*d\*e^2\*x^3 - 2\*d\*e^2\*x - d^2\*e + (c^2\*d^2\*e - e^3)\*x^2 + (c^2\*e^3\*x^4 + 2\*c^2\*d\*e^2\*x^3 - 2\*d\*e^2\*x - d^2\*e + (c^2\*d^2\*e - e^3)\*x^2)\*e^(log(c\*x + 1) + log(c\*x - 1))), x) - arc tan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/(e^3\*x^2 + 2\*d\*e^2\*x + d^2\*e) - 1/2\*a/(e^3\*x^2 + 2\*d\*e^2\*x + d^2\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^3,x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x+d)\*\*3,x)

[Out] Integral((a + b\*asec(c\*x))/(d + e\*x)\*\*3, x)

### 3.63 $\int (d + ex)^{3/2} (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=372

$$\frac{2(d + ex)^{5/2} (a + b \sec^{-1}(cx))}{5e} + \frac{4bd^3 \sqrt{1 - c^2 x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{5cex \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{d + ex}} + \frac{28bd \sqrt{1 - c^2 x^2} \sqrt{d + ex} E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{15c^2 x \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{d + ex}}$$

[Out]  $2/5*(e*x+d)^{(5/2)*(a+b*\text{arcsec}(c*x))/e+4/15*b*e*(-c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/x/(1-1/c^2/x^2)^{(1/2)+28/15*b*d*\text{EllipticE}(1/2*(-c*x+1)^{(1/2)*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)}*(e*x+d)^{(1/2)*(-c^2*x^2+1)^{(1/2)}/c^2/x/(1-1/c^2/x^2)^{(1/2)}/(c*(e*x+d)/(c*d+e))^{(1/2)+4/15*b*(2*c^2*d^2+e^2)*\text{EllipticF}(1/2*(-c*x+1)^{(1/2)*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)}*(c*(e*x+d)/(c*d+e))^{(1/2)*(-c^2*x^2+1)^{(1/2)}/c^4/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)+4/5*b*d^3*\text{EllipticPi}(1/2*(-c*x+1)^{(1/2)*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)}*(c*(e*x+d)/(c*d+e))^{(1/2)*(-c^2*x^2+1)^{(1/2)}/c/e/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.76, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {5226, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d + ex)^{5/2} (a + b \sec^{-1}(cx))}{5e} + \frac{4b \sqrt{1 - c^2 x^2} (2c^2 d^2 + e^2) \sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{15c^4 x \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{d + ex}} + \frac{4bd^3 \sqrt{1 - c^2 x^2} \sqrt{\frac{c(d+ex)}{cd+e}}}{5cex \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $(4*b*e*\text{Sqrt}[d + e*x]*(1 - c^2*x^2))/(15*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) + (2*(d + e*x)^{(5/2)*(a + b*\text{ArcSec}[c*x])}/(5*e) + (28*b*d*\text{Sqrt}[d + e*x]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)])/(15*c^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d + e*x))/(c*d + e)] + (4*b*(2*c^2*d^2 + e^2)*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)])/(15*c^4*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x]) + (4*b*d^3*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)])/(5*c*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

**Rule 168**

$\text{Int}[1/(((a_.) + (b_.)*(x_.))*\text{Sqrt}[(c_.) + (d_.)*(x_.)]*\text{Sqrt}[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(g_.) + (h_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(\text{Simp}[b*c -$

$a*d - b*x^2, x] * \text{Sqrt}[\text{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]] * \text{Sqrt}[\text{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \&\& \text{GtQ}[(d*e - c*f)/d, 0]$

#### Rule 419

$\text{Int}[1/(\text{Sqrt}[a_] + (b_.)*(x_)^2]*\text{Sqrt}[(c_) + (d_.)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& !(\text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])]$

#### Rule 424

$\text{Int}[\text{Sqrt}[(a_) + (b_.)*(x_)^2]/\text{Sqrt}[(c_) + (d_.)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

#### Rule 537

$\text{Int}[1/(((a_) + (b_.)*(x_)^2)*\text{Sqrt}[(c_) + (d_.)*(x_)^2]*\text{Sqrt}[(e_) + (f_.)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticPi}[(b*c)/(a*d), \text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (c*f)/(d*e))]/(a*\text{Sqrt}[c]*\text{Sqrt}[e]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[d/c, 0] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[e, 0] \&\& !(!\text{GtQ}[f/e, 0] \&\& \text{SimplerSqrtQ}[-(f/e), -(d/c)])]$

#### Rule 538

$\text{Int}[1/(((a_) + (b_.)*(x_)^2)*\text{Sqrt}[(c_) + (d_.)*(x_)^2]*\text{Sqrt}[(e_) + (f_.)*(x_)^2]), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/((a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c]*\text{Sqrt}[e + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[c, 0]$

#### Rule 719

$\text{Int}[(d_) + (e_.)*(x_)^m]/\text{Sqrt}[(a_) + (c_.)*(x_)^2], x\_Symbol] \rightarrow \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/((c*\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m^2, 1/4]$

#### Rule 844

$\text{Int}[(d_.) + (e_.)*(x_)^m]*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^p, x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D$

Int[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 931

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)/(Sqrt[(f\_.) + (g\_.)\*(x\_)]\*Sqrt[(a\_.) + (c\_.)\*(x\_)^2]), x\_Symbol] := Simp[(2\*e^2\*(d + e\*x)^(m - 2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(c\*g\*(2\*m - 1)), x] - Dist[1/(c\*g\*(2\*m - 1)), Int[((d + e\*x)^(m - 3))\*Simp[a\*e^2\*(d\*g + 2\*e\*f\*(m - 2)) - c\*d^3\*g\*(2\*m - 1) + e\*(e\*(a\*e\*g\*(2\*m - 3)) + c\*d\*(2\*e\*f - 3\*d\*g\*(2\*m - 1)))\*x + 2\*e^2\*(c\*e\*f - 3\*c\*d\*g)\*(m - 1)\*x^2, x)/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[2\*m] && GeQ[m, 2]

### Rule 933

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(f\_.) + (g\_.)\*(x\_)]\*Sqrt[(a\_.) + (c\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[a + c\*x^2], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && !GtQ[a, 0]

### Rule 958

Int[((f\_.) + (g\_.)\*(x\_))^(n\_)/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (c\_.)\*(x\_)^2]), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

### Rule 1574

Int[(x\_)^(m\_)\*((a\_.) + (c\_.)\*(x\_)^(mn2\_.))^(p\_)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[(x^(2\*n\*FracPart[p]))\*(a + c/x^(2\*n))^FracPart[p])/(c + a\*x^(2\*n))^FracPart[p], Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

### Rule 1584

Int[(u\_.)\*(x\_)^(m\_)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

### Rule 5226

```

Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int (d+ex)^{3/2} (a+b \sec^{-1}(cx)) dx &= \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} - \frac{(2b) \int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{5ce} \\
&= \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{(d+ex)^{5/2}}{x\sqrt{-\frac{1}{c^2}+x^2}} dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{d}{x\sqrt{d+ex}}\right) dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} - \frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1-\frac{1}{c^2x^2}}} - \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}} dx}{5c\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{4be\sqrt{d+ex} (1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} - \frac{\left(6bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}} dx}{5c\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{4be\sqrt{d+ex} (1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} + \frac{12bd^2\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1-\frac{1}{c^2x^2}}}{5c^2\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{4be\sqrt{d+ex} (1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} + \frac{12bd\sqrt{d+ex} \sqrt{1-\frac{1}{c^2x^2}}}{5c^2\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{4be\sqrt{d+ex} (1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} + \frac{12bd\sqrt{d+ex} \sqrt{1-\frac{1}{c^2x^2}}}{5c^2\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{4be\sqrt{d+ex} (1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2} (a+b \sec^{-1}(cx))}{5e} + \frac{28bd\sqrt{d+ex} \sqrt{1-\frac{1}{c^2x^2}}}{15c^2\sqrt{1-\frac{1}{c^2x^2}}}
\end{aligned}$$



**Mathematica [C]** time = 1.48, size = 333, normalized size = 0.90

$$\frac{1}{15} \left( \frac{6a(d+ex)^{5/2}}{e} - \frac{4bex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}{c} + \frac{4ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}} \left( (9c^2d^2 - 7cde + e^2) F\left(i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)\right)}{c^3e\sqrt{1-\frac{1}{c^2x^2}}}\right)}{c} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

[Out] ((-4\*b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])/c + (6\*a\*(d + e\*x)^(5/2))/e + (6\*b\*(d + e\*x)^(5/2)\*ArcSec[c\*x])/e + ((4\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-c\*d + e)]\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*(-7\*c\*d\*(c\*d - e)\*EllipticE[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] + (9\*c^2\*d^2 - 7\*c\*d\*e + e^2)\*EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] - 3\*c^2\*d^2\*EllipticPi[1 + e/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e))]/(c^3\*e\*Sqrt[-(c/(c\*d + e))]\*Sqrt[1 - 1/(c^2\*x^2)]\*x))/15

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(3/2)\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(3/2)\*(a+b\*arcsec(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x + d)^(3/2)\*(b\*arcsec(c\*x) + a), x)

**maple [B]** time = 0.20, size = 810, normalized size = 2.18

$$\frac{2(ex+d)^{\frac{5}{2}}a}{5} + 2b \left( \frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5} - \frac{2 \left( \sqrt{\frac{c}{dc-e}} (ex+d)^{\frac{5}{2}} c^2 + 9d^2 \sqrt{-\frac{(ex+d)c-dc+e}{dc-e}} \sqrt{-\frac{(ex+d)c-dc-e}{dc+e}} \operatorname{EllipticF}\left(\sqrt{ex+d} \sqrt{\frac{c}{dc-e}}, \sqrt{\frac{dc-e}{dc+e}}\right) c^2 - 7 \sqrt{-\frac{(ex+d)c-dc+e}{dc-e}} \right)}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x)`

[Out]  $2/e*(1/5*(e*x+d)^{(5/2)}*a+b*(1/5*(e*x+d)^{(5/2)}*arcsec(c*x)-2/15/c^3*((c/(c*d-e))^{(1/2)}*(e*x+d)^{(5/2)}*c^2+9*d^2*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*c^2-7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*c^2*d^2-3*d^2*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticPi((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)})*c^2-2*(c/(c*d-e))^{(1/2)}*(e*x+d)^{(3/2)}*c^2*d+7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*c*d*e-7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*c*d*e+(c/(c*d-e))^{(1/2)}*(e*x+d)^{(1/2)}*c^2*d^2+(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*e^2-(c/(c*d-e))^{(1/2)}*(e*x+d)^{(1/2)}*e^2)/(c/(c*d-e))^{(1/2)}/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{(1/2))}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(c\*d-e>0)', see `assume?` for more details)Is c\*d-e positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right) (d + e x)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2),x)`

[Out] `int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*asec(c*x)),x)
```

```
[Out] Timed out
```

### 3.64 $\int \sqrt{d+ex} \left( a + b \sec^{-1}(cx) \right) dx$

**Optimal.** Leaf size=315

$$\frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} + \frac{4bd^2 \sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3cex \sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}} + \frac{4bd \sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3c^2x \sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}}$$

[Out]  $2/3*(e*x+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/e+4/3*b*\text{EllipticE}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c^2/x/(1-1/c^2/x^2)^{(1/2)}/(c*(e*x+d)/(c*d+e))^{(1/2)}+4/3*b*d*\text{EllipticF}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(c*(e*x+d)/(c*d+e))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c^2/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/3*b*d^2*\text{EllipticPi}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(c*(e*x+d)/(c*d+e))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/e/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.46, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5226, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} + \frac{4bd^2 \sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3cex \sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}} + \frac{4bd \sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3c^2x \sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[d + e*x]*(a + b*ArcSec[c*x]), x]`

[Out]  $(2*(d+e*x)^{(3/2)}*(a+b*\text{ArcSec}[c*x]))/(3*e) + (4*b*\text{Sqrt}[d+e*x]*\text{Sqrt}[1-c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1-c*x]/\text{Sqrt}[2]], (2*e)/(c*d+e)))/(3*c^2*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d+e*x))/(c*d+e)]) + (4*b*d*\text{Sqrt}[(c*(d+e*x))/(c*d+e)]*\text{Sqrt}[1-c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1-c*x]/\text{Sqrt}[2]], (2*e)/(c*d+e)))/(3*c^2*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[d+e*x]) + (4*b*d^2*\text{Sqrt}[(c*(d+e*x))/(c*d+e)]*\text{Sqrt}[1-c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1-c*x]/\text{Sqrt}[2]], (2*e)/(c*d+e)))/(3*c*e*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[d+e*x])$

#### Rule 168

`Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]`

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a]/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]
```

### Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

### Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

### Rule 5226

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^(m_.)), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{d+ex} (a+b \sec^{-1}(cx)) dx &= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} - \frac{(2b) \int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{3ce} \\
&= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{(d+ex)^{3/2}}{x\sqrt{-\frac{1}{c^2}+x^2}} dx}{3ce\sqrt{1-\frac{1}{c^2x^2}} x} \\
&= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \left(\frac{2de}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{2d}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right) dx}{3ce\sqrt{1-\frac{1}{c^2x^2}} x} \\
&= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} - \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{3c\sqrt{1-\frac{1}{c^2x^2}} x} - \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}} dx}{3c\sqrt{1-\frac{1}{c^2x^2}} x} + \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}} dx}{3c\sqrt{1-\frac{1}{c^2x^2}} x} \\
&= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} + \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1-c^2x^2} E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}} x\sqrt{d+ex}} \\
&= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} + \frac{4b\sqrt{d+ex} \sqrt{1-c^2x^2} E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}} x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&= \frac{2(d+ex)^{3/2} (a+b \sec^{-1}(cx))}{3e} + \frac{4b\sqrt{d+ex} \sqrt{1-c^2x^2} E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}} x\sqrt{\frac{c(d+ex)}{cd+e}}}
\end{aligned}$$

**Mathematica** [C] time = 6.60, size = 277, normalized size = 0.88

$$2 \left( a(d+ex)^{3/2} + \frac{2ib \sqrt{\frac{e(cx+1)}{e-cd}} \sqrt{\frac{e-cex}{cd+e}} \left( (2cd-e)F\left(i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right)\right) \Big|_{\frac{cd+e}{cd-e}} + (e-cd)E\left(i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right)\right) \Big|_{\frac{cd+e}{cd-e}} - cd\Pi\left(\frac{e}{cd}+1; i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right)\right) \right)}{c^2x \sqrt{1-\frac{1}{c^2x^2}} \sqrt{-\frac{c}{cd+e}}} \right)$$


---

3e

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]\*(a + b\*ArcSec[c\*x]), x]

[Out] (2\*(a\*(d + e\*x)^(3/2) + b\*(d + e\*x)^(3/2)\*ArcSec[c\*x] + ((2\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-c\*d + e)]\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*((-c\*d + e)\*EllipticE[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] + (2\*c\*d - e)\*EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] - c\*d\*EllipticPi[1 + e/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d + e))]]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)))/(c^2\*Sqrt[-(c/(c\*d + e))]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)))/(3\*e)

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(1/2)\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} (b \operatorname{arcsec}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(1/2)\*(a+b\*arcsec(c\*x)), x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arcsec(c\*x) + a), x)

**maple** [A] time = 0.07, size = 388, normalized size = 1.23

$$\frac{2(ex+d)^{\frac{3}{2}}a}{3} + 2b \left( \frac{(ex+d)^{\frac{3}{2}} \operatorname{arcsec}(cx)}{3} - \frac{2 \left( 2d \operatorname{EllipticF}\left(\sqrt{ex+d} \sqrt{\frac{c}{dc-e}}, \sqrt{\frac{dc-e}{dc+e}}\right) c - \operatorname{EllipticE}\left(\sqrt{ex+d} \sqrt{\frac{c}{dc-e}}, \sqrt{\frac{dc-e}{dc+e}}\right) cd - d \operatorname{EllipticPi}\left(\sqrt{ex+d} \sqrt{\frac{c}{dc-e}}\right) \right)}{3c^2 \sqrt{\frac{c}{dc-e}} x \sqrt{e}} \right)$$


---

e



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arcsec(c*x)-2/3/c^2*(2*d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*(-(e*x+d)*c-d*c+e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more details)Is c*d-e positive or negative?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asec}(cx)) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a+b*asec(c*x)),x)
```

```
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x), x)
```

$$3.65 \quad \int \frac{a+b \sec^{-1}(cx)}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=212

$$\frac{2\sqrt{d+ex} (a+b \sec^{-1}(cx))}{e} + \frac{4b\sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}} + \frac{4bd\sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}}$$

[Out] 2\*(a+b\*arcsec(c\*x))\*(e\*x+d)^(1/2)/e+4\*b\*EllipticF(1/2\*(-c\*x+1)^(1/2)\*2^(1/2), 2^(1/2)\*(e/(c\*d+e))^(1/2))\*(c\*(e\*x+d)/(c\*d+e))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e\*x+d)^(1/2)+4\*b\*d\*EllipticPi(1/2\*(-c\*x+1)^(1/2)\*2^(1/2), 2, 2^(1/2)\*(e/(c\*d+e))^(1/2))\*(c\*(e\*x+d)/(c\*d+e))^(1/2)\*(-c^2\*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e\*x+d)^(1/2)

**Rubi [A]** time = 0.30, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5226, 1574, 944, 719, 419, 933, 168, 538, 537}

$$\frac{2\sqrt{d+ex} (a+b \sec^{-1}(cx))}{e} + \frac{4b\sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}} + \frac{4bd\sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/Sqrt[d + e\*x], x]

[Out] (2\*Sqrt[d + e\*x]\*(a + b\*ArcSec[c\*x]))/e + (4\*b\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)])/((c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x]) + (4\*b\*d\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)]/(c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x]))

### Rule 168

Int[1/(((a\_.) + (b\_.)\*(x\_))\*Sqrt[(c\_.) + (d\_.)\*(x\_)]\*Sqrt[(e\_.) + (f\_.)\*(x\_)]\*Sqrt[(g\_.) + (h\_.)\*(x\_)]), x\_Symbol] := Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt

$[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ !(\text{NegQ}[b/a] \ \&\& \ \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

### Rule 537

$\text{Int}[1/(((a_) + (b_)*(x_)^2)*\text{Sqrt}[(c_) + (d_)*(x_)^2]*\text{Sqrt}[(e_) + (f_)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticPi}[(b*c)/(a*d), \text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (c*f)/(d*e))]/(a*\text{Sqrt}[c]*\text{Sqrt}[e]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ !\text{GtQ}[d/c, 0] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[e, 0] \ \&\& \ !(\text{GtQ}[f/e, 0] \ \&\& \ \text{SimplerSqrtQ}[-(f/e), -(d/c)])$

### Rule 538

$\text{Int}[1/(((a_) + (b_)*(x_)^2)*\text{Sqrt}[(c_) + (d_)*(x_)^2]*\text{Sqrt}[(e_) + (f_)*(x_)^2]), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/((a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c]*\text{Sqrt}[e + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ !\text{GtQ}[c, 0]$

### Rule 719

$\text{Int}[(d_) + (e_)*(x_)^m]/\text{Sqrt}[(a_) + (c_)*(x_)^2], x\_Symbol] \rightarrow \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/(\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{EqQ}[m^2, 1/4]$

### Rule 933

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(f_) + (g_)*(x_)]*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 2]\}, \text{Dist}[\text{Sqrt}[1 + (c*x^2)/a]/\text{Sqrt}[a + c*x^2], \text{Int}[1/((d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 - q*x]*\text{Sqrt}[1 + q*x]), x], x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ !\text{GtQ}[a, 0]$

### Rule 944

$\text{Int}[\text{Sqrt}[(f_) + (g_)*(x_)]/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[1/(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]), x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[1/((d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]), x], x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

### Rule 1574

```

Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(
q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p]))*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !In
tegerQ[p] && !IntegerQ[q] && PosQ[n]

```

### Rule 5226

```

Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex}} dx &= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} - \frac{(2b) \int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}} x^2} dx}{ce} \\
&= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d+ex}}{x\sqrt{-\frac{1}{c^2}+x^2}} dx}{ce\sqrt{1-\frac{1}{c^2x^2}} x} \\
&= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{c\sqrt{1-\frac{1}{c^2x^2}} x} - \frac{\left(2bd\sqrt{-\frac{1}{c^2} + x^2}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} - \frac{\left(2bd\sqrt{1 - c^2x^2}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}} dx}{ce\sqrt{1-\frac{1}{c^2x^2}} x} + \frac{\left(4b\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}} x\sqrt{d + ex}} + \frac{\left(4bd\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}} x\sqrt{d + ex}} + \frac{\left(4bd\sqrt{\frac{c(d+ex)}{cd+e}}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2\sqrt{d + ex} (a + b \sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}} x\sqrt{d + ex}} + \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}}{ce\sqrt{1-\frac{1}{c^2x^2}}}
\end{aligned}$$

**Mathematica [C]** time = 2.81, size = 212, normalized size = 1.00

$$\frac{2 \left( a\sqrt{d + ex} + \frac{2ib\sqrt{\frac{e(cx+1)}{e-cd}} \sqrt{\frac{e-cex}{cd+e}} \left( F\left( i \sinh^{-1}\left( \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right) \middle| \frac{cd+e}{cd-e} \right) - \Pi\left( \frac{e}{cd} + 1; i \sinh^{-1}\left( \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right) \middle| \frac{cd+e}{cd-e} \right) \right)}{cx\sqrt{1-\frac{1}{c^2x^2}} \sqrt{-\frac{c}{cd+e}}} \right) + b \sec^{-1}(cx)\sqrt{d + ex}}{e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/Sqrt[d + e\*x],x]

[Out] (2\*(a\*Sqrt[d + e\*x] + b\*Sqrt[d + e\*x]\*ArcSec[c\*x] + ((2\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-(c\*d) + e)]\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*(EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] - EllipticPi[1 + e/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)])))/(c\*Sqrt[-(c/(c\*d + e))]\*Sqrt[1 - 1/(c^2\*x^2)]\*x))/e

**fricas** [F] time = 1.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/sqrt(e\*x + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/sqrt(e\*x + d), x)

**maple** [A] time = 0.07, size = 254, normalized size = 1.20

$$2a\sqrt{ex + d} + 2b \left( \sqrt{ex + d} \operatorname{arcsec}(cx) - \frac{2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}} \sqrt{-\frac{(ex+d)c-dc-e}{dc+e}} \left( \operatorname{EllipticF}\left(\sqrt{ex+d}, \sqrt{\frac{c}{dc-e}}, \sqrt{\frac{dc-e}{dc+e}}\right) - \operatorname{EllipticPi}\left(\sqrt{ex+d}, \sqrt{\frac{c}{dc-e}}\right)\right)}{c\sqrt{\frac{c^2(ex+d)^2 - 2c^2d(ex+d) + c^2d^2 - e^2}{c^2e^2x^2}}} x \sqrt{\frac{c}{dc-e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x+d)^(1/2),x)

[Out] 2/e\*(a\*(e\*x+d)^(1/2)+b\*((e\*x+d)^(1/2)\*arcsec(c\*x)-2/c\*(-((e\*x+d)\*c-d\*c+e)/(c\*d-e))^(1/2)\*(-((e\*x+d)\*c-d\*c-e)/(c\*d+e))^(1/2)\*(EllipticF((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2),((c\*d-e)/(c\*d+e))^(1/2))-EllipticPi((e\*x+d)^(1/2)\*(c/(c\*d-

$e)^{(1/2)}, 1/c*(c*d-e)/d, (c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)})) / ((c^2*(e*x+d)^2 - 2*c^2*d*(e*x+d) + c^2*d^2 - e^2) / c^2/e^2/x^2)^{(1/2)}/x/(c/(c*d-e))^{(1/2)})$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(c\*d-e>0)', see `assume?` for more details) Is c\*d-e positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(1/2), x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x+d)\*\*(1/2), x)

[Out] Integral((a + b\*asec(c\*x))/sqrt(d + e\*x), x)

$$3.66 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex)^{3/2}} dx$$

Optimal. Leaf size=119

$$\frac{2(a+b \sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

[Out]  $-2*(a+b*\text{arcsec}(c*x))/e/(e*x+d)^{(1/2)}-4*b*\text{EllipticPi}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(c*(e*x+d)/(c*d+e))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/e/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5226, 1574, 933, 168, 538, 537}

$$\frac{2(a+b \sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2} \sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x)^(3/2), x]

[Out]  $(-2*(a + b*\text{ArcSec}[c*x]))/(e*\text{Sqrt}[d + e*x]) - (4*b*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(c*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

Rule 168

Int[1/(((a\_.) + (b\_.)\*(x\_))\*Sqrt[(c\_.) + (d\_.)\*(x\_)]\*Sqrt[(e\_.) + (f\_.)\*(x\_)]\*Sqrt[(g\_.) + (h\_.)\*(x\_)]), x\_Symbol] := Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

Rule 537

Int[1/(((a\_.) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_.) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_.) + (f\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e))]/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplifierSqrtQ[-(f/e), -(d/c)])



Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

Rule 933

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]
```

Rule 1574

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(mn2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Dist[(x^(2*n*FracPart[p]))*(a + c/x^(2*n))^FracPart[p]/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 5226

```
Int[((a_) + ArcSec[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_))^(m_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSec[c*x]))/(e*(m + 1)), x] - Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex)^{3/2}} dx &= -\frac{2(a + b \sec^{-1}(cx))}{e\sqrt{d + ex}} + \frac{(2b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 \sqrt{d + ex}} dx}{ce} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x\sqrt{d + ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{ce\sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(2b\sqrt{1 - c^2 x^2}\right) \int \frac{1}{x\sqrt{1 - cx} \sqrt{1 + cx} \sqrt{d + ex}} dx}{ce\sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{e\sqrt{d + ex}} - \frac{\left(4b\sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{(1 - x^2)\sqrt{2 - x^2} \sqrt{d + \frac{e}{c} - \frac{ex^2}{c}} dx, x, \sqrt{1 - cx}\right)}{ce\sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{e\sqrt{d + ex}} - \frac{\left(4b\sqrt{\frac{c(d + ex)}{cd + e}} \sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{(1 - x^2)\sqrt{2 - x^2} \sqrt{1 - \frac{ex^2}{c(d + \frac{e}{c})}} dx, x, \sqrt{1 - cx}\right)}{ce\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{e\sqrt{d + ex}} - \frac{4b\sqrt{\frac{c(d + ex)}{cd + e}} \sqrt{1 - c^2 x^2} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd + e}\right)}{ce\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}}
\end{aligned}$$

**Mathematica [A]** time = 0.29, size = 124, normalized size = 1.04

$$\frac{2\left(\left(c^2 x^2 - 1\right)\left(a + b \sec^{-1}(cx)\right) + 2bcx\sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{1 - c^2 x^2} \sqrt{\frac{c(d + ex)}{cd + e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd + e}\right)\right)}{e\left(c^2 x^2 - 1\right)\sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x)^(3/2), x]

[Out] (-2\*((-1 + c^2\*x^2)\*(a + b\*ArcSec[c\*x]) + 2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)]))/(e\*Sqrt[d + e\*x]\*(-1 + c^2\*x^2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(e\*x + d)^(3/2), x)

**maple** [A] time = 0.07, size = 217, normalized size = 1.82

$$\frac{-\frac{2a}{\sqrt{ex+d}} + 2b \left( -\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}} - \frac{2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}} \sqrt{-\frac{(ex+d)c-dc-e}{dc+e}} \operatorname{EllipticPi}\left(\sqrt{ex+d}, \sqrt{\frac{c}{dc-e}}, \frac{dc-e}{cd}, \sqrt{\frac{c}{dc+e}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}} x d \sqrt{\frac{c}{dc-e}}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x+d)^(3/2),x)

[Out]  $2/e*(-1/(e*x+d)^{(1/2)}*a+b*(-1/(e*x+d)^{(1/2)}*\operatorname{arcsec}(c*x)-2/c/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{(1/2)}/x/d/(c/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticPi}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)}))$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(c\*d-e>0)', see 'assume?' for more details) Is c\*d-e positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(3/2), x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x+d)\*\*(3/2), x)

[Out] Integral((a + b\*asec(c\*x))/(d + e\*x)\*\*(3/2), x)

$$3.67 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=298

$$\frac{2(a+b \sec^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{1-c^2x^2}}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}}$$

[Out]  $-2/3*(a+b*\operatorname{arcsec}(c*x))/e/(e*x+d)^{(3/2)}-4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/3*b*\operatorname{EllipticE}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)},2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^{(1/2)}/(c*(e*x+d)/(c*d+e))^{(1/2)}-4/3*b*\operatorname{EllipticPi}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)},2,2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(c*(e*x+d)/(c*d+e))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/d/e/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.41, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5226, 1574, 958, 745, 21, 719, 424, 933, 168, 538, 537}

$$\frac{2(a+b \sec^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{1-c^2x^2}}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+b*\operatorname{ArcSec}[c*x])/(d+e*x)^{(5/2)},x]$

[Out]  $(-4*b*e*(1-c^2*x^2))/(3*c*d*(c^2*d^2-e^2)*\operatorname{Sqrt}[1-1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])-(2*(a+b*\operatorname{ArcSec}[c*x]))/(3*e*(d+e*x)^{(3/2)})+(4*b*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-c*x]/\operatorname{Sqrt}[2]],(2*e)/(c*d+e)))/(3*d*(c^2*d^2-e^2)*\operatorname{Sqrt}[1-1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c*(d+e*x))/(c*d+e)])-(4*b*\operatorname{Sqrt}[(c*(d+e*x))/(c*d+e)]*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{EllipticPi}[2,\operatorname{ArcSin}[\operatorname{Sqrt}[1-c*x]/\operatorname{Sqrt}[2]],(2*e)/(c*d+e)))/(3*c*d*e*\operatorname{Sqrt}[1-1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])$

**Rule 21**

$\operatorname{Int}[(u_*)*((a_)+(b_)*(v_))^{(m_)}*((c_)+(d_)*(v_))^{(n_)},x\_Symbol] := \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c+d*v)^{(m+n)},x],x] /; \operatorname{FreeQ}\{a,b,c,d,n\},x] \&\& \operatorname{EqQ}[b*c-a*d,0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c+d*x, a+b*x])$

**Rule 168**

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplifierSqrtQ[-(f/e), -(d/c)])
```

#### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

#### Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

#### Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
```

$m + 2*p + 3], 0])$

### Rule 933

$\text{Int}[1/(((d\_.) + (e\_.)*(x\_))*\text{Sqrt}[(f\_.) + (g\_.)*(x\_)]*\text{Sqrt}[(a\_.) + (c\_.)*(x\_)]^2), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 2]\}, \text{Dist}[\text{Sqrt}[1 + (c*x^2)/a]/\text{Sqrt}[a + c*x^2], \text{Int}[1/((d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 - q*x]*\text{Sqrt}[1 + q*x]), x], x]] /;$   $\text{FreeQ}\{a, c, d, e, f, g\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{!GtQ}[a, 0]$

### Rule 958

$\text{Int}(((f\_.) + (g\_.)*(x\_))^n)/(((d\_.) + (e\_.)*(x\_))*\text{Sqrt}[(a\_.) + (c\_.)*(x\_)]^2), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[1/(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]), (f + g*x)^{n + 1/2}/(d + e*x), x], x] /;$   $\text{FreeQ}\{a, c, d, e, f, g\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{IntegerQ}[n + 1/2]$

### Rule 1574

$\text{Int}[(x\_)^{m\_}*((a\_.) + (c\_.)*(x\_))^{mn2\_})^{p\_}*((d\_.) + (e\_.)*(x\_))^{n\_})^{q\_}, x\_Symbol] \rightarrow \text{Dist}[(x^{(2*n*FracPart[p])}*(a + c/x^{(2*n)})^{FracPart[p]})/(c + a*x^{(2*n)})^{FracPart[p]}, \text{Int}[x^{(m - 2*n*p)}*(d + e*x^n)^q*(c + a*x^{(2*n)})^p, x], x] /;$   $\text{FreeQ}\{a, c, d, e, m, n, p, q\}, x\} \&\& \text{EqQ}[mn2, -2*n] \&\& \text{!IntegerQ}[p] \&\& \text{!IntegerQ}[q] \&\& \text{PosQ}[n]$

### Rule 5226

$\text{Int}(((a\_.) + \text{ArcSec}[(c\_.)*(x\_)]*(b\_))*((d\_.) + (e\_.)*(x\_))^{m\_}), x\_Symbol] \rightarrow \text{Simp}(((d + e*x)^{m + 1}*(a + b*\text{ArcSec}[c*x]))/(e*(m + 1)), x) - \text{Dist}[b/(c*e*(m + 1)), \text{Int}[(d + e*x)^{m + 1}/(x^2*\text{Sqrt}[1 - 1/(c^2*x^2)]), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m\}, x\} \&\& \text{NeQ}[m, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex)^{5/2}} dx &= \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{(2b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 (d + ex)^{3/2}} dx}{3ce} \\
&= \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3ce \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \left( -\frac{e}{d(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} + \frac{1}{dx \sqrt{d + ex} \sqrt{-\frac{1}{c^2} + x^2}} \right) dx}{3ce \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3cd \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x \sqrt{d + ex}} dx}{3cde \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{4be(1 - c^2 x^2)}{3cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(4b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{-\frac{d}{2}}{\sqrt{d + ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3cd \left(d^2 - \frac{e^2}{c^2}\right) \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{4be(1 - c^2 x^2)}{3cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d + ex}}{\sqrt{-\frac{1}{c^2} + x^2}} dx}{3cd \left(d^2 - \frac{e^2}{c^2}\right) \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{4be(1 - c^2 x^2)}{3cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(4b \sqrt{\frac{c(d + ex)}{cd + e}} \sqrt{1 - c^2 x^2}\right) \int \frac{1}{\sqrt{d + ex}} dx}{3cde \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{4be(1 - c^2 x^2)}{3cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{4b \sqrt{d + ex} \sqrt{1 - c^2 x^2} E\left(\sin^{-1}\left(\frac{\sqrt{d + ex}}{\sqrt{1 - c^2 x^2}}\right)\right)}{3d(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2}} x}
\end{aligned}$$



**Mathematica [C]** time = 6.64, size = 326, normalized size = 1.09

$$2 \left( -\frac{a}{(d+ex)^{3/2}} + \frac{2bce^2x\sqrt{1-\frac{1}{c^2x^2}}}{(c^2d^3-de^2)\sqrt{d+ex}} - \frac{2ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left(cdF\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)\right)\frac{cd+e}{cd-e}\right)-cdE\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)\right)\frac{cd+e}{cd-e}\right)+(cd+e)I}{d^2x\sqrt{1-\frac{1}{c^2x^2}}\left(-\frac{c}{cd+e}\right)^{3/2}(cd+e)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x)^(5/2), x]

[Out] (2\*(-(a/(d + e\*x)^(3/2)) + (2\*b\*c\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/((c^2\*d^3 - d\*e^2)\*Sqrt[d + e\*x]) - (b\*ArcSec[c\*x])/(d + e\*x)^(3/2) - ((2\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-(c\*d) + e)]\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*(-(c\*d\*EllipticE[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e))] + c\*d\*EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] + (c\*d + e)\*EllipticPi[1 + e/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)])))/(d^2\*(-(c/(c\*d + e)))^(3/2)\*(c\*d + e)^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x)))/(3\*e)

**fricas [F]** time = 1.41, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex + d} (b \operatorname{arcsec}(cx) + a)}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arcsec(c\*x) + a)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(e\*x + d)^(5/2), x)

**maple [B]** time = 0.08, size = 886, normalized size = 2.97

$$-\frac{2a}{3(ex+d)^{\frac{3}{2}}} + 2b \left( -\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}} + \frac{2\sqrt{\frac{c}{dc-e}}(ex+d)^2c^2d}{3} - \frac{2\sqrt{\frac{(ex+d)c-dc+e}{dc-e}}\sqrt{\frac{(ex+d)c-dc-e}{dc+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}, \sqrt{\frac{c}{dc-e}}, \frac{dc-e}{cd}, \sqrt{\frac{c}{dc+e}}\right)\sqrt{ex+d}c^2d^2}{3} - \frac{2\sqrt{\frac{(ex+d)c-dc+e}{dc-e}}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/(e*x+d)^(5/2),x)`

[Out]  $2/e*(-1/3*a/(e*x+d)^{(3/2)}+b*(-1/3/(e*x+d)^{(3/2)}*\operatorname{arcsec}(c*x)+2/3/c*((c/(c*d-e))^{(1/2)}*(e*x+d)^2*c^2*d-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticPi}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)})*(e*x+d)^{(1/2)}*c^2*d^2-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*c^2*d^2+((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticE}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*c^2*d^2-2*(c/(c*d-e))^{(1/2)}*(e*x+d)*c^2*d^2-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*c*d*e+((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticE}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*c*d*e+(c/(c*d-e))^{(1/2)}*c^2*d^3+((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*\operatorname{EllipticPi}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)})*(e*x+d)^{(1/2)}*e^2-(c/(c*d-e))^{(1/2)}*d*e^2)/(c*d-e)/(e*x+d)^{(1/2)}/(c*d+e)/(c/(c*d-e))^{(1/2)}/d^2/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{(1/2))}$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(c\*d-e>0)', see 'assume?' for more details) Is c\*d-e positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(5/2), x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x+d)\*\*(5/2), x)

[Out] Integral((a + b\*asec(c\*x))/(d + e\*x)\*\*(5/2), x)

$$3.68 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex)^{7/2}} dx$$

**Optimal.** Leaf size=540

$$\frac{2(a+b \sec^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} - \frac{16bce(1-c^2x^2)}{15x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)^2\sqrt{d+ex}} - \frac{4be}{15cdx\sqrt{1-\frac{1}{c^2x^2}}}$$

[Out]  $-2/5*(a+b*\text{arcsec}(c*x))/e/(e*x+d)^{(5/2)}-4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^2)/x/(e*x+d)^{(3/2)}/(1-1/c^2/x^2)^{(1/2)}-16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2-e^2)^2/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/15*b*(7*c^2*d^2-3*e^2)*\text{EllipticE}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^{(1/2)}/(c*(e*x+d)/(c*d+e))^{(1/2)}-4/15*b*\text{EllipticF}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(c*(e*x+d)/(c*d+e))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/5*b*\text{EllipticPi}(1/2*(-c*x+1)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(c*d+e))^{(1/2)})*(c*(e*x+d)/(c*d+e))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/d^2/e/x/(1-1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.71, antiderivative size = 637, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {5226, 1574, 958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$\frac{2(a+b \sec^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{16bce(1-c^2x^2)}{15x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)^2\sqrt{d+ex}} - \frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4be}{15cdx\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x)^(7/2), x]

[Out]  $(-4*b*e*(1-c^2*x^2))/(15*c*d*(c^2*d^2-e^2)*\text{Sqrt}[1-1/(c^2*x^2)]*x*(d+e*x)^{(3/2)} - (16*b*c*e*(1-c^2*x^2))/(15*(c^2*d^2-e^2)^2*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[d+e*x]) - (4*b*e*(1-c^2*x^2))/(5*c*d^2*(c^2*d^2-e^2)*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[d+e*x]) - (2*(a+b*\text{ArcSec}[c*x]))/(5*e*(d+e*x)^{(5/2)}) + (16*b*c^2*\text{Sqrt}[d+e*x]*\text{Sqrt}[1-c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1-c*x]/\text{Sqrt}[2]], (2*e)/(c*d+e)))/(15*(c^2*d^2-e^2)^2*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d+e*x))/(c*d+e)]) + (4*b*\text{Sqrt}[d+e*x]*\text{Sqrt}[1-c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1-c*x]/\text{Sqrt}[2]], (2*e)/(c*d+e)))/(5*d^2*(c^2*d^2-e^2)*\text{Sqrt}[1-1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d+e*x))/(c*d+e)]) - (4*b*\text{Sqrt}[(c*(d+e*x))/(c*d+e)]*\text{Sqrt}[1-c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1-c$

```
*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]
*x*Sqrt[d + e*x]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*El
lipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(5*c*d^2*e*Sqr
t[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

### Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

### Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

, f}, x] && !GtQ[c, 0]

### Rule 719

Int[((d\_) + (e\_)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

### Rule 745

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*Simp[d\*(m + 1) - e\*(m + 2\*p + 3)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2\*p + 3], 0])

### Rule 835

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p\*Simp[(c\*d\*f + a\*e\*g)\*(m + 1) - c\*(e\*f - d\*g)\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 844

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 933

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(f\_) + (g\_)\*(x\_)]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[a + c\*x^2], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a

\*e<sup>2</sup>, 0] && !GtQ[a, 0]

### Rule 958

Int[((f\_.) + (g\_.)\*(x\_))^(n\_)/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

### Rule 1574

Int[(x\_)^(m\_.)\*((a\_.) + (c\_.)\*(x\_)^(mn2\_.))^(p\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[(x^(2\*n\*FracPart[p])\*(a + c/x^(2\*n))^FracPart[p])/(c + a\*x^(2\*n))^FracPart[p], Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

### Rule 5226

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcSec[c\*x]))/(e\*(m + 1)), x] - Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex)^{7/2}} dx &= -\frac{2(a + b \sec^{-1}(cx))}{5e(d + ex)^{5/2}} + \frac{(2b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 (d + ex)^{5/2}} dx}{5ce} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{5e(d + ex)^{5/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x(d + ex)^{5/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{5ce\sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{5e(d + ex)^{5/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \left[ -\frac{e}{d(d + ex)^{5/2} \sqrt{-\frac{1}{c^2} + x^2}} - \frac{e}{d^2(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} + \frac{e}{d^2 x \sqrt{-\frac{1}{c^2} + x^2}} \right] dx}{5ce\sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \sec^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{5cd^2\sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{5/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{5cd\sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= -\frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{4be(1 - c^2 x^2)}{5cd^2(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{5e(d + ex)^{5/2}} \\
&= -\frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{5cd^2(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \\
&= -\frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{5cd^2(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \\
&= -\frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{5cd^2(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \\
&= -\frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}} - \frac{2(a + b \sec^{-1}(cx))}{5cd^2(c^2 d^2 - e^2)\sqrt{1 - \frac{1}{c^2 x^2}} x\sqrt{d + ex}}
\end{aligned}$$



**Mathematica [C]** time = 8.91, size = 407, normalized size = 0.75

$$2 \left( -\frac{3a}{(d+ex)^{5/2}} + \frac{2bce^2x \sqrt{1 - \frac{1}{c^2x^2} (c^2d^2(8d+7ex) - e^2(4d+3ex))}}{(c^2d^3 - de^2)^2 (d+ex)^{3/2}} + \frac{2ib \sqrt{\frac{e(cx+1)}{e-cd}} \sqrt{\frac{e-cex}{cd+e}} (-cd(6c^2d^2 - cde - 3e^2)) F\left(i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right) \middle| \frac{cd+e}{cd-e}\right)}{d^3x \sqrt{\dots}} \right)$$

15e

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x)^(7/2), x]

[Out] (2\*((-3\*a)/(d + e\*x)^(5/2) + (2\*b\*c\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(-(e^2\*(4\*d + 3\*e\*x)) + c^2\*d^2\*(8\*d + 7\*e\*x)))/((c^2\*d^3 - d\*e^2)^2\*(d + e\*x)^(3/2)) - (3\*b\*ArcSec[c\*x])/(d + e\*x)^(5/2) + ((2\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-(c\*d) + e)]\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*(c\*d\*(7\*c^2\*d^2 - 3\*e^2)\*EllipticE[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] - c\*d\*(6\*c^2\*d^2 - c\*d\*e - 3\*e^2)\*EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] - 3\*(c\*d - e)\*(c\*d + e)^2\*EllipticPi[1 + e/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d + e))]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)]))/((d^3\*(c\*d - e)\*(-(c/(c\*d + e)))^(3/2)\*(c\*d + e)^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x)))/(15\*e)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(7/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex + d)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x+d)^(7/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(e\*x + d)^(7/2), x)

**maple [B]** time = 0.09, size = 1640, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^(7/2),x)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsec(c*x)+2/15/c*(7*(c/(c
*d-e))^(1/2)*(e*x+d)^3*c^4*d^3-6*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x
+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4+7*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1
/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(
1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4-3*(-((e*x+d)*c-d*c+e)/
(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d
)^(3/2)*c^4*d^4-13*(c/(c*d-e))^(1/2)*(e*x+d)^2*c^4*d^4-7*(-((e*x+d)*c-d*c+e
)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^3*d^3*e+7*(-((e
*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^3*
d^3*e-3*(c/(c*d-e))^(1/2)*(e*x+d)^3*c^2*d*e^2+5*(c/(c*d-e))^(1/2)*(e*x+d)*c
^4*d^5+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x
+d)^(3/2)*c^2*d^2*e^2-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c
-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*(e*x+d)^(3/2)*c^2*d^2*e^2+6*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-
((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^2*d^2*
e^2+5*(c/(c*d-e))^(1/2)*(e*x+d)^2*c^2*d^2*e^2+(c/(c*d-e))^(1/2)*c^4*d^6+3*(
-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*
c*d*e^3-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(
1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*
x+d)^(3/2)*c*d*e^3-8*(c/(c*d-e))^(1/2)*(e*x+d)*c^2*d^3*e^2-3*(-((e*x+d)*c-d
*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*
(e*x+d)^(3/2)*e^4-2*(c/(c*d-e))^(1/2)*c^2*d^4*e^2+3*(c/(c*d-e))^(1/2)*(e*x+
d)*d*e^4+(c/(c*d-e))^(1/2)*d^2*e^4)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(3/2)
/(c*d+e)/(c^2*d^2-e^2)/d^3/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c
^2/e^2/x^2)^(1/2))
```

```
maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(c\*d-e>0)', see `assume?` for more details) Is c\*d-e positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{(d + ex)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(7/2), x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x)^(7/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x+d)\*\*(7/2), x)

[Out] Timed out

### 3.69 $\int x^4 (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=206

$$\frac{1}{5}dx^5 (a + b \sec^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \sec^{-1}(cx)) - \frac{bex^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}} - \frac{bx(42c^2d+25e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{560c^6\sqrt{c^2x^2}} - \frac{bx^2\sqrt{c^2x^2-1}}{560c^6\sqrt{c^2x^2}}$$

[Out]  $\frac{1}{5}d*x^5*(a+b*\text{arcsec}(c*x))+\frac{1}{7}*e*x^7*(a+b*\text{arcsec}(c*x))-1/560*b*(42*c^2*d+25*e)*x*\text{arctanh}(c*x/(c^2*x^2-1)^{(1/2)})/c^6/(c^2*x^2)^{(1/2)}-1/560*b*(42*c^2*d+25*e)*x^2*(c^2*x^2-1)^{(1/2)}/c^5/(c^2*x^2)^{(1/2)}-1/840*b*(42*c^2*d+25*e)*x^4*(c^2*x^2-1)^{(1/2)}/c^3/(c^2*x^2)^{(1/2)}-1/42*b*e*x^6*(c^2*x^2-1)^{(1/2)}/c/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {14, 5238, 12, 459, 321, 217, 206}

$$\frac{1}{5}dx^5 (a + b \sec^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \sec^{-1}(cx)) - \frac{bx^4\sqrt{c^2x^2-1}(42c^2d+25e)}{840c^3\sqrt{c^2x^2}} - \frac{bx^2\sqrt{c^2x^2-1}(42c^2d+25e)}{560c^5\sqrt{c^2x^2}} - \frac{bx^2\sqrt{c^2x^2-1}}{560c^6\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]), x]

[Out]  $-(b*(42*c^2*d+25*e)*x^2*\text{Sqrt}[-1+c^2*x^2])/(560*c^5*\text{Sqrt}[c^2*x^2]) - (b*(42*c^2*d+25*e)*x^4*\text{Sqrt}[-1+c^2*x^2])/(840*c^3*\text{Sqrt}[c^2*x^2]) - (b*e*x^6*\text{Sqrt}[-1+c^2*x^2])/(42*c*\text{Sqrt}[c^2*x^2]) + (d*x^5*(a+b*\text{ArcSec}[c*x]))/5 + (e*x^7*(a+b*\text{ArcSec}[c*x]))/7 - (b*(42*c^2*d+25*e)*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1+c^2*x^2]])/(560*c^6*\text{Sqrt}[c^2*x^2])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 217

$Int[1/Sqrt[(a_) + (b_)*(x_)^2], x\_Symbol] \rightarrow Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[\{a, b\}, x] \&\& !GtQ[a, 0]$

### Rule 321

$Int[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow Simp[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; FreeQ[\{a, b, c, p\}, x] \&\& IGtQ[n, 0] \&\& GtQ[m, n - 1] \&\& NeQ[m + n*p + 1, 0] \&\& IntBinomialQ[a, b, c, n, m, p, x]$

### Rule 459

$Int[((e_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}*((c_) + (d_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow Simp[(d*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(b*e*(m + n*(p+1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p+1) + 1))/(b*(m + n*(p+1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[\{a, b, c, d, e, m, n, p\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[m + n*(p+1) + 1, 0]$

### Rule 5238

$Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow With[\{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]\}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[\{a, b, c, d, e, f, m, p\}, x] \&\& ((IGtQ[p, 0] \&\& !(ILtQ[(m - 1)/2, 0] \&\& GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] \&\& !(ILtQ[p, 0] \&\& GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] \&\& !ILtQ[(m - 1)/2, 0]))$

### Rubi steps

$$\begin{aligned}
\int x^4 (d + ex^2) (a + b \sec^{-1}(cx)) dx &= \frac{1}{5} dx^5 (a + b \sec^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^4(7d+5ex^2)}{35\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{1}{5} dx^5 (a + b \sec^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^4(7d+5ex^2)}{\sqrt{-1+c^2x^2}} dx}{35\sqrt{c^2x^2}} \\
&= -\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5} dx^5 (a + b \sec^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \sec^{-1}(cx)) + \frac{(b}{ \\
&= -\frac{b(42c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5} dx^5 (a + b \sec^{-1}(c \\
&= -\frac{b(42c^2d+25e)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\
&= -\frac{b(42c^2d+25e)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\
&= -\frac{b(42c^2d+25e)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 141, normalized size = 0.68

$$\frac{48ac^7x^5(7d+5ex^2) + 48bc^7x^5\sec^{-1}(cx)(7d+5ex^2) - 3b(42c^2d+25e)\log\left(x\left(\sqrt{1-\frac{1}{c^2x^2}}+1\right)\right) - bc^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{1680c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]), x]

[Out] (48\*a\*c^7\*x^5\*(7\*d + 5\*e\*x^2) - b\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2\*(75\*e + 2\*c^2\*(63\*d + 25\*e\*x^2) + c^4\*(84\*d\*x^2 + 40\*e\*x^4)) + 48\*b\*c^7\*x^5\*(7\*d + 5\*e\*x^2)\*ArcSec[c\*x] - 3\*b\*(42\*c^2\*d + 25\*e)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x])/ (1680\*c^7)

**fricas [A]** time = 1.93, size = 192, normalized size = 0.93

$$\frac{240ac^7ex^7 + 336ac^7dx^5 + 48(5bc^7ex^7 + 7bc^7dx^5 - 7bc^7d - 5bc^7e)\operatorname{arcsec}(cx) + 96(7bc^7d + 5bc^7e)\operatorname{arctan}\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)}{1680c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/1680\*(240\*a\*c^7\*e\*x^7 + 336\*a\*c^7\*d\*x^5 + 48\*(5\*b\*c^7\*e\*x^7 + 7\*b\*c^7\*d\*x^5 - 7\*b\*c^7\*d - 5\*b\*c^7\*e)\*arcsec(c\*x) + 96\*(7\*b\*c^7\*d + 5\*b\*c^7\*e)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + 3\*(42\*b\*c^2\*d + 25\*b\*e)\*log(-c\*x + sqrt(c^2\*x^2 - 1)) - (40\*b\*c^5\*e\*x^5 + 2\*(42\*b\*c^5\*d + 25\*b\*c^3\*e)\*x^3 + 3\*(42\*b\*c^3\*d + 25\*b\*c\*e)\*x)\*sqrt(c^2\*x^2 - 1))/c^7

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] Timed out

**maple** [A] time = 0.06, size = 338, normalized size = 1.64

$$\frac{ae x^7}{7} + \frac{a x^5 d}{5} + \frac{b \operatorname{arcsec}(cx) e x^7}{7} + \frac{b \operatorname{arcsec}(cx) x^5 d}{5} - \frac{b x^6 e}{42c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b x^4 e}{168c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b x^4 d}{20c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b x^2 d}{40c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x)

[Out] 1/7\*a\*e\*x^7+1/5\*a\*x^5\*d+1/7\*b\*arcsec(c\*x)\*e\*x^7+1/5\*b\*arcsec(c\*x)\*x^5\*d-1/42/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^6\*e-1/168/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4\*e-1/20/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4\*d-1/40/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2\*d-5/336/c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2\*e+3/40/c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d-3/40/c^6\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*(c^2\*x^2-1)^(1/2)\*d\*ln(c\*x+(c^2\*x^2-1)^(1/2))+5/112/c^7\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*e-5/112/c^8\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*e\*ln(c\*x+(c^2\*x^2-1)^(1/2))

**maxima** [A] time = 0.33, size = 296, normalized size = 1.44

$$\frac{1}{7} a e x^7 + \frac{1}{5} a d x^5 + \frac{1}{80} \left( 16 x^5 \operatorname{arcsec}(c x) + \frac{2 \left( 3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 5 \sqrt{-\frac{1}{c^2 x^2} + 1} \right)}{c^4 \left( \frac{1}{c^2 x^2} - 1 \right)^2 + 2 c^4 \left( \frac{1}{c^2 x^2} - 1 \right) + c^4} - \frac{3 \log \left( \sqrt{-\frac{1}{c^2 x^2} + 1} + 1 \right)}{c^4} + \frac{3 \log \left( \sqrt{-\frac{1}{c^2 x^2} + 1} - 1 \right)}{c^4} \right) b d +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{7}aex^7 + \frac{1}{5}adx^5 + \frac{1}{80}(16x^5\text{arcsec}(cx) + (2(3(-1/(c^2x^2) + 1)^{3/2} - 5\sqrt{-1/(c^2x^2) + 1)})/(c^4(1/(c^2x^2) - 1)^2 + 2c^4(1/(c^2x^2) - 1) + c^4) - 3\log(\sqrt{-1/(c^2x^2) + 1} + 1)/c^4 + 3\log(\sqrt{-1/(c^2x^2) + 1} - 1)/c^4)/c)bd + \frac{1}{672}(96x^7\text{arcsec}(cx) - (2(15(-1/(c^2x^2) + 1)^{5/2} - 40(-1/(c^2x^2) + 1)^{3/2} + 33\sqrt{-1/(c^2x^2) + 1)})/(c^6(1/(c^2x^2) - 1)^3 + 3c^6(1/(c^2x^2) - 1)^2 + 3c^6(1/(c^2x^2) - 1) + c^6) + 15\log(\sqrt{-1/(c^2x^2) + 1} + 1)/c^6 - 15\log(\sqrt{-1/(c^2x^2) + 1} - 1)/c^6)/c)bx^e$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 (ex^2 + d) \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(d + e\*x^2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^4\*(d + e\*x^2)\*(a + b\*acos(1/(c\*x))), x)

sympy [A] time = 11.68, size = 408, normalized size = 1.98

$$\frac{adx^5}{5} + \frac{aex^7}{7} + \frac{bdx^5 \operatorname{asec}(cx)}{5} + \frac{bex^7 \operatorname{asec}(cx)}{7} - \frac{bd \begin{cases} \left( \frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3 \operatorname{acosh}(cx)}{8c^4} \right) & \text{for } |c^2x^2| < 1 \\ \left( -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i \operatorname{asin}(cx)}{8c^4} \right) & \text{otherwise} \end{cases}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x\*\*2+d)\*(a+b\*asec(c\*x)),x)

[Out]  $a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*asec(c*x)/5 + b*e*x**7*asec(c*x)/7 - b*d * \operatorname{Piecewise}((c*x**5/(4*\sqrt{c**2*x**2 - 1}) + x**3/(8*c*\sqrt{c**2*x**2 - 1}) - 3*x/(8*c**3*\sqrt{c**2*x**2 - 1}) + 3*\operatorname{acosh}(c*x)/(8*c**4), \operatorname{Abs}(c**2*x**2) > 1), (-I*c*x**5/(4*\sqrt{-c**2*x**2 + 1}) - I*x**3/(8*c*\sqrt{-c**2*x**2 + 1}) + 3*I*x/(8*c**3*\sqrt{-c**2*x**2 + 1}) - 3*I*\operatorname{asin}(c*x)/(8*c**4), \operatorname{True}))/ (5*c) - b*e*\operatorname{Piecewise}((c*x**7/(6*\sqrt{c**2*x**2 - 1}) + x**5/(24*c*\sqrt{c**2*x**2 - 1}) + 5*x**3/(48*c**3*\sqrt{c**2*x**2 - 1}) - 5*x/(16*c**5*\sqrt{c**2*x**2 - 1}) + 5*\operatorname{acosh}(c*x)/(16*c**6), \operatorname{Abs}(c**2*x**2) > 1), (-I*c*x**7/(6*\sqrt{-c**2*x**2 + 1}) - I*x**5/(24*c*\sqrt{-c**2*x**2 + 1}) - 5*I*x**3/(48*c**3*\sqrt{-c**2*x**2 + 1}) + 5*I*x/(16*c**5*\sqrt{-c**2*x**2 + 1}) - 5*I*\operatorname{asin}(c*x)/(16*c**6), \operatorname{True}))/ (7*c)$



### 3.70 $\int x^2 (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=161

$$\frac{1}{3}dx^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}ex^5 (a + b \sec^{-1}(cx)) - \frac{bex^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}} - \frac{bx(20c^2d+9e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}} - \frac{bx^2\sqrt{c^2x^2-1}}{120c^4\sqrt{c^2x^2}}$$

[Out] 1/3\*d\*x^3\*(a+b\*arcsec(c\*x))+1/5\*e\*x^5\*(a+b\*arcsec(c\*x))-1/120\*b\*(20\*c^2\*d+9\*e)\*x\*arctanh(c\*x/(c^2\*x^2-1)^(1/2))/c^4/(c^2\*x^2)^(1/2)-1/120\*b\*(20\*c^2\*d+9\*e)\*x^2\*(c^2\*x^2-1)^(1/2)/c^3/(c^2\*x^2)^(1/2)-1/20\*b\*e\*x^4\*(c^2\*x^2-1)^(1/2)/c/(c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {14, 5238, 12, 459, 321, 217, 206}

$$\frac{1}{3}dx^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}ex^5 (a + b \sec^{-1}(cx)) - \frac{bx^2\sqrt{c^2x^2-1}(20c^2d+9e)}{120c^3\sqrt{c^2x^2}} - \frac{bx(20c^2d+9e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]), x]

[Out] -(b\*(20\*c^2\*d + 9\*e)\*x^2\*Sqrt[-1 + c^2\*x^2])/(120\*c^3\*Sqrt[c^2\*x^2]) - (b\*e\*x^4\*Sqrt[-1 + c^2\*x^2])/(20\*c\*Sqrt[c^2\*x^2]) + (d\*x^3\*(a + b\*ArcSec[c\*x]))/3 + (e\*x^5\*(a + b\*ArcSec[c\*x]))/5 - (b\*(20\*c^2\*d + 9\*e)\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(120\*c^4\*Sqrt[c^2\*x^2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v])]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 217

$Int[1/Sqrt[(a_) + (b_)*(x_)^2], x\_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[\{a, b\}, x] \&\& !GtQ[a, 0]$

### Rule 321

$Int[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := Simp[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; FreeQ[\{a, b, c, p\}, x] \&\& IGtQ[n, 0] \&\& GtQ[m, n - 1] \&\& NeQ[m + n*p + 1, 0] \&\& IntBinomialQ[a, b, c, n, m, p, x]$

### Rule 459

$Int[((e_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}*((c_) + (d_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := Simp[(d*(e*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[\{a, b, c, d, e, m, n, p\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[m + n*(p + 1) + 1, 0]$

### Rule 5238

$Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] := With[\{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]\}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[\{a, b, c, d, e, f, m, p\}, x] \&\& ((IGtQ[p, 0] \&\& !(ILtQ[(m - 1)/2, 0] \&\& GtQ[m + 2*p + 3, 0])) \parallel (IGtQ[(m + 1)/2, 0] \&\& !(ILtQ[p, 0] \&\& GtQ[m + 2*p + 3, 0])) \parallel (ILtQ[(m + 2*p + 1)/2, 0] \&\& !ILtQ[(m - 1)/2, 0]))$

### Rubi steps

$$\begin{aligned}
\int x^2 (d + ex^2) (a + b \sec^{-1}(cx)) dx &= \frac{1}{3} dx^3 (a + b \sec^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^2(5d+3ex^2)}{15\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{1}{3} dx^3 (a + b \sec^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^2(5d+3ex^2)}{\sqrt{-1+c^2x^2}} dx}{15\sqrt{c^2x^2}} \\
&= -\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \sec^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \sec^{-1}(cx)) + \dots \\
&= -\frac{b(20c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \sec^{-1}(cx)) \\
&= -\frac{b(20c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \sec^{-1}(cx)) \\
&= -\frac{b(20c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \sec^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 123, normalized size = 0.76

$$\frac{c^2x^2 \left( 8ac^3x(5d+3ex^2) - b\sqrt{1-\frac{1}{c^2x^2}} \left( c^2(20d+6ex^2) + 9e \right) \right) + 8bc^5x^3 \sec^{-1}(cx) (5d+3ex^2) - b(20c^2d+9e)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]),x]

[Out] (c^2\*x^2\*(8\*a\*c^3\*x\*(5\*d + 3\*e\*x^2) - b\*Sqrt[1 - 1/(c^2\*x^2)]\*(9\*e + c^2\*(2\*0\*d + 6\*e\*x^2))) + 8\*b\*c^5\*x^3\*(5\*d + 3\*e\*x^2)\*ArcSec[c\*x] - b\*(20\*c^2\*d + 9\*e)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x])/(120\*c^5)

**fricas [A]** time = 2.03, size = 170, normalized size = 1.06

$$\frac{24ac^5ex^5 + 40ac^5dx^3 + 8(3bc^5ex^5 + 5bc^5dx^3 - 5bc^5d - 3bc^5e) \operatorname{arcsec}(cx) + 16(5bc^5d + 3bc^5e) \operatorname{arctan}\left(-\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 -
5*b*c^5*d - 3*b*c^5*e)*arcsec(c*x) + 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-c*
x + sqrt(c^2*x^2 - 1)) + (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 - 1))
- (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

**giac [B]** time = 3.03, size = 9820, normalized size = 60.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/120*(40*b*c^2*d*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 - 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) +
1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) -
1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10) + 40*a*c^2*d/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 - 40*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c
^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x)
+ 1)^2) - 100*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(
c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*
x) + 1)^10)*(1/(c*x) + 1)^2) + 100*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-
1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 40*b*c^2*d*sqrt(-1/(c^
2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10)*(1/(c*x) + 1)) - 40*a*c^2*d*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/
```

$$\begin{aligned}
& (1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^2 \\
& ) - 80*b*c^2*d*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) \\
& ) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c \\
& ^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) \\
& + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^4 + 24*b \\
& *arccos(1/(c*x))*e/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6* \\
& (1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + \\
& 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5 \\
& /(1/(c*x) + 1)^{10} - 200*b*c^2*d*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c^2*x \\
& ^2) + 1} + 1/(c*x) + 1)))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + \\
& 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/ \\
& (c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) \\
& - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^4 - 9*b*e*\log(\text{abs}(\sqrt{-1/(c^2*x^2) \\
& ) + 1} + 1/(c*x) + 1)))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10* \\
& c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c* \\
& x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - \\
& 1)^5/(1/(c*x) + 1)^{10} + 200*b*c^2*d*(1/(c^2*x^2) - 1)^2*\log(\text{abs}(\sqrt{-1/(c \\
& ^2*x^2) + 1} - 1/(c*x) - 1)))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + \\
& 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/ \\
& (1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2* \\
& x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^4 + 9*b*e*\log(\text{abs}(\sqrt{-1/(c^2 \\
& *x^2) + 1} - 1/(c*x) - 1)))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + \\
& 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1 \\
& / (c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) \\
& ) - 1)^5/(1/(c*x) + 1)^{10} + 80*b*c^2*d*(-1/(c^2*x^2) + 1)^{(3/2)}/((c^6 + 5* \\
& c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) \\
& + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - \\
& 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) \\
& + 1)^3) - 80*a*c^2*d*(1/(c^2*x^2) - 1)^2)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1 \\
& / (c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2 \\
& *x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + \\
& c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^4) + 80*b*c^2*d*(1/ \\
& (c^2*x^2) - 1)^3*\arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + \\
& 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - \\
& 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/( \\
& c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^6) + 24*a*e/(c^6 + 5*c^6*(1 \\
& / (c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^ \\
& 4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/ \\
& (1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} - 120*b*(1/(c^2 \\
& *x^2) - 1)*arccos(1/(c*x))*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + \\
& 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3 \\
& / (1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2* \\
& x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^2) - 200*b*c^2*d*(1/(c^2*x^2) - \\
& 1)^3*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1)))/((c^6 + 5*c^6*(1/(c^2* \\
& x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10
\end{aligned}$$

$$\begin{aligned}
& *c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^6 - 45 \\
& *b*(1/(c^2*x^2) - 1)*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^2) + 200*b*c^2*d*(1/(c^2*x^2) - 1)^3*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^6) + 45*b*(1/(c^2*x^2) - 1)*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^2) - 30*b*\text{sqrt}(-1/(c^2*x^2) + 1)*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)) + 80*a*c^2*d*(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^6) + 40*b*c^2*d*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^8) - 120 \\
& *a*(1/(c^2*x^2) - 1)*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^2) + 240*b*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^4) - 100*b*c^2*d*(1/(c^2*x^2) - 1)^4*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^4) + 100*b*c^2*d*(1/(c^2*x^2) - 1)^4*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^2)
\end{aligned}$$



$$\begin{aligned}
& 1/(c*x) + 1)^{10}*(1/(c*x) + 1)^6 + 40*b*c^2*d*(1/(c^2*x^2) - 1)^4*\sqrt{-1/} \\
& (c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/ \\
& (c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1) \\
& ^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1 \\
& / (c*x) + 1)^{10})*(1/(c*x) + 1)^9 - 40*a*c^2*d*(1/(c^2*x^2) - 1)^5/((c^6 + 5 \\
& *c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x \\
& ) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) \\
& - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) \\
& + 1)^{10} - 240*a*(1/(c^2*x^2) - 1)^3*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/ \\
& (c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2* \\
& x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c \\
& ^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^6 + 120*b*(1/(c^2*x \\
& ^2) - 1)^4*arccos(1/(c*x))*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^ \\
& 2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3 \\
& / (1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2* \\
& x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^8 - 45*b*(1/(c^2*x^2) - 1)^4*e \\
& *log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) \\
& - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6* \\
& (1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + \\
& 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^8) + 45*b*(1 \\
& / (c^2*x^2) - 1)^4*e*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + \\
& 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c* \\
& x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) \\
& - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x \\
& ) + 1)^8) + 12*b*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)*e/((c^6 + 5*c^6 \\
& *(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + \\
& 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1) \\
& ^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1 \\
& )^7) + 120*a*(1/(c^2*x^2) - 1)^4*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) \\
& + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) \\
& - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1 \\
& / (c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^8) - 24*b*(1/(c^2*x^2) - \\
& 1)^5*arccos(1/(c*x))*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10 \\
& *c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c \\
& *x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - \\
& 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^{10} - 9*b*(1/(c^2*x^2) - 1)^5*e*log(a \\
& bs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/( \\
& 1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^ \\
& 2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + \\
& c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1)^{10} + 9*b*(1/(c^2* \\
& x^2) - 1)^5*e*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6* \\
& (1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1 \\
& )^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^ \\
& 4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10})*(1/(c*x) + 1) \\
& ^{10} + 30*b*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)*e/((c^6 + 5*c^6*(1/(
\end{aligned}$$



$$c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^9 - 24*a*(1/(c^2*x^2) - 1)^5*e/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}*(1/(c*x) + 1)^{10}))*c$$

**maple [B]** time = 0.05, size = 282, normalized size = 1.75

$$\frac{ae x^5}{5} + \frac{ad x^3}{3} + \frac{b \operatorname{arcsec}(cx) e x^5}{5} + \frac{b \operatorname{arcsec}(cx) d x^3}{3} - \frac{b x^4 e}{20c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b x^2 e}{40c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{bd x^2}{6c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{bd}{6c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b \sqrt{c^2 x^2 - 1}}{6c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x)`

[Out]  $1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/5*b*\operatorname{arcsec}(c*x)*e*x^5 + 1/3*b*\operatorname{arcsec}(c*x)*d*x^3 - 1/20/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^4*e - 1/40/c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^2*e - 1/6/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*d*x^2 + 1/6/c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*d - 1/6/c^4*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*(c^2*x^2-1)^{(1/2)}*d*\ln(c*x+(c^2*x^2-1)^{(1/2)}) + 3/40/c^5*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*e - 3/40/c^6*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*e*\ln(c*x+(c^2*x^2-1)^{(1/2)})$

**maxima [A]** time = 0.33, size = 232, normalized size = 1.44

$$\frac{1}{5} a e x^5 + \frac{1}{3} a d x^3 + \frac{1}{12} \left( 4 x^3 \operatorname{arcsec}(c x) - \frac{2 \sqrt{-\frac{1}{c^2 x^2} + 1} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} + 1\right) - \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2}}{c^2 \left(\frac{1}{c^2 x^2} - 1\right) + c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} + 1\right) - \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2} \right) b d + \frac{1}{80} 16 x^5 \operatorname{arcsec}(c x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")`

[Out]  $1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*\operatorname{arcsec}(c*x) - (2*\sqrt{-1/(c^2*x^2) + 1})/(c^2*(1/(c^2*x^2) - 1) + c^2) + \log(\sqrt{-1/(c^2*x^2) + 1} + 1)/c^2 - \log(\sqrt{-1/(c^2*x^2) + 1} - 1)/c^2)/c)*b*d + 1/80*(16*x^5*\operatorname{arcsec}(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^{(3/2)} - 5*\sqrt{-1/(c^2*x^2) + 1}))/c^4*(1/(c^2*x^2) - 1))$

$-1)^2 + 2c^4(1/(c^2x^2) - 1) + c^4) - 3\log(\sqrt{-1/(c^2x^2) + 1} + 1) / c^4 + 3\log(\sqrt{-1/(c^2x^2) + 1} - 1) / c^4) / c) * b * e$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (ex^2 + d) \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))), x)`

[Out] `int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))), x)`

**sympy [A]** time = 7.30, size = 294, normalized size = 1.83

$$\frac{adx^3}{3} + \frac{aex^5}{5} + \frac{bdx^3 \operatorname{asec}(cx)}{3} + \frac{bex^5 \operatorname{asec}(cx)}{5} - \frac{bd \left( \begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise} \end{cases} \right)}{3c} - \frac{be \left( \begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise} \end{cases} \right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d)*(a+b*asec(c*x)), x)`

[Out] `a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*asec(c*x)/3 + b*e*x**5*asec(c*x)/5 - b*d *Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c) - b*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)`

### 3.71 $\int (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=109

$$dx (a + b \sec^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \sec^{-1}(cx)) - \frac{bx (6c^2d + e) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}} - \frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

[Out] d\*x\*(a+b\*arcsec(c\*x))+1/3\*e\*x^3\*(a+b\*arcsec(c\*x))-1/6\*b\*(6\*c^2\*d+e)\*x\*arctanh(c\*x/(c^2\*x^2-1)^(1/2))/c^2/(c^2\*x^2)^(1/2)-1/6\*b\*e\*x^2\*(c^2\*x^2-1)^(1/2)/c/(c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5228, 12, 388, 217, 206}

$$dx (a + b \sec^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \sec^{-1}(cx)) - \frac{bx (6c^2d + e) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}} - \frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)\*(a + b\*ArcSec[c\*x]),x]

[Out] -(b\*e\*x^2\*sqrt[-1 + c^2\*x^2])/(6\*c\*sqrt[c^2\*x^2]) + d\*x\*(a + b\*ArcSec[c\*x]) + (e\*x^3\*(a + b\*ArcSec[c\*x]))/3 - (b\*(6\*c^2\*d + e)\*x\*ArcTanh[(c\*x)/sqrt[-1 + c^2\*x^2]])/(6\*c^2\*sqrt[c^2\*x^2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

### Rule 5228

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

### Rubi steps

$$\begin{aligned}
\int (d + ex^2)(a + b \sec^{-1}(cx)) dx &= dx(a + b \sec^{-1}(cx)) + \frac{1}{3}ex^3(a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{3d+ex^2}{3\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= dx(a + b \sec^{-1}(cx)) + \frac{1}{3}ex^3(a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{3d+ex^2}{\sqrt{-1+c^2x^2}} dx}{3\sqrt{c^2x^2}} \\
&= -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a + b \sec^{-1}(cx)) + \frac{1}{3}ex^3(a + b \sec^{-1}(cx)) + \frac{(b(-6c^2d - 3d^2))}{6c^2\sqrt{c^2x^2}} \\
&= -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a + b \sec^{-1}(cx)) + \frac{1}{3}ex^3(a + b \sec^{-1}(cx)) + \frac{(b(-6c^2d - 3d^2))}{6c^2\sqrt{c^2x^2}} \\
&= -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a + b \sec^{-1}(cx)) + \frac{1}{3}ex^3(a + b \sec^{-1}(cx)) - \frac{b(6c^2d + 3d^2)}{6c^2\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.28, size = 150, normalized size = 1.38

$$adx + \frac{1}{3}aex^3 - \frac{bdx\sqrt{1 - \frac{1}{c^2x^2}} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} - \frac{bex^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{6c} - \frac{be \log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 1\right)\right)}{6c^3} + bdx \sec^{-1}(cx) + \frac{1}{3}bex^3 \sec^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSec[c*x]), x]
```

```
[Out] a*d*x + (a*e*x^3)/3 - (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*
x*ArcSec[c*x] + (b*e*x^3*ArcSec[c*x])/3 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]]/Sqrt[-1 + c^2*x^2] - (b*e*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)]))]/(6*c^3)
```

**fricas** [A] time = 2.09, size = 141, normalized size = 1.29

$$\frac{2ac^3ex^3 + 6ac^3dx - \sqrt{c^2x^2 - 1}bcex + 2(bc^3ex^3 + 3bc^3dx - 3bc^3d - bc^3e) \operatorname{arcsec}(cx) + 4(3bc^3d + bc^3e) \operatorname{arctan}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*x
^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arcsec(c*x) + 4*(3*b*c^3*d + b*c^3*
e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^2*d + b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)))/c^3
```

**giac** [B] time = 2.05, size = 4069, normalized size = 37.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*c^2*d*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6) - 6*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(
c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*log(a
bs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6) + 6*a*c^2*d/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(c^4 +
3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 18*
b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((
c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2)
+ 18*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1
)^2) + 6*a*c^2*d*(1/(c^2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
```

$$\begin{aligned}
& + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1) \\
& ^3/(1/(cx) + 1)^6*(1/(cx) + 1)^2 - 6b*c^2*d*(1/(c^2x^2) - 1)^2*\arccos \\
& (1/(cx))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x \\
& ^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(c \\
& *x) + 1)^4) + 2b*\arccos(1/(cx))*e/(c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) \\
& + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1) \\
& ^3/(1/(cx) + 1)^6) - 18b*c^2*d*(1/(c^2x^2) - 1)^2*\log(\text{abs}(\text{sqrt}(-1/(c^2x \\
& ^2) + 1) + 1/(cx) + 1))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + \\
& 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) \\
& ) + 1)^6)*(1/(cx) + 1)^4) - b*e*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) + 1/(cx) + \\
& 1))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - \\
& 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6) + 18b*c^2* \\
& d*(1/(c^2x^2) - 1)^2*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) - 1/(cx) - 1))/((c^4 \\
& + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(c \\
& *x) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) + 1)^4) + b* \\
& e*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) - 1/(cx) - 1))/((c^4 + 3c^4(1/(c^2x^2) \\
& - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/( \\
& c^2x^2) - 1)^3/(1/(cx) + 1)^6) - 6a*c^2*d*(1/(c^2x^2) - 1)^2/((c^4 + 3c \\
& ^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) \\
& + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) + 1)^4) - 6b*c^ \\
& 2*d*(1/(c^2x^2) - 1)^3*\arccos(1/(cx))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/ \\
& (cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) \\
& - 1)^3/(1/(cx) + 1)^6)*(1/(cx) + 1)^6) + 2a*e/(c^4 + 3c^4(1/(c^2x^2) \\
& - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/ \\
& (c^2x^2) - 1)^3/(1/(cx) + 1)^6) - 6b*(1/(c^2x^2) - 1)*\arccos(1/(cx))*e \\
& /((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^ \\
& 2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) + 1)^ \\
& 2) - 6b*c^2*d*(1/(c^2x^2) - 1)^3*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) + 1/(cx) \\
& + 1))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\
& - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) \\
& + 1)^6) - 3b*(1/(c^2x^2) - 1)*e*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) + 1/(cx) \\
& + 1))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\
& - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) \\
& + 1)^2) + 6b*c^2*d*(1/(c^2x^2) - 1)^3*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) - 1 \\
& /((cx) - 1))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^ \\
& 2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1 \\
& /((cx) + 1)^6) + 3b*(1/(c^2x^2) - 1)*e*\log(\text{abs}(\text{sqrt}(-1/(c^2x^2) + 1) - 1 \\
& /((cx) - 1))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^ \\
& 2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1 \\
& /((cx) + 1)^2) - 2b*\text{sqrt}(-1/(c^2x^2) + 1)*e/((c^4 + 3c^4(1/(c^2x^2) - \\
& 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^ \\
& 2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) + 1)) - 6a*c^2*d*(1/(c^2x^2) - 1) \\
& ^3/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1 \\
& )^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)*(1/(cx) + 1 \\
& )^6) - 6a*(1/(c^2x^2) - 1)*e/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1
\end{aligned}$$

$$\begin{aligned} &)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/( \\ &1/(cx) + 1)^6*(1/(cx) + 1)^2 + 6*b*(1/(c^2x^2) - 1)^2*\arccos(1/(cx))* \\ &e/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1) \\ &^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6*(1/(cx) + 1) \\ &^4) - 3*b*(1/(c^2x^2) - 1)^2*e*\log(\text{abs}(\sqrt{-1/(c^2x^2) + 1} + 1/(cx) + \\ &1)))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1) \\ &^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6*(1/(cx) + 1) \\ &^4) + 3*b*(1/(c^2x^2) - 1)^2*e*\log(\text{abs}(\sqrt{-1/(c^2x^2) + 1} - 1/(cx) \\ &- 1)))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\ &- 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6*(1/(cx) \\ &+ 1)^4) + 6*a*(1/(c^2x^2) - 1)^2*e/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) \\ &) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1) \\ &)^3/(1/(cx) + 1)^6*(1/(cx) + 1)^4) - 2*b*(1/(c^2x^2) - 1)^3*\arccos(1/(c \\ &x))*e/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\ &- 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6*(1/(cx) \\ &+ 1)^6) - b*(1/(c^2x^2) - 1)^3*e*\log(\text{abs}(\sqrt{-1/(c^2x^2) + 1} + 1/(cx) \\ &+ 1)))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\ &- 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6*(1/(cx) \\ &+ 1)^6) + b*(1/(c^2x^2) - 1)^3*e*\log(\text{abs}(\sqrt{-1/(c^2x^2) + 1} - 1/(cx) \\ &- 1)))/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\ &- 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6*(1/(cx) \\ &+ 1)^6) + 2*b*(1/(c^2x^2) - 1)^2*\sqrt{-1/(c^2x^2) + 1}*e/((c^4 + 3c^4(1/(c^2x^2) \\ &- 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) \\ &- 1)^3/(1/(cx) + 1)^6*(1/(cx) + 1)^5) - 2*a*(1/(c^2 \\ &x^2) - 1)^3*e/((c^4 + 3c^4(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 3c^4(1/(c^2x^2) \\ &- 1)^2/(1/(cx) + 1)^4 + c^4(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6)* \\ &(1/(cx) + 1)^6))*c \end{aligned}$$

**maple [B]** time = 0.05, size = 195, normalized size = 1.79

$$\frac{ae x^3}{3} + adx + \frac{b \operatorname{arcsec}(cx) x^3 e}{3} + b \operatorname{arcsec}(cx) x d - \frac{b \sqrt{c^2 x^2 - 1} d \ln \left( cx + \sqrt{c^2 x^2 - 1} \right)}{c^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x} - \frac{be x^2}{6c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{be}{6c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b}{6c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arcsec(c\*x)),x)

[Out]  $\frac{1}{3} a e x^3 + a d x + \frac{1}{3} b \operatorname{arcsec}(c x) x^3 e + b \operatorname{arcsec}(c x) x d - \frac{1}{c^2} \frac{b}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} \frac{d \ln \left( cx + \sqrt{c^2 x^2 - 1} \right)}{x \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{1}{6} \frac{b e x^2}{c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{1}{6} \frac{b e}{c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{1}{6} \frac{b}{c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$

**maxima** [A] time = 0.31, size = 154, normalized size = 1.41

$$\frac{1}{3} aex^3 + \frac{1}{12} \left( 4x^3 \operatorname{arcsec}(cx) - \frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2} \right) be+adx + \frac{(2cx \operatorname{arcsec}(cx) - \log(\dots))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*e\*x^3 + 1/12\*(4\*x^3\*arcsec(c\*x) - (2\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) - 1) + c^2) + log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b\*e + a\*d\*x + 1/2\*(2\*c\*x\*arcsec(c\*x) - log(sqrt(-1/(c^2\*x^2) + 1) + 1) + log(-sqrt(-1/(c^2\*x^2) + 1) + 1))\*b\*d/c

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (ex^2 + d) \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int((d + e\*x^2)\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 5.72, size = 153, normalized size = 1.40

$$adx + \frac{aex^3}{3} + bdx \operatorname{asec}(cx) + \frac{bex^3 \operatorname{asec}(cx)}{3} - \frac{bd \begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c} + \frac{be \begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{ia}{3c} \end{cases}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*asec(c\*x)),x)

[Out] a\*d\*x + a\*e\*x\*\*3/3 + b\*d\*x\*asec(c\*x) + b\*e\*x\*\*3\*asec(c\*x)/3 - b\*d\*Piecewise((acosh(c\*x), Abs(c\*\*2\*x\*\*2) > 1), (-I\*asin(c\*x), True))/c - b\*e\*Piecewise((x\*sqrt(c\*\*2\*x\*\*2 - 1)/(2\*c) + acosh(c\*x)/(2\*c\*\*2), Abs(c\*\*2\*x\*\*2) > 1), (-I\*c\*x\*\*3/(2\*sqrt(-c\*\*2\*x\*\*2 + 1)) + I\*x/(2\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)) - I\*asin(c\*x)/(2\*c\*\*2), True))/(3\*c)



$$3.72 \quad \int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=87

$$-\frac{d(a+b \sec^{-1}(cx))}{x} + ex(a+b \sec^{-1}(cx)) + \frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} - \frac{bex \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

[Out]  $-d*(a+b*\operatorname{arcsec}(c*x))/x+e*x*(a+b*\operatorname{arcsec}(c*x))-b*e*x*\operatorname{arctanh}(c*x/(c^2*x^2-1)^{(1/2)})/(c^2*x^2)^{(1/2)}+b*c*d*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5238, 451, 217, 206}

$$-\frac{d(a+b \sec^{-1}(cx))}{x} + ex(a+b \sec^{-1}(cx)) + \frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} - \frac{bex \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x^2,x]

[Out]  $(b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/ \operatorname{Sqrt}[c^2*x^2] - (d*(a + b*\operatorname{ArcSec}[c*x]))/x + e*x*(a + b*\operatorname{ArcSec}[c*x]) - (b*e*x*\operatorname{ArcTanh}[(c*x)/\operatorname{Sqrt}[-1 + c^2*x^2]])/\operatorname{Sqrt}[c^2*x^2]$

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))
```

### Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \sec^{-1}(cx))}{x^2} dx &= -\frac{d(a + b \sec^{-1}(cx))}{x} + ex(a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{-d+ex^2}{x^2\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{x} + ex(a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{1}{\sqrt{-1+c^2x^2}}}{\sqrt{c^2x^2}} \\ &= \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{x} + ex(a + b \sec^{-1}(cx)) - \frac{(bcx) \text{Subst}\left(\frac{1}{\sqrt{-1+c^2x^2}}, x, cx\right)}{\sqrt{c^2x^2}} \\ &= \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{x} + ex(a + b \sec^{-1}(cx)) - \frac{bex \tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 104, normalized size = 1.20

$$-\frac{ad}{x} + aex + bcd\sqrt{\frac{c^2x^2 - 1}{c^2x^2}} - \frac{bex\sqrt{1 - \frac{1}{c^2x^2}} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right)}{\sqrt{c^2x^2 - 1}} - \frac{bd \sec^{-1}(cx)}{x} + bex \sec^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2, x]
```

[Out]  $-\frac{(a*d)}{x} + a*e*x + b*c*d*\text{Sqrt}\left[\frac{-1 + c^2*x^2}{c^2*x^2}\right] - (b*d*\text{ArcSec}[c*x]) / x + b*e*x*\text{ArcSec}[c*x] - (b*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]]) / \text{Sqrt}[-1 + c^2*x^2]$

**fricas** [A] time = 0.45, size = 123, normalized size = 1.41

$$\frac{bc^2dx + acex^2 + bex \log\left(-cx + \sqrt{c^2x^2 - 1}\right) + \sqrt{c^2x^2 - 1}bcd - acd - 2(bcd - bce)x \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) +}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")`

[Out]  $(b*c^2*d*x + a*c*e*x^2 + b*e*x*\log(-c*x + \text{sqrt}(c^2*x^2 - 1)) + \text{sqrt}(c^2*x^2 - 1)*b*c*d - a*c*d - 2*(b*c*d - b*c*e)*x*\arctan(-c*x + \text{sqrt}(c^2*x^2 - 1)) + (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*\text{arcsec}(c*x))/(c*x)$

**giac** [B] time = 0.62, size = 1098, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")`

[Out]  $-(b*c^2*d*\arccos(1/(c*x)))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + a*c^2*d/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c^2*d*(1/(c^2*x^2) - 1)*\arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*c^2*d*\text{sqrt}(-1/(c^2*x^2) + 1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + 2*a*c^2*d*(1/(c^2*x^2) - 1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + b*c^2*d*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - b*\arccos(1/(c*x))*e/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - b*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) + a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - a*e/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*(1/(c^2*x^2) - 1)*\arccos(1/(c*x))*e/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 2*a*(1/(c^2*x^2) - 1)*e/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - b*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))*e/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - b*(1/(c^2*x^2) - 1)^2*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + b*(1/(c^2*x^2) - 1)^2*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + b*(1/(c^2*x^2) - 1)^2*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))$

+ 1) - 1/(c\*x) - 1))/((c^2 - c^2\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)\*(1/(c\*x) + 1)^4) - a\*(1/(c^2\*x^2) - 1)^2\*e/((c^2 - c^2\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4)\*(1/(c\*x) + 1)^4))\*c

**maple [A]** time = 0.06, size = 137, normalized size = 1.57

$$aex - \frac{ad}{x} + b \operatorname{arcsec}(cx) ex - \frac{b \operatorname{arcsec}(cx) d}{x} + \frac{cbd}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{bd}{cx^2 \sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{b\sqrt{c^2x^2-1} e \ln\left(cx + \sqrt{c^2x^2-1}\right)}{c^2 \sqrt{\frac{c^2x^2-1}{c^2x^2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^2,x)

[Out] a\*e\*x-a\*d/x+b\*arcsec(c\*x)\*e\*x-b\*arcsec(c\*x)\*d/x+c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d-b/c/x^2/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d-b/c^2\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*e\*ln(c\*x+(c^2\*x^2-1)^(1/2))

**maxima [A]** time = 0.32, size = 89, normalized size = 1.02

$$\left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bd+aex+\frac{\left(2cx \operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2c} - \frac{a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^2,x, algorithm="maxima")

[Out] (c\*sqrt(-1/(c^2\*x^2) + 1) - arcsec(c\*x)/x)\*b\*d + a\*e\*x + 1/2\*(2\*c\*x\*arcsec(c\*x) - log(sqrt(-1/(c^2\*x^2) + 1) + 1) + log(-sqrt(-1/(c^2\*x^2) + 1) + 1))\*b\*e/c - a\*d/x

**mupad [B]** time = 0.79, size = 72, normalized size = 0.83

$$aex - \frac{d\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right) - bcx \sqrt{1 - \frac{1}{c^2x^2}}\right)}{x} - \frac{be \operatorname{atanh}\left(\frac{1}{\sqrt{1 - \frac{1}{c^2x^2}}}\right)}{c} + bex \operatorname{acos}\left(\frac{1}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x^2,x)

[Out] a\*e\*x - (d\*(a + b\*acos(1/(c\*x)) - b\*c\*x\*(1 - 1/(c^2\*x^2))^(1/2)))/x - (b\*e\*atanh(1/(1 - 1/(c^2\*x^2))^(1/2)))/c + b\*e\*x\*acos(1/(c\*x))

sympy [A] time = 5.11, size = 73, normalized size = 0.84

$$-\frac{ad}{x} + aex + bcd\sqrt{1 - \frac{1}{c^2x^2}} - \frac{bd \operatorname{asec}(cx)}{x} + bex \operatorname{asec}(cx) - \frac{be \begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*asec(c\*x))/x\*\*2,x)

[Out] -a\*d/x + a\*e\*x + b\*c\*d\*sqrt(1 - 1/(c\*\*2\*x\*\*2)) - b\*d\*asec(c\*x)/x + b\*e\*x\*asec(c\*x) - b\*e\*Piecewise((acosh(c\*x), Abs(c\*\*2\*x\*\*2) > 1), (-I\*asin(c\*x), True))/c

$$3.73 \quad \int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=105

$$-\frac{d(a+b \sec^{-1}(cx))}{3x^3} - \frac{e(a+b \sec^{-1}(cx))}{x} + \frac{bc\sqrt{c^2x^2-1}(2c^2d+9e)}{9\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

[Out]  $-1/3*d*(a+b*\text{arcsec}(c*x))/x^3-e*(a+b*\text{arcsec}(c*x))/x+1/9*b*c*(2*c^2*d+9*e)*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}+1/9*b*c*d*(c^2*x^2-1)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5238, 12, 453, 264}

$$-\frac{d(a+b \sec^{-1}(cx))}{3x^3} - \frac{e(a+b \sec^{-1}(cx))}{x} + \frac{bc\sqrt{c^2x^2-1}(2c^2d+9e)}{9\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]`

[Out]  $(b*c*(2*c^2*d + 9*e)*\text{Sqrt}[-1 + c^2*x^2])/(9*\text{Sqrt}[c^2*x^2]) + (b*c*d*\text{Sqrt}[-1 + c^2*x^2])/(9*x^2*\text{Sqrt}[c^2*x^2]) - (d*(a + b*\text{ArcSec}[c*x]))/(3*x^3) - (e*(a + b*\text{ArcSec}[c*x]))/x$

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

### Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

### Rule 264

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]`

### Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1), x) + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrate[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \sec^{-1}(cx))}{x^4} dx &= -\frac{d(a + b \sec^{-1}(cx))}{3x^3} - \frac{e(a + b \sec^{-1}(cx))}{x} - \frac{(bcx) \int \frac{-d-3ex^2}{3x^4\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= -\frac{d(a + b \sec^{-1}(cx))}{3x^3} - \frac{e(a + b \sec^{-1}(cx))}{x} - \frac{(bcx) \int \frac{-d-3ex^2}{x^4\sqrt{-1+c^2x^2}} dx}{3\sqrt{c^2x^2}} \\ &= \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{3x^3} - \frac{e(a + b \sec^{-1}(cx))}{x} - \frac{(bc(-2c^2d - 3ex^2))}{9x^2\sqrt{c^2x^2}} \\ &= \frac{bc(2c^2d + 9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{3x^3} - \frac{e(a + b \sec^{-1}(cx))}{x} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 69, normalized size = 0.66

$$\frac{-3a(d + 3ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(2c^2dx^2 + d + 9ex^2) - 3b \sec^{-1}(cx)(d + 3ex^2)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4, x]
```

[Out]  $(-3*a*(d + 3*e*x^2) + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])*x*(d + 2*c^2*d*x^2 + 9*e*x^2) - 3*b*(d + 3*e*x^2)*\text{ArcSec}[c*x]/(9*x^3)$

**fricas** [A] time = 2.06, size = 67, normalized size = 0.64

$$\frac{9 a e x^2 + 3 a d + 3 (3 b e x^2 + b d) \operatorname{arcsec}(c x) - \sqrt{c^2 x^2 - 1} ((2 b c^2 d + 9 b e) x^2 + b d)}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")`

[Out]  $-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*\operatorname{arcsec}(c*x) - \sqrt{c^2*x^2 - 1}*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3$

**giac** [A] time = 0.16, size = 116, normalized size = 1.10

$$\frac{1}{9} \left( 2 b c^2 d \sqrt{-\frac{1}{c^2 x^2} + 1} + 9 b \sqrt{-\frac{1}{c^2 x^2} + 1} e - \frac{9 b \arccos\left(\frac{1}{c x}\right) e}{c x} + \frac{b d \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^2} - \frac{9 a e}{c x} - \frac{3 b d \arccos\left(\frac{1}{c x}\right)}{c x^3} - \frac{3 a d}{c x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")`

[Out]  $1/9*(2*b*c^2*d*\sqrt{-1/(c^2*x^2) + 1} + 9*b*\sqrt{-1/(c^2*x^2) + 1}*e - 9*b*\arccos(1/(c*x))*e/(c*x) + b*d*\sqrt{-1/(c^2*x^2) + 1}/x^2 - 9*a*e/(c*x) - 3*b*d*\arccos(1/(c*x))/(c*x^3) - 3*a*d/(c*x^3))*c$

**maple** [A] time = 0.06, size = 121, normalized size = 1.15

$$c^3 \left( \frac{a \left( -\frac{d}{3 c x^3} - \frac{e}{c x} \right)}{c^2} + \frac{b \left( -\frac{\operatorname{arcsec}(c x) d}{3 c x^3} - \frac{\operatorname{arcsec}(c x) e}{c x} + \frac{(c^2 x^2 - 1)(2 c^4 d x^2 + 9 c^2 e x^2 + c^2 d)}{9 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} c^4 x^4} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x)`

[Out]  $c^3*(a/c^2*(-1/3*c*d/x^3 - e/c/x) + b/c^2*(-1/3*\operatorname{arcsec}(c*x)/c*d/x^3 - \operatorname{arcsec}(c*x)*e/c/x + 1/9*(c^2*x^2 - 1)*(2*c^4*d*x^2 + 9*c^2*e*x^2 + c^2*d)/((c^2*x^2 - 1)/c^2/x^2))^(1/2)/c^4/x^4)$



**maxima** [A] time = 0.32, size = 94, normalized size = 0.90

$$\left( c\sqrt{-\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsec}(cx)}{x} \right) be - \frac{1}{9} bd \left( \frac{c^4 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} + \frac{3 \operatorname{arcsec}(cx)}{x^3} \right) - \frac{ae}{x} - \frac{ad}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^4,x, algorithm="maxima")

[Out] (c\*sqrt(-1/(c^2\*x^2) + 1) - arcsec(c\*x)/x)\*b\*e - 1/9\*b\*d\*((c^4\*(-1/(c^2\*x^2) + 1)^(3/2) - 3\*c^4\*sqrt(-1/(c^2\*x^2) + 1))/c + 3\*arcsec(c\*x)/x^3) - a\*e/x - 1/3\*a\*d/x^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x^4,x)

[Out] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x^4, x)

**sympy** [A] time = 4.54, size = 150, normalized size = 1.43

$$-\frac{ad}{3x^3} - \frac{ae}{x} + bce\sqrt{1 - \frac{1}{c^2x^2}} - \frac{bd \operatorname{asec}(cx)}{3x^3} - \frac{be \operatorname{asec}(cx)}{x} + \frac{bd \begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*asec(c\*x))/x\*\*4,x)

[Out] -a\*d/(3\*x\*\*3) - a\*e/x + b\*c\*e\*sqrt(1 - 1/(c\*\*2\*x\*\*2)) - b\*d\*asec(c\*x)/(3\*x\*\*3) - b\*e\*asec(c\*x)/x + b\*d\*Piecewise((2\*c\*\*3\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*x) + c\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*x\*\*3), Abs(c\*\*2\*x\*\*2) > 1), (2\*I\*c\*\*3\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*x) + I\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*x\*\*3), True))/(3\*c)

$$3.74 \quad \int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=152

$$-\frac{d(a+b \sec^{-1}(cx))}{5x^5} - \frac{e(a+b \sec^{-1}(cx))}{3x^3} + \frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}} + \frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}}$$

[Out]  $-1/5*d*(a+b*\text{arcsec}(c*x))/x^5-1/3*e*(a+b*\text{arcsec}(c*x))/x^3+2/225*b*c^3*(12*c^2*d+25*e)*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}+1/25*b*c*d*(c^2*x^2-1)^{(1/2)}/x^4/(c^2*x^2)^{(1/2)}+1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {14, 5238, 12, 453, 271, 264}

$$-\frac{d(a+b \sec^{-1}(cx))}{5x^5} - \frac{e(a+b \sec^{-1}(cx))}{3x^3} + \frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x^6,x]

[Out]  $(2*b*c^3*(12*c^2*d + 25*e)*\text{Sqrt}[-1 + c^2*x^2])/(225*\text{Sqrt}[c^2*x^2]) + (b*c*d*\text{Sqrt}[-1 + c^2*x^2])/(25*x^4*\text{Sqrt}[c^2*x^2]) + (b*c*(12*c^2*d + 25*e)*\text{Sqrt}[-1 + c^2*x^2])/(225*x^2*\text{Sqrt}[c^2*x^2]) - (d*(a + b*\text{ArcSec}[c*x]))/(5*x^5) - (e*(a + b*\text{ArcSec}[c*x]))/(3*x^3)$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n},

p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \sec^{-1}(cx))}{x^6} dx &= -\frac{d(a + b \sec^{-1}(cx))}{5x^5} - \frac{e(a + b \sec^{-1}(cx))}{3x^3} - \frac{(bcx) \int \frac{-3d-5ex^2}{15x^6 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(a + b \sec^{-1}(cx))}{5x^5} - \frac{e(a + b \sec^{-1}(cx))}{3x^3} - \frac{(bcx) \int \frac{-3d-5ex^2}{x^6 \sqrt{-1+c^2x^2}} dx}{15\sqrt{c^2x^2}} \\
&= \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{5x^5} - \frac{e(a + b \sec^{-1}(cx))}{3x^3} - \frac{(bc(-12c^2d - 25e)\sqrt{-1+c^2x^2})}{225x^2\sqrt{c^2x^2}} \\
&= \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d + 25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{5x^5} - \frac{e(a + b \sec^{-1}(cx))}{3x^3} \\
&= \frac{2bc^3(12c^2d + 25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d + 25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 94, normalized size = 0.62

$$\frac{-15a(3d + 5ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(25ex^2(2c^2x^2 + 1) + 3d(8c^4x^4 + 4c^2x^2 + 3)) - 15b \sec^{-1}(cx)(3d + 5ex^2)}{225x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x^6,x]

[Out] (-15\*a\*(3\*d + 5\*e\*x^2) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(25\*e\*x^2\*(1 + 2\*c^2\*x^2) + 3\*d\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(3\*d + 5\*e\*x^2)\*ArcSec[c\*x])/(225\*x^5)

**fricas [A]** time = 1.07, size = 89, normalized size = 0.59

$$\frac{75 a e x^2 + 45 a d + 15 (5 b e x^2 + 3 b d) \operatorname{arcsec}(c x) - (2 (12 b c^4 d + 25 b c^2 e) x^4 + (12 b c^2 d + 25 b e) x^2 + 9 b d) \sqrt{c^2 x^2 - 1}}{225 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="fricas")

[Out] -1/225\*(75\*a\*e\*x^2 + 45\*a\*d + 15\*(5\*b\*e\*x^2 + 3\*b\*d)\*arcsec(c\*x) - (2\*(12\*b\*c^4\*d + 25\*b\*c^2\*e)\*x^4 + (12\*b\*c^2\*d + 25\*b\*e)\*x^2 + 9\*b\*d)\*sqrt(c^2\*x^2 - 1))/x^5

**giac** [A] time = 0.16, size = 162, normalized size = 1.07

$$\frac{1}{225} \left( 24bc^4d\sqrt{-\frac{1}{c^2x^2} + 1} + 50bc^2\sqrt{-\frac{1}{c^2x^2} + 1}e + \frac{12bc^2d\sqrt{-\frac{1}{c^2x^2} + 1}}{x^2} + \frac{25b\sqrt{-\frac{1}{c^2x^2} + 1}e}{x^2} - \frac{75b\arccos\left(\frac{1}{cx}\right)e}{cx^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="giac")

[Out] 1/225\*(24\*b\*c^4\*d\*sqrt(-1/(c^2\*x^2) + 1) + 50\*b\*c^2\*sqrt(-1/(c^2\*x^2) + 1)\*e + 12\*b\*c^2\*d\*sqrt(-1/(c^2\*x^2) + 1)/x^2 + 25\*b\*sqrt(-1/(c^2\*x^2) + 1)\*e/x^2 - 75\*b\*arccos(1/(c\*x))\*e/(c\*x^3) + 9\*b\*d\*sqrt(-1/(c^2\*x^2) + 1)/x^4 - 75\*a\*e/(c\*x^3) - 45\*b\*d\*arccos(1/(c\*x))/(c\*x^5) - 45\*a\*d/(c\*x^5))\*c

**maple** [A] time = 0.06, size = 140, normalized size = 0.92

$$c^5 \left( \frac{a \left( -\frac{e}{3c^3x^3} - \frac{d}{5c^3x^5} \right)}{c^2} + \frac{b \left( -\frac{\arccos(cx)e}{3c^3x^3} - \frac{\arccos(cx)d}{5c^3x^5} + \frac{(c^2x^2-1)(24x^4c^6d+50c^4ex^4+12c^4dx^2+25c^2ex^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}c^6x^6} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^6,x)

[Out] c^5\*(a/c^2\*(-1/3\*e/c^3/x^3-1/5/c^3\*d/x^5)+b/c^2\*(-1/3\*arcsec(c\*x)\*e/c^3/x^3-1/5\*arcsec(c\*x)/c^3\*d/x^5+1/225\*(c^2\*x^2-1)\*(24\*c^6\*d\*x^4+50\*c^4\*e\*x^4+12\*c^4\*d\*x^2+25\*c^2\*e\*x^2+9\*c^2\*d)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))

**maxima** [A] time = 0.32, size = 137, normalized size = 0.90

$$\frac{1}{75}bd \left( \frac{3c^6\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} - 10c^6\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 15c^6\sqrt{-\frac{1}{c^2x^2} + 1}}{c} - \frac{15\arccos(cx)}{x^5} \right) - \frac{1}{9}be \left( \frac{c^4\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3c^4\sqrt{-\frac{1}{c^2x^2} + 1}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="maxima")

[Out] 1/75\*b\*d\*((3\*c^6\*(-1/(c^2\*x^2) + 1)^(5/2) - 10\*c^6\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*c^6\*sqrt(-1/(c^2\*x^2) + 1))/c - 15\*arcsec(c\*x)/x^5) - 1/9\*b\*e\*((c^4\*(

$$-1/(c^2x^2 + 1)^{3/2} - 3c^4\sqrt{-1/(c^2x^2 + 1)}/c + 3\operatorname{arcsec}(cx)/x^3 - 1/3ae/x^3 - 1/5ad/x^5$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x^6, x)

[Out] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x^6, x)

**sympy** [A] time = 9.29, size = 279, normalized size = 1.84

$$\frac{\frac{ad}{5x^5} - \frac{ae}{3x^3} - \frac{bd \operatorname{asec}(cx)}{5x^5} - \frac{be \operatorname{asec}(cx)}{3x^3}}{5c} + \frac{bd \begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}}{5c} + \frac{be \begin{cases} \frac{2c^3}{5x^5} \\ \frac{2ic^3}{5x^5} \end{cases}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*asec(c\*x))/x\*\*6, x)

[Out] -a\*d/(5\*x\*\*5) - a\*e/(3\*x\*\*3) - b\*d\*asec(c\*x)/(5\*x\*\*5) - b\*e\*asec(c\*x)/(3\*x\*\*3) + b\*d\*Piecewise((8\*c\*\*5\*sqrt(c\*\*2\*x\*\*2 - 1)/(15\*x) + 4\*c\*\*3\*sqrt(c\*\*2\*x\*\*2 - 1)/(15\*x\*\*3) + c\*sqrt(c\*\*2\*x\*\*2 - 1)/(5\*x\*\*5), Abs(c\*\*2\*x\*\*2) > 1), (8\*I\*c\*\*5\*sqrt(-c\*\*2\*x\*\*2 + 1)/(15\*x) + 4\*I\*c\*\*3\*sqrt(-c\*\*2\*x\*\*2 + 1)/(15\*x\*\*3) + I\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)/(5\*x\*\*5), True))/(5\*c) + b\*e\*Piecewise((2\*c\*\*3\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*x) + c\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*x\*\*3), Abs(c\*\*2\*x\*\*2) > 1), (2\*I\*c\*\*3\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*x) + I\*c\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*x\*\*3), True))/(3\*c)

$$3.75 \quad \int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=197

$$\frac{d(a+b \sec^{-1}(cx))}{7x^7} - \frac{e(a+b \sec^{-1}(cx))}{5x^5} + \frac{bc\sqrt{c^2x^2-1}(30c^2d+49e)}{1225x^4\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}} + \frac{8bc^5\sqrt{c^2x^2-1}(30c^2d+49e)}{3675\sqrt{c^2x^2}}$$

[Out]  $-1/7*d*(a+b*\text{arcsec}(c*x))/x^7-1/5*e*(a+b*\text{arcsec}(c*x))/x^5+8/3675*b*c^5*(30*c^2*d+49*e)*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}+1/49*b*c*d*(c^2*x^2-1)^{(1/2)}/x^6/(c^2*x^2)^{(1/2)}+1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^{(1/2)}/x^4/(c^2*x^2)^{(1/2)}+4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.12, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {14, 5238, 12, 453, 271, 264}

$$\frac{d(a+b \sec^{-1}(cx))}{7x^7} - \frac{e(a+b \sec^{-1}(cx))}{5x^5} + \frac{8bc^5\sqrt{c^2x^2-1}(30c^2d+49e)}{3675\sqrt{c^2x^2}} + \frac{4bc^3\sqrt{c^2x^2-1}(30c^2d+49e)}{3675x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x^8, x]

[Out]  $(8*b*c^5*(30*c^2*d+49*e)*\text{Sqrt}[-1+c^2*x^2]/(3675*\text{Sqrt}[c^2*x^2])+(b*c*d*\text{Sqrt}[-1+c^2*x^2])/(49*x^6*\text{Sqrt}[c^2*x^2])+(b*c*(30*c^2*d+49*e)*\text{Sqrt}[-1+c^2*x^2])/(1225*x^4*\text{Sqrt}[c^2*x^2])+(4*b*c^3*(30*c^2*d+49*e)*\text{Sqrt}[-1+c^2*x^2])/(3675*x^2*\text{Sqrt}[c^2*x^2])-(d*(a+b*\text{ArcSec}[c*x]))/(7*x^7)-(e*(a+b*\text{ArcSec}[c*x]))/(5*x^5)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_)+(b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c\*x)^(m+1)\*(a+b\*x^n)^(p+1)/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n,

p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps



$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \sec^{-1}(cx))}{x^8} dx &= -\frac{d(a + b \sec^{-1}(cx))}{7x^7} - \frac{e(a + b \sec^{-1}(cx))}{5x^5} - \frac{(bcx) \int \frac{-5d-7ex^2}{35x^8\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(a + b \sec^{-1}(cx))}{7x^7} - \frac{e(a + b \sec^{-1}(cx))}{5x^5} - \frac{(bcx) \int \frac{-5d-7ex^2}{x^8\sqrt{-1+c^2x^2}} dx}{35\sqrt{c^2x^2}} \\
&= \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{7x^7} - \frac{e(a + b \sec^{-1}(cx))}{5x^5} - \frac{(bc(-30c^2d - 105e))}{35\sqrt{c^2x^2}} \\
&= \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{bc(30c^2d + 49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d(a + b \sec^{-1}(cx))}{7x^7} - \frac{e(a + b \sec^{-1}(cx))}{5x^5} \\
&= \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{bc(30c^2d + 49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{4bc^3(30c^2d + 49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} \\
&= \frac{8bc^5(30c^2d + 49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{bc(30c^2d + 49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 110, normalized size = 0.56

$$\frac{-105a(5d + 7ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(49ex^2(8c^4x^4 + 4c^2x^2 + 3) + 15d(16c^6x^6 + 8c^4x^4 + 6c^2x^2 + 5)) - 105b \sec^{-1}(cx)}{3675x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x^8,x]

[Out] (-105\*a\*(5\*d + 7\*e\*x^2) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(49\*e\*x^2\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4) + 15\*d\*(5 + 6\*c^2\*x^2 + 8\*c^4\*x^4 + 16\*c^6\*x^6)) - 105\*b\*(5\*d + 7\*e\*x^2)\*ArcSec[c\*x])/(3675\*x^7)

**fricas [A]** time = 1.49, size = 110, normalized size = 0.56

$$\frac{735 aex^2 + 525 ad + 105(7bex^2 + 5bd) \operatorname{arcsec}(cx) - (8(30bc^6d + 49bc^4e)x^6 + 4(30bc^4d + 49bc^2e)x^4 + 3(30b^2c^6d + 49b^2c^4e)x^2 + 75b^2d)\sqrt{c^2x^2 - 1}}{3675x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="fricas")

[Out] -1/3675\*(735\*a\*e\*x^2 + 525\*a\*d + 105\*(7\*b\*e\*x^2 + 5\*b\*d)\*arcsec(c\*x) - (8\*(30\*b\*c^6\*d + 49\*b\*c^4\*e)\*x^6 + 4\*(30\*b\*c^4\*d + 49\*b\*c^2\*e)\*x^4 + 3\*(30\*b\*c^6\*d + 49\*b\*c^4\*e)\*x^2 + 75\*b\*d)\*sqrt(c^2\*x^2 - 1))/x^7

**giac** [A] time = 0.16, size = 207, normalized size = 1.05

$$\frac{1}{3675} \left( 240 bc^6 d \sqrt{-\frac{1}{c^2 x^2} + 1} + 392 bc^4 \sqrt{-\frac{1}{c^2 x^2} + 1} e + \frac{120 bc^4 d \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^2} + \frac{196 bc^2 \sqrt{-\frac{1}{c^2 x^2} + 1} e}{x^2} + \frac{90 bc^2 d \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="giac")

[Out] 1/3675\*(240\*b\*c^6\*d\*sqrt(-1/(c^2\*x^2) + 1) + 392\*b\*c^4\*sqrt(-1/(c^2\*x^2) + 1)\*e + 120\*b\*c^4\*d\*sqrt(-1/(c^2\*x^2) + 1)/x^2 + 196\*b\*c^2\*sqrt(-1/(c^2\*x^2) + 1)\*e/x^2 + 90\*b\*c^2\*d\*sqrt(-1/(c^2\*x^2) + 1)/x^4 + 147\*b\*sqrt(-1/(c^2\*x^2) + 1)\*e/x^4 - 735\*b\*arccos(1/(c\*x))\*e/(c\*x^5) + 75\*b\*d\*sqrt(-1/(c^2\*x^2) + 1)/x^6 - 735\*a\*e/(c\*x^5) - 525\*b\*d\*arccos(1/(c\*x))/(c\*x^7) - 525\*a\*d/(c\*x^7))\*c

**maple** [A] time = 0.06, size = 158, normalized size = 0.80

$$c^7 \left( \frac{a \left( -\frac{d}{7c^5 x^7} - \frac{e}{5c^5 x^5} \right)}{c^2} + \frac{b \left( -\frac{\operatorname{arcsec}(cx)d}{7c^5 x^7} - \frac{\operatorname{arcsec}(cx)e}{5c^5 x^5} + \frac{(c^2 x^2 - 1)(240c^8 d x^6 + 392c^6 e x^6 + 120x^4 c^6 d + 196c^4 e x^4 + 90c^4 d x^2 + 147c^2 e x^2 + 75c^2 d)}{3675 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} c^8 x^8} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^8,x)

[Out] c^7\*(a/c^2\*(-1/7/c^5\*d/x^7-1/5\*e/c^5/x^5)+b/c^2\*(-1/7\*arcsec(c\*x)/c^5\*d/x^7-1/5\*arcsec(c\*x)\*e/c^5/x^5+1/3675\*(c^2\*x^2-1)\*(240\*c^8\*d\*x^6+392\*c^6\*e\*x^6+120\*c^6\*d\*x^4+196\*c^4\*e\*x^4+90\*c^4\*d\*x^2+147\*c^2\*e\*x^2+75\*c^2\*d)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^8/x^8))

**maxima** [A] time = 0.32, size = 172, normalized size = 0.87

$$-\frac{1}{245} bd \left( \frac{5c^8 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{7}{2}} - 21c^8 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} + 35c^8 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 35c^8 \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} + \frac{35 \operatorname{arcsec}(cx)}{x^7} \right) + \frac{1}{75} be \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="maxima")

[Out]  $-1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^{(7/2)} - 21*c^8*(-1/(c^2*x^2) + 1)^{(5/2)} + 35*c^8*(-1/(c^2*x^2) + 1)^{(3/2)} - 35*c^8*\sqrt{-1/(c^2*x^2) + 1})/c + 35*\operatorname{arcsec}(c*x)/x^7) + 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^{(5/2)} - 10*c^6*(-1/(c^2*x^2) + 1)^{(3/2)} + 15*c^6*\sqrt{-1/(c^2*x^2) + 1})/c - 15*\operatorname{arcsec}(c*x)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8, x)`

[Out] `int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8, x)`

**sympy [A]** time = 55.10, size = 371, normalized size = 1.88

$$\frac{ad}{7x^7} - \frac{ae}{5x^5} - \frac{bd \operatorname{asec}(cx)}{7x^7} - \frac{be \operatorname{asec}(cx)}{5x^5} + \frac{bd \begin{cases} \frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise} \end{cases}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(a+b*asec(c*x))/x**8, x)`

[Out]  $-a*d/(7*x**7) - a*e/(5*x**5) - b*d*\operatorname{asec}(c*x)/(7*x**7) - b*e*\operatorname{asec}(c*x)/(5*x**5) + b*d*\operatorname{Piecewise}((16*c**7*\sqrt{c**2*x**2 - 1})/(35*x) + 8*c**5*\sqrt{c**2*x**2 - 1})/(35*x**3) + 6*c**3*\sqrt{c**2*x**2 - 1})/(35*x**5) + c*\sqrt{c**2*x**2 - 1})/(7*x**7), \operatorname{Abs}(c**2*x**2) > 1), (16*I*c**7*\sqrt{-c**2*x**2 + 1})/(35*x) + 8*I*c**5*\sqrt{-c**2*x**2 + 1})/(35*x**3) + 6*I*c**3*\sqrt{-c**2*x**2 + 1})/(35*x**5) + I*c*\sqrt{-c**2*x**2 + 1})/(7*x**7), \operatorname{True}))/7*c + b*e*\operatorname{Piecewise}((8*c**5*\sqrt{c**2*x**2 - 1})/(15*x) + 4*c**3*\sqrt{c**2*x**2 - 1})/(15*x**3) + c*\sqrt{c**2*x**2 - 1})/(5*x**5), \operatorname{Abs}(c**2*x**2) > 1), (8*I*c**5*\sqrt{-c**2*x**2 + 1})/(15*x) + 4*I*c**3*\sqrt{-c**2*x**2 + 1})/(15*x**3) + I*c*\sqrt{-c**2*x**2 + 1})/(5*x**5), \operatorname{True}))/5*c$

### 3.76 $\int x^5 (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=196

$$\frac{1}{6}dx^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}ex^8 (a + b \sec^{-1}(cx)) - \frac{bx (c^2x^2 - 1)^{5/2} (4c^2d + 9e)}{120c^7 \sqrt{c^2x^2}} - \frac{bx (c^2x^2 - 1)^{3/2} (8c^2d + 9e)}{72c^7 \sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2}}{72c^7 \sqrt{c^2x^2}}$$

[Out]  $\frac{1}{6}d*x^6*(a+b*\text{arcsec}(c*x))+\frac{1}{8}*e*x^8*(a+b*\text{arcsec}(c*x))-1/72*b*(8*c^2*d+9*e)*x*(c^2*x^2-1)^{(3/2)}/c^7/(c^2*x^2)^{(1/2)}-1/120*b*(4*c^2*d+9*e)*x*(c^2*x^2-1)^{(5/2)}/c^7/(c^2*x^2)^{(1/2)}-1/56*b*e*x*(c^2*x^2-1)^{(7/2)}/c^7/(c^2*x^2)^{(1/2)}-1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^{(1/2)}/c^7/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5238, 12, 446, 77}

$$\frac{1}{6}dx^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}ex^8 (a + b \sec^{-1}(cx)) - \frac{bx (c^2x^2 - 1)^{5/2} (4c^2d + 9e)}{120c^7 \sqrt{c^2x^2}} - \frac{bx (c^2x^2 - 1)^{3/2} (8c^2d + 9e)}{72c^7 \sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2}}{72c^7 \sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]`

[Out]  $-(b*(4*c^2*d + 3*e)*x*\text{Sqrt}[-1 + c^2*x^2])/(24*c^7*\text{Sqrt}[c^2*x^2]) - (b*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^{(3/2)})/(72*c^7*\text{Sqrt}[c^2*x^2]) - (b*(4*c^2*d + 9*e)*x*(-1 + c^2*x^2)^{(5/2)})/(120*c^7*\text{Sqrt}[c^2*x^2]) - (b*e*x*(-1 + c^2*x^2)^{(7/2)})/(56*c^7*\text{Sqrt}[c^2*x^2]) + (d*x^6*(a + b*\text{ArcSec}[c*x])/6 + (e*x^8*(a + b*\text{ArcSec}[c*x])/8)$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

#### Rule 77

`Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],`

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
c, d, e, f])))
```

### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
 \int x^5 (d + ex^2) (a + b \sec^{-1}(cx)) dx &= \frac{1}{6} dx^6 (a + b \sec^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^5(4d+3ex^2)}{24\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
 &= \frac{1}{6} dx^6 (a + b \sec^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^5(4d+3ex^2)}{\sqrt{-1+c^2x^2}} dx}{24\sqrt{c^2x^2}} \\
 &= \frac{1}{6} dx^6 (a + b \sec^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \sec^{-1}(cx)) - \frac{(bcx) \text{Subst} \left( \int \frac{x^2(4d+3e)}{\sqrt{-1+c^2}} dx \right)}{48\sqrt{c^2x^2}} \\
 &= \frac{1}{6} dx^6 (a + b \sec^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \sec^{-1}(cx)) - \frac{(bcx) \text{Subst} \left( \int \left( \frac{4c^2d+3e}{c^6\sqrt{-1+c^2}} \right) dx \right)}{48\sqrt{c^2x^2}} \\
 &= -\frac{b(4c^2d + 3e)x\sqrt{-1 + c^2x^2}}{24c^7\sqrt{c^2x^2}} - \frac{b(8c^2d + 9e)x(-1 + c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}} - \frac{b(4c^2d + 3e)}{48c^6}
 \end{aligned}$$

**Mathematica** [A] time = 0.24, size = 118, normalized size = 0.60

$$\frac{1}{24}ax^6(4d + 3ex^2) - \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}(c^6(84dx^4 + 45ex^6) + 2c^4(56dx^2 + 27ex^4) + 8c^2(28d + 9ex^2) + 144e)}{2520c^7} + \frac{1}{24}bx^6$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]), x]

[Out] (a\*x^6\*(4\*d + 3\*e\*x^2))/24 - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(144\*e + 8\*c^2\*(28\*d + 9\*e\*x^2) + 2\*c^4\*(56\*d\*x^2 + 27\*e\*x^4) + c^6\*(84\*d\*x^4 + 45\*e\*x^6)))/(2520\*c^7) + (b\*x^6\*(4\*d + 3\*e\*x^2)\*ArcSec[c\*x])/24

**fricas** [A] time = 1.98, size = 128, normalized size = 0.65

$$\frac{315ac^8ex^8 + 420ac^8dx^6 + 105(3bc^8ex^8 + 4bc^8dx^6) \operatorname{arcsec}(cx) - (45bc^6ex^6 + 6(14bc^6d + 9bc^4e)x^4 + 224bc^2d + 8(14bc^4d + 9bc^2e)x^2 + 144b^2e)\sqrt{c^2x^2 - 1}}{2520c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] 1/2520\*(315\*a\*c^8\*e\*x^8 + 420\*a\*c^8\*d\*x^6 + 105\*(3\*b\*c^8\*e\*x^8 + 4\*b\*c^8\*d\*x^6)\*arcsec(c\*x) - (45\*b\*c^6\*e\*x^6 + 6\*(14\*b\*c^6\*d + 9\*b\*c^4\*e)\*x^4 + 224\*b\*c^2\*d + 8\*(14\*b\*c^4\*d + 9\*b\*c^2\*e)\*x^2 + 144\*b^2\*e)\*sqrt(c^2\*x^2 - 1)/c^8

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)), x, algorithm="giac")

[Out] Timed out

**maple** [A] time = 0.05, size = 152, normalized size = 0.78

$$\frac{a\left(\frac{1}{8}e^8x^8 + \frac{1}{6}c^8dx^6\right)}{c^2} + \frac{b\left(\frac{\operatorname{arcsec}(cx)e^8x^8}{8} + \frac{\operatorname{arcsec}(cx)c^8x^6d}{6} - \frac{(c^2x^2-1)(45e^6x^6+84x^4c^6d+54c^4e^4x^4+112c^4dx^2+72c^2e^2x^2+224c^2d+144e)}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2} + \frac{b\left(\frac{\operatorname{arcsec}(cx)e^8x^8}{8} + \frac{\operatorname{arcsec}(cx)c^8x^6d}{6} - \frac{(c^2x^2-1)(45e^6x^6+84x^4c^6d+54c^4e^4x^4+112c^4dx^2+72c^2e^2x^2+224c^2d+144e)}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)), x)

[Out]  $\frac{1}{c^6} \left( \frac{a}{c^2} \left( \frac{1}{8} e^{cx^8} + \frac{1}{6} c^8 d x^6 \right) + \frac{b}{c^2} \left( \frac{1}{8} \operatorname{arcsec}(cx) e^{cx^8} + \frac{1}{6} \operatorname{arcsec}(cx) c^8 x^6 d - \frac{1}{2520} (c^2 x^2 - 1) (45 c^6 e^{cx^6} + 84 c^6 d x^4 + 54 c^4 e^{cx^4} + 112 c^4 d x^2 + 72 c^2 e^{cx^2} + 224 c^2 d + 144 e) \right) \right) / \left( \frac{c^2 x^2 - 1}{c^2/x^2} \right)^{(1/2)/c/x}$

**maxima** [A] time = 0.33, size = 185, normalized size = 0.94

$$\frac{1}{8} a e x^8 + \frac{1}{6} a d x^6 + \frac{1}{90} \left( 15 x^6 \operatorname{arcsec}(cx) - \frac{3 c^4 x^5 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} + 10 c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 15 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5} \right) b d + \frac{1}{280} \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{8} a e x^8 + \frac{1}{6} a d x^6 + \frac{1}{90} (15 x^6 \operatorname{arcsec}(cx) - (3 c^4 x^5 (-1/(c^2 x^2) + 1)^{(5/2)} + 10 c^2 x^3 (-1/(c^2 x^2) + 1)^{(3/2)} + 15 x \sqrt{-1/(c^2 x^2) + 1})/c^5) b d + \frac{1}{280} (35 x^8 \operatorname{arcsec}(cx) - (5 c^6 x^7 (-1/(c^2 x^2) + 1)^{(7/2)} + 21 c^4 x^5 (-1/(c^2 x^2) + 1)^{(5/2)} + 35 c^2 x^3 (-1/(c^2 x^2) + 1)^{(3/2)} + 35 x \sqrt{-1/(c^2 x^2) + 1})/c^7) b e$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 (e x^2 + d) \left( a + b \operatorname{acos}\left(\frac{1}{c x}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))),x)`

[Out] `int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))), x)`

**sympy** [A] time = 7.92, size = 364, normalized size = 1.86

$$\frac{a d x^6}{6} + \frac{a e x^8}{8} + \frac{b d x^6 \operatorname{asec}(cx)}{6} + \frac{b e x^8 \operatorname{asec}(cx)}{8} - \frac{b d \left\{ \begin{array}{ll} \frac{x^4 \sqrt{c^2 x^2 - 1}}{5c} + \frac{4 x^2 \sqrt{c^2 x^2 - 1}}{15 c^3} + \frac{8 \sqrt{c^2 x^2 - 1}}{15 c^5} & \text{for } |c^2 x^2| > 1 \\ \frac{i x^4 \sqrt{-c^2 x^2 + 1}}{5c} + \frac{4 i x^2 \sqrt{-c^2 x^2 + 1}}{15 c^3} + \frac{8 i \sqrt{-c^2 x^2 + 1}}{15 c^5} & \text{otherwise} \end{array} \right.}{6c} b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(e*x**2+d)*(a+b*asec(c*x)),x)`

[Out]  $a d x^6/6 + a e x^8/8 + b d x^6 \operatorname{asec}(cx)/6 + b e x^8 \operatorname{asec}(cx)/8 - b d * \operatorname{Piecewise}((x^4 \sqrt{c^2 x^2 - 1})/(5c) + 4 x^2 \sqrt{c^2 x^2 - 1}/(15$

```

*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt
(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt
(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) - b*e*Piecewise((x**6*sqrt(c**2*x*
*2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c**2*x**
2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2) > 1), (
I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(35*c**3)
+ 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*
c**7), True))/(8*c)

```



### 3.77 $\int x^3 (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=153

$$\frac{1}{4}dx^4(a + b \sec^{-1}(cx)) + \frac{1}{6}ex^6(a + b \sec^{-1}(cx)) - \frac{bx(c^2x^2 - 1)^{3/2}(3c^2d + 4e)}{36c^5\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2 - 1}(3c^2d + 2e)}{12c^5\sqrt{c^2x^2}} - \frac{bex(c^2x^2 - 1)^{5/2}}{30c^5\sqrt{c^2x^2}}$$

[Out]  $\frac{1}{4}d*x^4*(a+b*\text{arcsec}(c*x))+\frac{1}{6}*e*x^6*(a+b*\text{arcsec}(c*x))-\frac{1}{36}*b*(3*c^2*d+4*e)*x*(c^2*x^2-1)^{(3/2)}/c^5/(c^2*x^2)^{(1/2)}-\frac{1}{30}*b*e*x*(c^2*x^2-1)^{(5/2)}/c^5/(c^2*x^2)^{(1/2)}-\frac{1}{12}*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^{(1/2)}/c^5/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5238, 12, 446, 77}

$$\frac{1}{4}dx^4(a + b \sec^{-1}(cx)) + \frac{1}{6}ex^6(a + b \sec^{-1}(cx)) - \frac{bx(c^2x^2 - 1)^{3/2}(3c^2d + 4e)}{36c^5\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2 - 1}(3c^2d + 2e)}{12c^5\sqrt{c^2x^2}} - \frac{bex(c^2x^2 - 1)^{5/2}}{30c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(d + e*x^2)*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-(b*(3*c^2*d + 2*e)*x*\text{Sqrt}[-1 + c^2*x^2])/(12*c^5*\text{Sqrt}[c^2*x^2]) - (b*(3*c^2*d + 4*e)*x*(-1 + c^2*x^2)^{(3/2)})/(36*c^5*\text{Sqrt}[c^2*x^2]) - (b*e*x*(-1 + c^2*x^2)^{(5/2)})/(30*c^5*\text{Sqrt}[c^2*x^2]) + (d*x^4*(a + b*\text{ArcSec}[c*x]))/4 + (e*x^6*(a + b*\text{ArcSec}[c*x]))/6$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_)+(b_)*(v_)] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{InverseFunctionQ}[v]$

#### Rule 77

$\text{Int}[(a_)+(b_)*(x_)*((c_)+(d_)*(x_))^{(n_.)}*((e_)+(f_)*(x_))^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ ((\text{ILtQ}[n, 0])$

```
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
c, d, e, f])))
```

### Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int x^3 (d + ex^2) (a + b \sec^{-1}(cx)) dx &= \frac{1}{4} dx^4 (a + b \sec^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^3(3d+2ex^2)}{12\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{1}{4} dx^4 (a + b \sec^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \sec^{-1}(cx)) - \frac{(bcx) \int \frac{x^3(3d+2ex^2)}{\sqrt{-1+c^2x^2}} dx}{12\sqrt{c^2x^2}} \\
&= \frac{1}{4} dx^4 (a + b \sec^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \sec^{-1}(cx)) - \frac{(bcx) \operatorname{Subst}\left(\int \frac{x(3d+2ex)}{\sqrt{-1+c^2x}}\right)}{24\sqrt{c^2x^2}} \\
&= \frac{1}{4} dx^4 (a + b \sec^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \sec^{-1}(cx)) - \frac{(bcx) \operatorname{Subst}\left(\int \left(\frac{3c^2d+2ex}{c^4\sqrt{-1+c^2x}}\right)\right)}{24\sqrt{c^2x^2}} \\
&= -\frac{b(3c^2d+2e)x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}} - \frac{b(3c^2d+4e)x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.27, size = 98, normalized size = 0.64

$$\frac{1}{180}x \left( 15ax^3(3d + 2ex^2) - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}(3c^4(5dx^2 + 2ex^4) + c^2(30d + 8ex^2) + 16e)}{c^5} + 15bx^3 \sec^{-1}(cx)(3d + 2ex^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]),x]

[Out] (x\*(15\*a\*x^3\*(3\*d + 2\*e\*x^2) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*(16\*e + c^2\*(30\*d + 8\*e\*x^2) + 3\*c^4\*(5\*d\*x^2 + 2\*e\*x^4))))/c^5 + 15\*b\*x^3\*(3\*d + 2\*e\*x^2)\*ArcSec[c\*x])/180

**fricas [A]** time = 0.65, size = 107, normalized size = 0.70

$$\frac{30ac^6ex^6 + 45ac^6dx^4 + 15(2bc^6ex^6 + 3bc^6dx^4) \operatorname{arcsec}(cx) - (6bc^4ex^4 + 30bc^2d + (15bc^4d + 8bc^2e)x^2 + 16b^2e)}{180c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/180\*(30\*a\*c^6\*e\*x^6 + 45\*a\*c^6\*d\*x^4 + 15\*(2\*b\*c^6\*e\*x^6 + 3\*b\*c^6\*d\*x^4)\*arcsec(c\*x) - (6\*b\*c^4\*e\*x^4 + 30\*b\*c^2\*d + (15\*b\*c^4\*d + 8\*b\*c^2\*e)\*x^2 + 16\*b\*e)\*sqrt(c^2\*x^2 - 1))/c^6

**giac [B]** time = 0.31, size = 7840, normalized size = 51.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] 1/180\*(45\*b\*c^2\*d\*arccos(1/(c\*x))/(c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12) + 45\*a\*c^2\*d/(c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12 - 90\*b\*c^2\*d\*(1/(c^2\*x^2) - 1)\*arccos(1/(c\*x))/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12))



$$\begin{aligned}
& c^2x^2) - 1)/(1/(cx) + 1)^2 + 15c^7*(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 \\
& + 20c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + 15c^7*(1/(c^2x^2) - 1)^4/( \\
& 1/(cx) + 1)^8 + 6c^7*(1/(c^2x^2) - 1)^5/(1/(cx) + 1)^{10} + c^7*(1/(c^2x \\
& ^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + 1)) + 180*a*c^2*d*(1/(c^2x^2) - 1) \\
& ^3/((c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7*(1/(c^2x^2) - \\
& 1)^2/(1/(cx) + 1)^4 + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + 15*c^7* \\
& (1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1)^5/(1/(cx) + \\
& 1)^{10} + c^7*(1/(c^2x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + 1)^6) - 45*b*c \\
& ^2*d*(1/(c^2x^2) - 1)^4*arccos(1/(cx))/((c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1 \\
& / (cx) + 1)^2 + 15*c^7*(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 20*c^7*(1/(c^2 \\
& *x^2) - 1)^3/(1/(cx) + 1)^6 + 15*c^7*(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8 + \\
& 6*c^7*(1/(c^2x^2) - 1)^5/(1/(cx) + 1)^{10} + c^7*(1/(c^2x^2) - 1)^6/(1/(c \\
& *x) + 1)^{12}*(1/(cx) + 1)^8) - 180*a*(1/(c^2x^2) - 1)*e/((c^7 + 6*c^7*(1/ \\
& (c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7*(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 \\
& + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + 15*c^7*(1/(c^2x^2) - 1)^4/ \\
& (1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1)^5/(1/(cx) + 1)^{10} + c^7*(1/(c^2* \\
& x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + 1)^2) + 450*b*(1/(c^2x^2) - 1)^2* \\
& arccos(1/(cx))*e/((c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7* \\
& (1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + \\
& 1)^6 + 15*c^7*(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1 \\
& )^5/(1/(cx) + 1)^{10} + c^7*(1/(c^2x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + \\
& 1)^4) - 540*b*c^2*d*(1/(c^2x^2) - 1)^3*sqrt(-1/(c^2x^2) + 1)/((c^7 + 6*c \\
& ^7*(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7*(1/(c^2x^2) - 1)^2/(1/(cx) \\
& + 1)^4 + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + 15*c^7*(1/(c^2x^2) - \\
& 1)^4/(1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1)^5/(1/(cx) + 1)^{10} + c^7*(1 \\
& / (c^2x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + 1)^7) + 140*b*(-1/(c^2x^2) \\
& + 1)^{(3/2)}*e/((c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7*(1/(c \\
& ^2x^2) - 1)^2/(1/(cx) + 1)^4 + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 \\
& + 15*c^7*(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1)^5/( \\
& 1/(cx) + 1)^{10} + c^7*(1/(c^2x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + 1)^3 \\
& ) - 45*a*c^2*d*(1/(c^2x^2) - 1)^4/((c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1/(cx) \\
& + 1)^2 + 15*c^7*(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 20*c^7*(1/(c^2x^2) \\
& - 1)^3/(1/(cx) + 1)^6 + 15*c^7*(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8 + 6*c^7 \\
& *(1/(c^2x^2) - 1)^5/(1/(cx) + 1)^{10} + c^7*(1/(c^2x^2) - 1)^6/(1/(cx) + \\
& 1)^{12}*(1/(cx) + 1)^8) - 90*b*c^2*d*(1/(c^2x^2) - 1)^5*arccos(1/(cx))/(( \\
& c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7*(1/(c^2x^2) - 1)^2/ \\
& (1/(cx) + 1)^4 + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + 1)^6 + 15*c^7*(1/(c \\
& ^2x^2) - 1)^4/(1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1)^5/(1/(cx) + 1)^{10} \\
& + c^7*(1/(c^2x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + 1)^{10}) + 450*a*(1/( \\
& c^2x^2) - 1)^2*e/((c^7 + 6*c^7*(1/(c^2x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7* \\
& (1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 20*c^7*(1/(c^2x^2) - 1)^3/(1/(cx) + \\
& 1)^6 + 15*c^7*(1/(c^2x^2) - 1)^4/(1/(cx) + 1)^8 + 6*c^7*(1/(c^2x^2) - 1 \\
& )^5/(1/(cx) + 1)^{10} + c^7*(1/(c^2x^2) - 1)^6/(1/(cx) + 1)^{12}*(1/(cx) + \\
& 1)^4) - 600*b*(1/(c^2x^2) - 1)^3*arccos(1/(cx))*e/((c^7 + 6*c^7*(1/(c^2* \\
& x^2) - 1)/(1/(cx) + 1)^2 + 15*c^7*(1/(c^2x^2) - 1)^2/(1/(cx) + 1)^4 + 20
\end{aligned}$$

$$\begin{aligned}
& *c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^6 - 330*b*c^2*d*(1/(c^2*x^2) - 1)^4 * \sqrt{-1/(c^2*x^2) + 1}/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^9) - 312*b*(1/(c^2*x^2) - 1)^2*\sqrt{-1/(c^2*x^2) + 1}*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^5) - 90*a*c^2*d*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^10) + 45*b*c^2*d*(1/(c^2*x^2) - 1)^6*\arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^12) - 600*a*(1/(c^2*x^2) - 1)^3*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^6) + 450*b*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^11) - 312*b*(1/(c^2*x^2) - 1)^3*\sqrt{-1/(c^2*x^2) + 1}*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^7) + 45*a*c^2*d*(1/(c^2*x^2) - 1)^6/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12})*(1/(c*x) + 1)^12) + 450*a*(1/(c^2*x^2) - 1)^4*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
\end{aligned}$$

+ 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^8 - 180\*b\*(1/(c^2\*x^2) - 1)^5\*arccos(1/(c\*x))\*e/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^10) - 140\*b\*(1/(c^2\*x^2) - 1)^4\*sqrt(-1/(c^2\*x^2) + 1)\*e/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^9) - 180\*a\*(1/(c^2\*x^2) - 1)^5\*e/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^10) + 30\*b\*(1/(c^2\*x^2) - 1)^6\*arccos(1/(c\*x))\*e/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^12) - 60\*b\*(1/(c^2\*x^2) - 1)^5\*sqrt(-1/(c^2\*x^2) + 1)\*e/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^11) + 30\*a\*(1/(c^2\*x^2) - 1)^6\*e/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12\*(1/(c\*x) + 1)^12))\*c

**maple [A]** time = 0.05, size = 134, normalized size = 0.88

$$\frac{a\left(\frac{1}{6}e c^6 x^6 + \frac{1}{4}x^4 c^6 d\right)}{c^2} + \frac{b\left(\frac{\operatorname{arccos}(cx)e c^6 x^6}{6} + \frac{\operatorname{arccos}(cx)c^6 x^4 d}{4} - \frac{(c^2 x^2 - 1)(6c^4 e x^4 + 15c^4 d x^2 + 8c^2 e x^2 + 30c^2 d + 16e)}{180 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} cx}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)), x)

[Out] 1/c^4\*(a/c^2\*(1/6\*e\*c^6\*x^6+1/4\*x^4\*c^6\*d)+b/c^2\*(1/6\*arcsec(c\*x)\*e\*c^6\*x^6+1/4\*arcsec(c\*x)\*c^6\*x^4\*d-1/180\*(c^2\*x^2-1)\*(6\*c^4\*e\*x^4+15\*c^4\*d\*x^2+8\*c^2\*e\*x^2+30\*c^2\*d+16\*e)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x))

**maxima** [A] time = 0.31, size = 144, normalized size = 0.94

$$\frac{1}{6} aex^6 + \frac{1}{4} adx^4 + \frac{1}{12} \left( 3x^4 \operatorname{arcsec}(cx) - \frac{c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 3x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^3} \right) bd + \frac{1}{90} \left( 15x^6 \operatorname{arcsec}(cx) - \frac{3c^4 x^5 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} + 10c^2 x^3 \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5} \right) b^2 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/6\*a\*e\*x^6 + 1/4\*a\*d\*x^4 + 1/12\*(3\*x^4\*arcsec(c\*x) - (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*d + 1/90\*(15\*x^6\*arcsec(c\*x) - (3\*c^4\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 10\*c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^5)\*b^2\*e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (e x^2 + d) \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(d + e\*x^2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^3\*(d + e\*x^2)\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 5.29, size = 272, normalized size = 1.78

$$\frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4 \operatorname{asec}(cx)}{4} + \frac{bex^6 \operatorname{asec}(cx)}{6} - \frac{bd \left( \begin{cases} \frac{x^2 \sqrt{c^2 x^2 - 1}}{3c} + \frac{2\sqrt{c^2 x^2 - 1}}{3c^3} & \text{for } |c^2 x^2| > 1 \\ \frac{ix^2 \sqrt{-c^2 x^2 + 1}}{3c} + \frac{2i\sqrt{-c^2 x^2 + 1}}{3c^3} & \text{otherwise} \end{cases} \right)}{4c} - \frac{be \left( \begin{cases} \frac{x^4 \sqrt{c^2 x^2 - 1}}{5c} + \frac{4x^2 \sqrt{c^2 x^2 - 1}}{15c^3} & \text{for } |c^2 x^2| > 1 \\ \frac{ix^4 \sqrt{-c^2 x^2 + 1}}{5c} + \frac{4ix^2 \sqrt{-c^2 x^2 + 1}}{15c^3} & \text{otherwise} \end{cases} \right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x\*\*2+d)\*(a+b\*asec(c\*x)),x)

[Out] a\*d\*x\*\*4/4 + a\*e\*x\*\*6/6 + b\*d\*x\*\*4\*asec(c\*x)/4 + b\*e\*x\*\*6\*asec(c\*x)/6 - b\*d \*Piecewise((x\*\*2\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*c) + 2\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*c\*\*3), Abs(c\*\*2\*x\*\*2) > 1), (I\*x\*\*2\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*c) + 2\*I\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*c\*\*3), True))/(4\*c) - b\*e\*Piecewise((x\*\*4\*sqrt(c\*\*2\*x\*\*2 - 1)/(5\*c) + 4\*x\*\*2\*sqrt(c\*\*2\*x\*\*2 - 1)/(15\*c\*\*3) + 8\*sqrt(c\*\*2\*x\*\*2 - 1)/(15\*c\*\*5), Abs(c\*\*2\*x\*\*2) > 1), (I\*x\*\*4\*sqrt(-c\*\*2\*x\*\*2 + 1)/(5\*c) + 4\*I\*x\*\*2\*sqrt(-c\*\*2\*x\*\*2 + 1)/(15\*c\*\*3) + 8\*I\*sqrt(-c\*\*2\*x\*\*2 + 1)/(15\*c\*\*5), True))/(6\*c)



### 3.78 $\int x (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=138

$$\frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} - \frac{bcd^2 x \tan^{-1}(\sqrt{c^2 x^2 - 1})}{4e\sqrt{c^2 x^2}} - \frac{bx\sqrt{c^2 x^2 - 1} (2c^2 d + e)}{4c^3\sqrt{c^2 x^2}} - \frac{bex (c^2 x^2 - 1)^{3/2}}{12c^3\sqrt{c^2 x^2}}$$

[Out] 1/4\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/e-1/12\*b\*e\*x\*(c^2\*x^2-1)^(3/2)/c^3/(c^2\*x^2)^(1/2)-1/4\*b\*c\*d^2\*x\*arctan((c^2\*x^2-1)^(1/2))/e/(c^2\*x^2)^(1/2)-1/4\*b\*(2\*c^2\*d+e)\*x\*(c^2\*x^2-1)^(1/2)/c^3/(c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5236, 446, 88, 63, 205}

$$\frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} - \frac{bcd^2 x \tan^{-1}(\sqrt{c^2 x^2 - 1})}{4e\sqrt{c^2 x^2}} - \frac{bx\sqrt{c^2 x^2 - 1} (2c^2 d + e)}{4c^3\sqrt{c^2 x^2}} - \frac{bex (c^2 x^2 - 1)^{3/2}}{12c^3\sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]),x]

[Out] -(b\*(2\*c^2\*d + e)\*x\*sqrt[-1 + c^2\*x^2])/(4\*c^3\*sqrt[c^2\*x^2]) - (b\*e\*x\*(-1 + c^2\*x^2)^(3/2))/(12\*c^3\*sqrt[c^2\*x^2]) + ((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/(4\*e) - (b\*c\*d^2\*x\*ArcTan[sqrt[-1 + c^2\*x^2]])/(4\*e\*sqrt[c^2\*x^2])

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 88

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 5236

Int[((a\_) + ArcSec[(c\_)\*(x\_)])\*(b\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSec[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int x(d + ex^2)(a + b \sec^{-1}(cx)) dx &= \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} - \frac{(bcx) \int \frac{(d+ex^2)^2}{x\sqrt{-1+c^2x^2}} dx}{4e\sqrt{c^2x^2}} \\
 &= \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{(d+ex^2)^2}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{8e\sqrt{c^2x^2}} \\
 &= \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} - \frac{(bcx) \operatorname{Subst}\left(\int \left(\frac{e(2c^2d+e)}{c^2\sqrt{-1+c^2x}} + \frac{d^2}{x\sqrt{-1+c^2x}} + \frac{e^2\sqrt{-1+c^2x}}{c^2}\right) dx, x, x^2\right)}{8e\sqrt{c^2x^2}} \\
 &= -\frac{b(2c^2d + e)x\sqrt{-1 + c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1 + c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} \\
 &= -\frac{b(2c^2d + e)x\sqrt{-1 + c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1 + c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e} \\
 &= -\frac{b(2c^2d + e)x\sqrt{-1 + c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1 + c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{4e}
 \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 79, normalized size = 0.57

$$\frac{x \left( 3ac^3x(2d + ex^2) + 3bc^3x \sec^{-1}(cx)(2d + ex^2) - b\sqrt{1 - \frac{1}{c^2x^2}}(c^2(6d + ex^2) + 2e) \right)}{12c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]),x]

[Out] (x\*(3\*a\*c^3\*x\*(2\*d + e\*x^2) - b\*Sqrt[1 - 1/(c^2\*x^2)]\*(2\*e + c^2\*(6\*d + e\*x^2)) + 3\*b\*c^3\*x\*(2\*d + e\*x^2)\*ArcSec[c\*x]))/(12\*c^3)

**fricas [A]** time = 0.67, size = 86, normalized size = 0.62

$$\frac{3ac^4ex^4 + 6ac^4dx^2 + 3(bc^4ex^4 + 2bc^4dx^2) \operatorname{arcsec}(cx) - (bc^2ex^2 + 6bc^2d + 2be)\sqrt{c^2x^2 - 1}}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/12\*(3\*a\*c^4\*e\*x^4 + 6\*a\*c^4\*d\*x^2 + 3\*(b\*c^4\*e\*x^4 + 2\*b\*c^4\*d\*x^2)\*arcsec(c\*x) - (b\*c^2\*e\*x^2 + 6\*b\*c^2\*d + 2\*b\*e)\*sqrt(c^2\*x^2 - 1))/c^4

**giac [B]** time = 0.23, size = 3360, normalized size = 24.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] 1/12\*(6\*b\*c^2\*d\*arccos(1/(c\*x))/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) + 6\*a\*c^2\*d/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8) - 12\*b\*c^2\*d\*sqrt(-1/(c^2\*x^2) + 1)/((c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8)\*(1/(c\*x) + 1)) - 12\*b\*c^2\*d\*(1/(c^2\*x^2) - 1)^2\*arccos(1/(c\*x))/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8)\*(1/(c\*x) + 1)^4) + 3\*b\*arccos(1/(c\*x))\*e/(c^5 + 4\*c^5\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 6\*c^5\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 4\*c^5\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + c^5\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8)

$$\begin{aligned}
& 2) - 1)^4 / ((1/(c*x) + 1)^8) + 36*b*c^2*d*(-1/(c^2*x^2) + 1)^{(3/2)} / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^3 - 12*a*c^2*d*(1/(c^2*x^2) - 1)^2 / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^4 + 3*a*e/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^2 - 36*b*c^2*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^5 - 6*b*sqrt(-1/(c^2*x^2) + 1)*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)) + 6*b*c^2*d*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x)) / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^8 - 12*a*(1/(c^2*x^2) - 1)*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^2 + 18*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^4 - 12*b*c^2*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^7 + 10*b*(-1/(c^2*x^2) + 1)^{(3/2)}*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^3 + 6*a*c^2*d*(1/(c^2*x^2) - 1)^4 / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^8) + 18*a*(1/(c^2*x^2) - 1)^2*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^4 - 12*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^6 - 10*b*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)*e / ((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) * (1/(c*x) + 1)^2
\end{aligned}$$

$$\begin{aligned}
& + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^5 - \\
& 12*a*(1/(c^2*x^2) - 1)^3*e/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^6) + \\
& 3*b*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))*e/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^6) - \\
& 6*b*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)*e/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^6) + \\
& 3*a*(1/(c^2*x^2) - 1)^4*e/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^6) + \\
& 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^8)*c
\end{aligned}$$

**maple [A]** time = 0.05, size = 115, normalized size = 0.83

$$\frac{a\left(\frac{1}{4}c^4ex^4 + \frac{1}{2}c^4dx^2\right)}{c^2} + \frac{b\left(\frac{\operatorname{arcsec}(cx)e^{c^4x^4}}{4} + \frac{\operatorname{arcsec}(cx)c^4dx^2}{2} - \frac{(c^2x^2-1)(c^2ex^2+6c^2d+2e)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}cx}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x)

[Out] 1/c^2\*(a/c^2\*(1/4\*c^4\*e\*x^4+1/2\*c^4\*d\*x^2)+b/c^2\*(1/4\*arcsec(c\*x)\*e\*c^4\*x^4+1/2\*arcsec(c\*x)\*c^4\*d\*x^2-1/12\*(c^2\*x^2-1)\*(c^2\*e\*x^2+6\*c^2\*d+2\*e)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x))

**maxima [A]** time = 0.32, size = 100, normalized size = 0.72

$$\frac{1}{4}aex^4 + \frac{1}{2}adx^2 + \frac{1}{2}\left(x^2 \operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c}\right)bd + \frac{1}{12}\left(3x^4 \operatorname{arcsec}(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2} + 1}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/4\*a\*e\*x^4 + 1/2\*a\*d\*x^2 + 1/2\*(x^2\*arcsec(c\*x) - x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*b\*d + 1/12\*(3\*x^4\*arcsec(c\*x) - (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x (e x^2 + d) \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(d + e*x^2)*(a + b*acos(1/(c*x))),x)`

[Out] `int(x*(d + e*x^2)*(a + b*acos(1/(c*x))), x)`

**sympy** [A] time = 3.66, size = 177, normalized size = 1.28

$$\frac{adx^2}{2} + \frac{aex^4}{4} + \frac{bdx^2 \operatorname{asec}(cx)}{2} + \frac{bex^4 \operatorname{asec}(cx)}{4} - \frac{bd \begin{cases} \frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2| > 1 \\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}}{2c} - \frac{be \begin{cases} \frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2| > 1 \\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise} \end{cases}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d)*(a+b*asec(c*x)),x)`

[Out] `a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asec(c*x)/2 + b*e*x**4*asec(c*x)/4 - b*d*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) - b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)`

$$3.79 \quad \int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x} dx$$

Optimal. Leaf size=124

$$-d \log\left(\frac{1}{x}\right)(a+b \sec^{-1}(cx)) + \frac{1}{2}ex^2(a+b \sec^{-1}(cx)) - \frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c} - \frac{1}{2}ibd \operatorname{Li}_2\left(e^{2i \operatorname{csc}^{-1}(cx)}\right) - \frac{1}{2}ibd \operatorname{csc}^{-1}(cx)^2 + bd$$

[Out]  $-1/2*I*b*d*\operatorname{arccsc}(c*x)^2 + 1/2*e*x^2*(a+b*\operatorname{arcsec}(c*x)) + b*d*\operatorname{arccsc}(c*x)*\ln(1-(I/c/x+(1-1/c^2/x^2)^{(1/2)})^2) - b*d*\operatorname{arccsc}(c*x)*\ln(1/x) - d*(a+b*\operatorname{arcsec}(c*x))*\ln(1/x) - 1/2*I*b*d*\operatorname{polylog}(2, (I/c/x+(1-1/c^2/x^2)^{(1/2)})^2) - 1/2*b*e*x*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.28, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$ , Rules used = {5240, 14, 4732, 6742, 264, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}ibd \operatorname{PolyLog}(2, e^{2i \operatorname{csc}^{-1}(cx)}) - d \log\left(\frac{1}{x}\right)(a+b \sec^{-1}(cx)) + \frac{1}{2}ex^2(a+b \sec^{-1}(cx)) - \frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c} - \frac{1}{2}ibd \operatorname{csc}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] `Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x, x]`

[Out]  $-(b*e*\sqrt{1-1/(c^2*x^2)}*x)/(2*c) - (I/2)*b*d*\operatorname{ArcCsc}[c*x]^2 + (e*x^2*(a + b*\operatorname{ArcSec}[c*x])/2 + b*d*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcCsc}[c*x])}] - b*d*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[x^{-1}] - d*(a + b*\operatorname{ArcSec}[c*x])* \operatorname{Log}[x^{-1}] - (I/2)*b*d*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcCsc}[c*x])}])$

#### Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

#### Rule 264

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]`

#### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2326

```
Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)]/Sqrt[(d_) + (e_)*(x_)^2], x_Symb
ol] := Simp[(ArcSin[Rt[-e, 2]*x]/Sqrt[d]]*(a + b*Log[c*x^n])/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[Rt[-e, 2]*x]/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3717

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)
^m)*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

### Rule 4625

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)]^(n_)/(x_), x_Symbol] := Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

### Rule 4732

```
Int[((a_) + ArcCos[(c_)*(x_)])*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_
)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCos[c*x], u, x] + Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```



Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \sec^{-1}(cx))}{x} dx &= -\text{Subst} \left( \int \frac{(e + dx^2)(a + b \cos^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x} \right) \\
&= \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) - d (a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{b \text{Subst} \left( \int \frac{-\frac{e}{2x^2} + d \log(x)}{\sqrt{1 - \frac{x^2}{c^2}}} dx \right)}{c} \\
&= \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) - d (a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{b \text{Subst} \left( \int \left( -\frac{e}{2x^2 \sqrt{1 - \frac{x^2}{c^2}}} \right) dx \right)}{c} \\
&= \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) - d (a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{(bd) \text{Subst} \left( \int \frac{\log(x)}{\sqrt{1 - \frac{x^2}{c^2}}} dx \right)}{c} \\
&= -\frac{be\sqrt{1 - \frac{1}{c^2 x^2}} x}{2c} + \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) - bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d (a + b \sec^{-1}(cx)) \\
&= -\frac{be\sqrt{1 - \frac{1}{c^2 x^2}} x}{2c} + \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) - bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d (a + b \sec^{-1}(cx)) \\
&= -\frac{be\sqrt{1 - \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{2} ibd \csc^{-1}(cx)^2 + \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) - bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d (a + b \sec^{-1}(cx)) \\
&= -\frac{be\sqrt{1 - \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{2} ibd \csc^{-1}(cx)^2 + \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) + bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d (a + b \sec^{-1}(cx)) \\
&= -\frac{be\sqrt{1 - \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{2} ibd \csc^{-1}(cx)^2 + \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) + bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d (a + b \sec^{-1}(cx)) \\
&= -\frac{be\sqrt{1 - \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{2} ibd \csc^{-1}(cx)^2 + \frac{1}{2} ex^2 (a + b \sec^{-1}(cx)) + bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d (a + b \sec^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 115, normalized size = 0.93

$$ad \log(x) + \frac{1}{2} aex^2 - \frac{bex\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{2c} + \frac{1}{2} ibd \text{Li}_2(-e^{2i \sec^{-1}(cx)}) + \frac{1}{2} ibd \sec^{-1}(cx)^2 - bd \sec^{-1}(cx) \log(1 + e^{2i \sec^{-1}(cx)}) + \frac{1}{2} bex^2$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x,x]

[Out] (a\*e\*x^2)/2 - (b\*e\*x\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(2\*c) + (b\*e\*x^2\*ArcSec[c\*x])/2 + (I/2)\*b\*d\*ArcSec[c\*x]^2 - b\*d\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] + a\*d\*Log[x] + (I/2)\*b\*d\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]

**fricas** [F] time = 1.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2 + ad + (bex^2 + bd) \operatorname{arcsec}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))/x, x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]Undef/Undefined Inf encountered in limitLimit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple** [A] time = 2.62, size = 142, normalized size = 1.15

$$\frac{ax^2e}{2} + da \ln(cx) + \frac{ibd \operatorname{arcsec}(cx)^2}{2} + \frac{b \operatorname{arcsec}(cx) x^2 e}{2} - \frac{b \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x e}{2c} - \frac{ibe}{2c^2} - bd \operatorname{arcsec}(cx) \ln\left(1 + \left(\frac{1}{cx} + i \sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x,x)

[Out] 1/2\*a\*x^2\*e+d\*a\*ln(c\*x)+1/2\*I\*b\*d\*arcsec(c\*x)^2+1/2\*b\*arcsec(c\*x)\*x^2\*e-1/2\*b/c\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x\*e-1/2\*I\*b/c^2\*e-b\*d\*arcsec(c\*x)\*ln(1+(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)+1/2\*I\*b\*d\*polylog(2,-(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} aex^2 + ad \log(x) - \frac{-2i bc^2 ex^2 \log(c) - 2i bc^2 d \log(-cx + 1) \log(x) - 2i bc^2 d \log(x)^2 - 2i bc^2 d \operatorname{Li}_2(cx) - 2i bc^2 d \operatorname{Li}_2(-cx + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x,x, algorithm="maxima")

[Out] 1/2\*a\*e\*x^2 + a\*d\*log(x) - 1/4\*(-2\*I\*b\*c^2\*e\*x^2\*log(c) - 2\*I\*b\*c^2\*d\*log(-c\*x + 1)\*log(x) - 2\*I\*b\*c^2\*d\*log(x)^2 - 2\*I\*b\*c^2\*d\*dilog(c\*x) - 2\*I\*b\*c^2\*d\*dilog(-c\*x) + I\*(b\*e\*(log(c\*x + 1)/c^2 + log(c\*x - 1)/c^2) + 8\*b\*d\*integrate(1/2\*log(x)/(c^2\*x^3 - x), x))\*c^2 + 4\*c^2\*integrate(1/2\*(b\*e\*x^2 + 2\*b\*d\*log(x))\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/(c^2\*x^3 - x), x) - I\*b\*e\*log(c\*x - 1) - 2\*(b\*c^2\*e\*x^2 + 2\*b\*c^2\*d\*log(x))\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + (I\*b\*c^2\*e\*x^2 + 2\*I\*b\*c^2\*d\*log(x))\*log(c^2\*x^2) + (-2\*I\*b\*c^2\*d\*log(x) - I\*b\*e)\*log(c\*x + 1) + (-2\*I\*b\*c^2\*e\*x^2 - 4\*I\*b\*c^2\*d\*log(c))\*log(x))/c^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x,x)

[Out] int(((d + e\*x^2)\*(a + b\*acos(1/(c\*x))))/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*asec(c\*x))/x,x)

[Out] Integral((a + b\*asec(c\*x))\*(d + e\*x\*\*2)/x, x)

$$3.80 \quad \int \frac{(d+ex^2)(a+b \sec^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=137

$$-\frac{d(a+b \sec^{-1}(cx))}{2x^2} - e \log\left(\frac{1}{x}\right) (a+b \sec^{-1}(cx)) + \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{1}{2}ibe \operatorname{Li}_2\left(e^{2i \csc^{-1}(cx)}\right) - \frac{1}{2}ibe \operatorname{cs}$$

[Out]  $-1/4*b*c^2*d*\operatorname{arccsc}(c*x) - 1/2*I*b*e*\operatorname{arccsc}(c*x)^2 - 1/2*d*(a+b*\operatorname{arcsec}(c*x))/x^2 + b*e*\operatorname{arccsc}(c*x)*\ln(1-(I/c/x+(1-1/c^2/x^2)^{(1/2)})^2) - b*e*\operatorname{arccsc}(c*x)*\ln(1/x) - e*(a+b*\operatorname{arcsec}(c*x))*\ln(1/x) - 1/2*I*b*e*\operatorname{polylog}(2, (I/c/x+(1-1/c^2/x^2)^{(1/2)})^2) + 1/4*b*c*d*(1-1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.29, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$ , Rules used = {5240, 14, 4732, 12, 6742, 321, 216, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}ibe \operatorname{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) - \frac{d(a+b \sec^{-1}(cx))}{2x^2} - e \log\left(\frac{1}{x}\right) (a+b \sec^{-1}(cx)) + \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x^2)*(a + b*\operatorname{ArcSec}[c*x])/x^3, x]$

[Out]  $(b*c*d*\operatorname{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) - (b*c^2*d*\operatorname{ArcCsc}[c*x])/4 - (I/2)*b*e*\operatorname{ArcCsc}[c*x]^2 - (d*(a + b*\operatorname{ArcSec}[c*x]))/(2*x^2) + b*e*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcCsc}[c*x])}] - b*e*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[x^{(-1)}] - e*(a + b*\operatorname{ArcSec}[c*x])* \operatorname{Log}[x^{(-1)}] - (I/2)*b*e*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcCsc}[c*x])}]$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /;$   $\operatorname{FreeQ}[b, x]$

### Rule 14

$\operatorname{Int}[(u_*)((c_*)(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /;$   $\operatorname{FreeQ}[\{c, m\}, x] \ \&\& \ \operatorname{SumQ}[u] \ \&\& \ !\operatorname{LinearQ}[u, x] \ \&\& \ !\operatorname{MatchQ}[u, (a_*) + (b_*)(v_)] /;$   $\operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{InverseFunctionQ}[v]$

### Rule 216

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_*) + (b_*)(x_)^2], x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSin}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[-b, 2], x] /;$   $\operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{NegQ}[b]$

Rule 321

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))*((c_.) + (d_.)*(x_)^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] := Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n])/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)
^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] := Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCos[c*x], u, x] + Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \sec^{-1}(cx))}{x^3} dx &= -\text{Subst} \left( \int \frac{(e + dx^2)(a + b \cos^{-1}(\frac{x}{c}))}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{d(a + b \sec^{-1}(cx))}{2x^2} - e(a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{b \text{Subst} \left( \int \frac{dx^2 + 2e \log(x)}{2\sqrt{1-\frac{x^2}{c^2}}} dx \right)}{c} \\
&= -\frac{d(a + b \sec^{-1}(cx))}{2x^2} - e(a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{b \text{Subst} \left( \int \frac{dx^2 + 2e \log(x)}{\sqrt{1-\frac{x^2}{c^2}}} dx \right)}{2c} \\
&= -\frac{d(a + b \sec^{-1}(cx))}{2x^2} - e(a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{b \text{Subst} \left( \int \left( \frac{dx^2}{\sqrt{1-\frac{x^2}{c^2}}} + \frac{2e}{\sqrt{1-\frac{x^2}{c^2}}} \right) dx \right)}{2c} \\
&= -\frac{d(a + b \sec^{-1}(cx))}{2x^2} - e(a + b \sec^{-1}(cx)) \log\left(\frac{1}{x}\right) - \frac{(bd) \text{Subst} \left( \int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}} dx \right)}{2c} \\
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{d(a + b \sec^{-1}(cx))}{2x^2} - be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - e(a + b \sec^{-1}(cx)) \\
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{d(a + b \sec^{-1}(cx))}{2x^2} - be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) \\
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \sec^{-1}(cx))}{2x^2} - be \\
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \sec^{-1}(cx))}{2x^2} + be \\
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \sec^{-1}(cx))}{2x^2} + be \\
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \sec^{-1}(cx))}{2x^2} + be
\end{aligned}$$



**Mathematica [A]** time = 0.10, size = 132, normalized size = 0.96

$$-\frac{ad}{2x^2} + ae \log(x) + \frac{bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d \sin^{-1}\left(\frac{1}{cx}\right) - \frac{bd \sec^{-1}(cx)}{2x^2} + \frac{1}{2}ibe\text{Li}_2\left(-e^{2i \sec^{-1}(cx)}\right) + \frac{1}{2}ibe \sec^{-1}(cx)^2 - be \sec^{-1}(cx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out]  $-\frac{1}{2}*(a*d)/x^2 + (b*c*d*\text{Sqrt}[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*\text{ArcSec}[c*x])/(2*x^2) + (I/2)*b*e*\text{ArcSec}[c*x]^2 - (b*c^2*d*\text{ArcSin}[1/(c*x)])/4 - b*e*\text{ArcSec}[c*x]*\text{Log}[1 + E^((2*I)*\text{ArcSec}[c*x])] + a*e*\text{Log}[x] + (I/2)*b*e*\text{PolyLog}[2, -E^((2*I)*\text{ArcSec}[c*x])]$

**fricas [F]** time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^3,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))/x^3, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \text{arcsec}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^3, x)

**maple [A]** time = 0.85, size = 145, normalized size = 1.06

$$ae \ln(cx) - \frac{da}{2x^2} + \frac{ib \text{arcsec}(cx)^2 e}{2} + \frac{cbd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x} + \frac{c^2bd \text{arcsec}(cx)}{4} - \frac{b \text{arcsec}(cx) d}{2x^2} - be \text{arcsec}(cx) \ln\left(1 + \left(\frac{1}{cx} + i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arcsec(c\*x))/x^3,x)

[Out]  $a*e*\ln(cx) - 1/2*d*a/x^2 + 1/2*I*b*\operatorname{arcsec}(cx)^2*e + 1/4*c*b*d/x*((c^2*x^2-1)/c^2/x^2)^{(1/2)} + 1/4*c^2*b*d*\operatorname{arcsec}(cx) - 1/2*b*\operatorname{arcsec}(cx)*d/x^2 - b*e*\operatorname{arcsec}(cx)*\ln(1+(1/c/x+I*(1-1/c^2/x^2)^{(1/2}))^2) + 1/2*I*b*e*\operatorname{polylog}(2, -(1/c/x+I*(1-1/c^2/x^2)^{(1/2}))^2)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(c^2 \int \frac{\sqrt{cx+1} \sqrt{cx-1} \log(x)}{c^4 x^3 - c^2 x} dx - \arctan\left(\sqrt{cx+1} \sqrt{cx-1}\right) \log(x)\right) b e - \frac{1}{4} b d \left( \frac{c^4 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^2 x^2 \left(\frac{1}{c^2 x^2} - 1\right) - 1} - c^3 \arctan\left(cx \sqrt{-\frac{1}{c^2 x^2} + 1}\right) \right) / c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsec(cx))/x^3,x, algorithm="maxima")`

[Out]  $-(c^2*\operatorname{integrate}(\sqrt{cx+1}*\sqrt{cx-1}*\log(x)/(c^4*x^3 - c^2*x), x) - \arctan(\sqrt{cx+1}*\sqrt{cx-1})*\log(x))*b*e - 1/4*b*d*((c^4*x*\sqrt{-1/(c^2*x^2) + 1})/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*\arctan(cx*\sqrt{-1/(c^2*x^2) + 1}))/c + 2*\operatorname{arcsec}(cx)/x^2) + a*e*\log(x) - 1/2*a*d/x^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3,x)`

[Out] `int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(a+b*asec(cx))/x**3,x)`

[Out] `Integral((a + b*asec(cx))*(d + e*x**2)/x**3, x)`

### 3.81 $\int x^2 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=252

$$\frac{1}{3}d^2x^3(a + b \sec^{-1}(cx)) + \frac{2}{5}dex^5(a + b \sec^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \sec^{-1}(cx)) - \frac{be^2x^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}} - \frac{bex^4\sqrt{c^2x^2-1}}{840c^3\sqrt{c^2x^2}}$$

[Out]  $\frac{1}{3}d^2x^3(a+b*\text{arcsec}(c*x))+\frac{2}{5}d*e*x^5*(a+b*\text{arcsec}(c*x))+\frac{1}{7}e^2*x^7*(a+b*\text{arcsec}(c*x))-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*\text{arctanh}(c*x/(c^2*x^2-1)^{(1/2)})/c^6/(c^2*x^2)^{(1/2)}-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x^2*(c^2*x^2-1)^{(1/2)}/c^5/(c^2*x^2)^{(1/2)}-1/840*b*e*(84*c^2*d+25*e)*x^4*(c^2*x^2-1)^{(1/2)}/c^3/(c^2*x^2)^{(1/2)}-1/42*b*e^2*x^6*(c^2*x^2-1)^{(1/2)}/c/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.24, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {270, 5238, 12, 1267, 459, 321, 217, 206}

$$\frac{1}{3}d^2x^3(a + b \sec^{-1}(cx)) + \frac{2}{5}dex^5(a + b \sec^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \sec^{-1}(cx)) - \frac{bx^2\sqrt{c^2x^2-1}(280c^4d^2 + 252c^2de + 75e^2)}{1680c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(d + e*x^2)^2*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-(b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x^2*\text{Sqrt}[-1 + c^2*x^2])/(1680*c^5*\text{Sqrt}[c^2*x^2]) - (b*e*(84*c^2*d + 25*e)*x^4*\text{Sqrt}[-1 + c^2*x^2])/(840*c^3*\text{Sqrt}[c^2*x^2]) - (b*e^2*x^6*\text{Sqrt}[-1 + c^2*x^2])/(42*c*\text{Sqrt}[c^2*x^2]) + (d^2*x^3*(a + b*\text{ArcSec}[c*x]))/3 + (2*d*e*x^5*(a + b*\text{ArcSec}[c*x]))/5 + (e^2*x^7*(a + b*\text{ArcSec}[c*x]))/7 - (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/(1680*c^6*\text{Sqrt}[c^2*x^2])$

#### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

#### Rule 206

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 270

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

Rule 1267

Int[((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Simp[(c^p\*(f\*x)^(m + 4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*f^(4\*p - 1)\*(m + 4\*p + 2\*q + 1)), x] + Dist[1/(e\*(m + 4\*p + 2\*q + 1)), Int[(f\*x)^m\*(d + e\*x^2)^q\*ExpandToSum[e\*(m + 4\*p + 2\*q + 1)\*((a + b\*x^2 + c\*x^4)^p - c^p\*x^(4\*p)) - d\*c^p\*(m + 4\*p - 1)\*x^(4\*p - 2), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4\*p + 2\*q + 1, 0]

Rule 5238

Int[((a\_) + ArcSec[(c\_)\*(x\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2

\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
 \int x^2 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx &= \frac{1}{3}d^2x^3 (a + b \sec^{-1}(cx)) + \frac{2}{5}dex^5 (a + b \sec^{-1}(cx)) + \frac{1}{7}e^2x^7 (a + b \sec^{-1}(cx)) \\
 &= \frac{1}{3}d^2x^3 (a + b \sec^{-1}(cx)) + \frac{2}{5}dex^5 (a + b \sec^{-1}(cx)) + \frac{1}{7}e^2x^7 (a + b \sec^{-1}(cx)) \\
 &= -\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3 (a + b \sec^{-1}(cx)) + \frac{2}{5}dex^5 (a + b \sec^{-1}(cx)) \\
 &= -\frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3 (a + b \sec^{-1}(cx)) \\
 &= -\frac{b(280c^4d^2+252c^2de+75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} - \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} \\
 &= -\frac{b(280c^4d^2+252c^2de+75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} - \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} \\
 &= -\frac{b(280c^4d^2+252c^2de+75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} - \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.37, size = 186, normalized size = 0.74

$$\frac{c^2x^2 \left( 16ac^5x(35d^2 + 42dex^2 + 15e^2x^4) - b\sqrt{1 - \frac{1}{c^2x^2}} \left( 8c^4(35d^2 + 21dex^2 + 5e^2x^4) + 2c^2e(126d + 25ex^2) + 75e^2 \right) \right)}{1680c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out] (c^2\*x^2\*(16\*a\*c^5\*x\*(35\*d^2 + 42\*d\*e\*x^2 + 15\*e^2\*x^4) - b\*Sqrt[1 - 1/(c^2\*x^2)]\*(75\*e^2 + 2\*c^2\*e\*(126\*d + 25\*e\*x^2) + 8\*c^4\*(35\*d^2 + 21\*d\*e\*x^2 + 5\*e^2\*x^4))) + 16\*b\*c^7\*x^3\*(35\*d^2 + 42\*d\*e\*x^2 + 15\*e^2\*x^4)\*ArcSec[c\*x] - b\*(280\*c^4\*d^2 + 252\*c^2\*d\*e + 75\*e^2)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x]/(1680\*c^7)

**fricas** [A] time = 4.04, size = 273, normalized size = 1.08

$$240 ac^7 e^2 x^7 + 672 ac^7 dex^5 + 560 ac^7 d^2 x^3 + 16 (15 bc^7 e^2 x^7 + 42 bc^7 dex^5 + 35 bc^7 d^2 x^3 - 35 bc^7 d^2 - 42 bc^7 de - 15 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] 1/1680\*(240\*a\*c^7\*e^2\*x^7 + 672\*a\*c^7\*d\*e\*x^5 + 560\*a\*c^7\*d^2\*x^3 + 16\*(15\*b\*c^7\*e^2\*x^7 + 42\*b\*c^7\*d\*e\*x^5 + 35\*b\*c^7\*d^2\*x^3 - 35\*b\*c^7\*d^2 - 42\*b\*c^7\*d\*e - 15\*b\*c^7\*e^2)\*arcsec(c\*x) + 32\*(35\*b\*c^7\*d^2 + 42\*b\*c^7\*d\*e + 15\*b\*c^7\*e^2)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + (280\*b\*c^4\*d^2 + 252\*b\*c^2\*d\*e + 75\*b\*e^2)\*log(-c\*x + sqrt(c^2\*x^2 - 1)) - (40\*b\*c^5\*e^2\*x^5 + 2\*(84\*b\*c^5\*d\*e + 25\*b\*c^3\*e^2)\*x^3 + (280\*b\*c^5\*d^2 + 252\*b\*c^3\*d\*e + 75\*b\*c\*e^2)\*x)\*sqrt(c^2\*x^2 - 1)/c^7

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.05, size = 494, normalized size = 1.96

$$\frac{a e^2 x^7}{7} + \frac{2 a e d x^5}{5} + \frac{a x^3 d^2}{3} + \frac{b \operatorname{arcsec}(c x) e^2 x^7}{7} + \frac{2 b \operatorname{arcsec}(c x) e d x^5}{5} + \frac{b \operatorname{arcsec}(c x) x^3 d^2}{3} - \frac{b x^6 e^2}{42 c \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b x^4 e^2}{168 c^3 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x)

[Out] 1/7\*a\*e^2\*x^7+2/5\*a\*e\*d\*x^5+1/3\*a\*x^3\*d^2+1/7\*b\*arcsec(c\*x)\*e^2\*x^7+2/5\*b\*a\*arcsec(c\*x)\*e\*d\*x^5+1/3\*b\*arcsec(c\*x)\*x^3\*d^2-1/42/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^6\*e^2-1/168/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4\*e^2-1/10/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4\*e\*d-1/20/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2\*e\*d-1/6/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d^2\*x^2+1/6/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d^2-1/6/c^4\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*d^2\*ln(c\*x+(c^2\*x^2-1)^(1/2))-5/336/c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2\*e^2+3/20/c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*e\*d-3/20/c^6\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*e\*d\*ln(c\*x+(c^2\*x^2-1)^(1/2))+5/112/c^7\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)

$2*x^2-1)/c^2/x^2)^{(1/2)}*e^{-5/112/c^8*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*e^2*\ln(c*x+(c^2*x^2-1)^{(1/2)})$

**maxima** [A] time = 0.34, size = 405, normalized size = 1.61

$$\frac{1}{7}ae^2x^7 + \frac{2}{5}adex^5 + \frac{1}{3}ad^2x^3 + \frac{1}{12} \left( 4x^3 \operatorname{arcsec}(cx) - \frac{2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c} \right) bd^2 + \frac{1}{40} \left( 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{7}a*e^2*x^7 + \frac{2}{5}a*d*e*x^5 + \frac{1}{3}a*d^2*x^3 + \frac{1}{12}*(4*x^3*\operatorname{arcsec}(c*x) - (2*\sqrt{-1/(c^2*x^2)+1})/(c^2*(1/(c^2*x^2)-1)+c^2) + \log(\sqrt{-1/(c^2*x^2)+1})+1)/c^2 - \log(\sqrt{-1/(c^2*x^2)+1}-1)/c^2)/c)*b*d^2 + \frac{1}{40}*(16*x^5*\operatorname{arcsec}(c*x) + (2*(3*(-1/(c^2*x^2)+1)^{(3/2)} - 5*\sqrt{-1/(c^2*x^2)+1}))/c^4*(1/(c^2*x^2)-1)^2 + 2*c^4*(1/(c^2*x^2)-1)+c^4) - 3*\log(\sqrt{-1/(c^2*x^2)+1})+1)/c^4 + 3*\log(\sqrt{-1/(c^2*x^2)+1}-1)/c^4)/c)*b*d*e + \frac{1}{672}*(96*x^7*\operatorname{arcsec}(c*x) - (2*(15*(-1/(c^2*x^2)+1)^{(5/2)} - 40*(-1/(c^2*x^2)+1)^{(3/2)} + 33*\sqrt{-1/(c^2*x^2)+1}))/c^6*(1/(c^2*x^2)-1)^3 + 3*c^6*(1/(c^2*x^2)-1)^2 + 3*c^6*(1/(c^2*x^2)-1)+c^6) + 15*\log(\sqrt{-1/(c^2*x^2)+1})+1)/c^6 - 15*\log(\sqrt{-1/(c^2*x^2)+1}-1)/c^6)/c)*b*e^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (ex^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^2\*(d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 13.68, size = 542, normalized size = 2.15

$$\frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} + \frac{bd^2x^3 \operatorname{asec}(cx)}{3} + \frac{2bdex^5 \operatorname{asec}(cx)}{5} + \frac{be^2x^7 \operatorname{asec}(cx)}{7} - \frac{bd^2 \left\{ \begin{array}{l} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - i \end{array} \right.}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x)),x)

[Out]  $a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*asec(c*x)/3 + 2*b*d*e*x**5*asec(c*x)/5 + b*e**2*x**7*asec(c*x)/7 - b*d**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c) - 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c) - b*e**2*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)$



### 3.82 $\int (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=191

$$d^2x(a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3(a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b \sec^{-1}(cx)) - \frac{be^2x^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}} - \frac{bx(120c^4d^2 + 40c^2de + 9e^2)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}$$

[Out]  $d^2x*(a+b*\text{arcsec}(c*x))+2/3*d*e*x^3*(a+b*\text{arcsec}(c*x))+1/5*e^2*x^5*(a+b*\text{arcsec}(c*x))-1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*\text{arctanh}(c*x/(c^2*x^2-1)^{(1/2)})/c^4/(c^2*x^2)^{(1/2)}-1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^{(1/2)}/c^3/(c^2*x^2)^{(1/2)}-1/20*b*e^2*x^4*(c^2*x^2-1)^{(1/2)}/c/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {194, 5228, 12, 1159, 388, 217, 206}

$$d^2x(a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3(a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b \sec^{-1}(cx)) - \frac{bx(120c^4d^2 + 40c^2de + 9e^2)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out]  $-(b*e*(40*c^2*d + 9*e)*x^2*\text{Sqrt}[-1 + c^2*x^2])/(120*c^3*\text{Sqrt}[c^2*x^2]) - (b*e^2*x^4*\text{Sqrt}[-1 + c^2*x^2])/(20*c*\text{Sqrt}[c^2*x^2]) + d^2*x*(a + b*\text{ArcSec}[c*x]) + (2*d*e*x^3*(a + b*\text{ArcSec}[c*x]))/3 + (e^2*x^5*(a + b*\text{ArcSec}[c*x]))/5 - (b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/(120*c^4*\text{Sqrt}[c^2*x^2])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel \text{LtQ}[b, 0]$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

### Rule 388

$\text{Int}[(a_) + (b_.)*(x_)^{(n_)}]^{(p_)}*((c_) + (d_.)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n*(p+1) + 1, 0]$

### Rule 1159

$\text{Int}[(d_) + (e_.)*(x_)^2]^{(q_)}*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(c^p*x^{(4*p-1)}*(d + e*x^2)^{(q+1)})/(e*(4*p + 2*q + 1)), x] + \text{Dist}[1/(e*(4*p + 2*q + 1)), \text{Int}[(d + e*x^2)^q*\text{ExpandToSum}[e*(4*p + 2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^{(4*p-2)} - e*c^p*(4*p + 2*q + 1)*x^{(4*p)}, x], x], x] \text{ /; FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{!LtQ}[q, -1]$

### Rule 5228

$\text{Int}[(a_.) + \text{ArcSec}[(c_.)*(x_)]*(b_.)]*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcSec}[c*x], u, x] - \text{Dist}[(b*c*x)/\text{Sqrt}[c^2*x^2], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x], x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \&\& (\text{IGtQ}[p, 0] \parallel \text{ILtQ}[p + 1/2, 0])$

### Rubi steps

$$\begin{aligned}
\int (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx &= d^2x (a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \sec^{-1}(cx)) - \\
&= d^2x (a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \sec^{-1}(cx)) - \\
&= -\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \sec^{-1}(cx)) - \\
&= -\frac{be(40c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \sec^{-1}(cx)) - \\
&= -\frac{be(40c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \sec^{-1}(cx)) - \\
&= -\frac{be(40c^2d+9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \sec^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \sec^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \sec^{-1}(cx)) -
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 153, normalized size = 0.80

$$\frac{c^2x \left( 8ac^3 (15d^2 + 10dex^2 + 3e^2x^4) - bex\sqrt{1 - \frac{1}{c^2x^2}} (c^2 (40d + 6ex^2) + 9e) \right) + 8bc^5x \sec^{-1}(cx) (15d^2 + 10dex^2 + 3e^2x^4)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out] (c^2\*x\*(8\*a\*c^3\*(15\*d^2 + 10\*d\*e\*x^2 + 3\*e^2\*x^4) - b\*e\*Sqrt[1 - 1/(c^2\*x^2)])\*x\*(9\*e + c^2\*(40\*d + 6\*e\*x^2))) + 8\*b\*c^5\*x\*(15\*d^2 + 10\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcSec[c\*x] - b\*(120\*c^4\*d^2 + 40\*c^2\*d\*e + 9\*e^2)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x]/(120\*c^5)

**fricas [A]** time = 2.28, size = 237, normalized size = 1.24

$$\frac{24ac^5e^2x^5 + 80ac^5dex^3 + 120ac^5d^2x + 8(3bc^5e^2x^5 + 10bc^5dex^3 + 15bc^5d^2x - 15bc^5d^2 - 10bc^5de - 3bc^5e^2)}{120c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out]  $\frac{1}{120}*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*\text{arcsec}(c*x) + 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*\text{arctan}(-c*x + \sqrt{c^2*x^2 - 1}) + (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*\log(-c*x + \sqrt{c^2*x^2 - 1}) - (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b*c*e^2)*x)*\sqrt{c^2*x^2 - 1})/c^5$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")`

[Out] Timed out

**maple** [B] time = 0.05, size = 372, normalized size = 1.95

$$\frac{a e^2 x^5}{5} + \frac{2 a x^3 d e}{3} + a x d^2 + \frac{b \operatorname{arcsec}(c x) e^2 x^5}{5} + \frac{2 b \operatorname{arcsec}(c x) x^3 d e}{3} + b \operatorname{arcsec}(c x) x d^2 - \frac{b \sqrt{c^2 x^2 - 1} d^2 \ln\left(c x + \sqrt{c^2 x^2 - 1}\right)}{c^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsec(c*x)),x)`

[Out]  $\frac{1}{5} a e^2 x^5 + \frac{2}{3} a x^3 d e + a x d^2 + \frac{1}{5} b \operatorname{arcsec}(c x) e^2 x^5 + \frac{2}{3} b \operatorname{arcsec}(c x) x^3 d e + b \operatorname{arcsec}(c x) x d^2 - \frac{1}{c^2} b \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} \ln\left(c x + \sqrt{c^2 x^2 - 1}\right) - \frac{1}{20} \frac{b}{c} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} x^4 e^2 - \frac{1}{40} \frac{b}{c^3} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} x^2 e^2 - \frac{1}{3} \frac{b}{c} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} e d x^2 + \frac{1}{3} \frac{b}{c^3} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} e d - \frac{1}{3} \frac{b}{c^4} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} e^2 + \frac{3}{40} \frac{b}{c^5} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} e^2 - \frac{3}{40} \frac{b}{c^6} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} e^2 + \frac{3}{40} \frac{b}{c^5} \frac{(c^2 x^2 - 1)^{1/2}}{(c^2 x^2 - 1)^{1/2} / x^2} e^2 \ln\left(c x + \sqrt{c^2 x^2 - 1}\right)$

**maxima** [A] time = 0.34, size = 296, normalized size = 1.55

$$\frac{1}{5} a e^2 x^5 + \frac{2}{3} a d e x^3 + \frac{1}{6} \left( 4 x^3 \operatorname{arcsec}(c x) - \frac{2 \sqrt{-\frac{1}{c^2 x^2} + 1} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right) - \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2} \right) b d e + \frac{1}{80} \left( 16 x^5 \operatorname{arcsec}(c x) - \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right) - \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{5}a^2e^{2x^5} + \frac{2}{3}a^2d^2e^{2x^3} + \frac{1}{6}(4x^3\text{arcsec}(cx) - (2\sqrt{-1/(c^2x^2)} + 1)/c^2 + 1)/(c^2(1/(c^2x^2) - 1) + c^2) + \log(\sqrt{-1/(c^2x^2)} + 1)/c^2 - \log(\sqrt{-1/(c^2x^2)} + 1) - 1)/c^2/c * b^2d^2e^{2x} + 1/80(16x^5\text{arcsec}(cx) + (2(3(-1/(c^2x^2) + 1)^{3/2} - 5\sqrt{-1/(c^2x^2)} + 1)))/(c^4(1/(c^2x^2) - 1)^2 + 2c^4(1/(c^2x^2) - 1) + c^4) - 3\log(\sqrt{-1/(c^2x^2)} + 1) + 1)/c^4 + 3\log(\sqrt{-1/(c^2x^2)} + 1) - 1)/c^4/c * b^2e^{2x} + a^2d^2x + 1/2 * (2cx\text{arcsec}(cx) - \log(\sqrt{-1/(c^2x^2)} + 1) + 1) + \log(-\sqrt{-1/(c^2x^2)} + 1) + 1) * b^2d^2/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (ex^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))),x)

[Out] int((d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 9.79, size = 355, normalized size = 1.86

$$ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x \operatorname{asec}(cx) + \frac{2bdex^3 \operatorname{asec}(cx)}{3} + \frac{be^2x^5 \operatorname{asec}(cx)}{5} - \frac{bd^2 \begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x)),x)

[Out]  $a^2d^2x + 2a^2d^2e^{2x^3}/3 + a^2e^{2x^5}/5 + b^2d^2x \operatorname{asec}(cx) + 2b^2d^2e^{2x^3} \operatorname{asec}(cx)/3 + b^2e^{2x^5} \operatorname{asec}(cx)/5 - b^2d^2 \operatorname{Piecewise}(\operatorname{acosh}(cx), \operatorname{Abs}(c^2x^2) > 1), (-I \operatorname{asin}(cx), \operatorname{True}))/c - 2b^2d^2e^{2x^3} \operatorname{Piecewise}((x\sqrt{c^2x^2 - 1})/(2c) + \operatorname{acosh}(cx)/(2c^2), \operatorname{Abs}(c^2x^2) > 1), (-I c x^3/(2\sqrt{-c^2x^2 + 1}) + I x/(2c\sqrt{-c^2x^2 + 1}) - I \operatorname{asin}(cx)/(2c^2), \operatorname{True}))/3c - b^2e^{2x^5} \operatorname{Piecewise}(c^5/(4\sqrt{c^2x^2 - 1}) + x^3/(8c\sqrt{c^2x^2 - 1}) - 3x/(8c^3\sqrt{c^2x^2 - 1}) + 3\operatorname{acosh}(cx)/(8c^4), \operatorname{Abs}(c^2x^2) > 1), (-I c x^5/(4\sqrt{-c^2x^2 + 1}) - I x^3/(8c\sqrt{-c^2x^2 + 1}) + 3I x/(8c^3\sqrt{-c^2x^2 + 1}) - 3I \operatorname{asin}(cx)/(8c^4), \operatorname{True}))/5c$

$$3.83 \quad \int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=162

$$-\frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \sec^{-1}(cx)) + \frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} - \frac{bex (12c^2d + e) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}}$$

[Out]  $-d^2*(a+b*\text{arcsec}(c*x))/x+2*d*e*x*(a+b*\text{arcsec}(c*x))+1/3*e^2*x^3*(a+b*\text{arcsec}(c*x))-1/6*b*e*(12*c^2*d+e)*x*\text{arctanh}(c*x/(c^2*x^2-1)^{(1/2)})/c^2/(c^2*x^2)^{(1/2)}+b*c*d^2*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}-1/6*b*e^2*x^2*(c^2*x^2-1)^{(1/2)}/c/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 5238, 12, 1265, 388, 217, 206}

$$-\frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \sec^{-1}(cx)) + \frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} - \frac{bex (12c^2d + e) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcSec}[c*x])/x^2, x]$

[Out]  $(b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/ \text{Sqrt}[c^2*x^2] - (b*e^2*x^2*\text{Sqrt}[-1 + c^2*x^2]) / (6*c*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcSec}[c*x])/x + 2*d*e*x*(a + b*\text{ArcSec}[c*x]) + (e^2*x^3*(a + b*\text{ArcSec}[c*x]))/3 - (b*e*(12*c^2*d + e)*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]]) / (6*c^2*\text{Sqrt}[c^2*x^2])$

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 206

$\text{Int}[(a_)+(b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_)+(b_.)*(x_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rule 1265

Int[((f\_.)\*(x\_))^(m\_)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, f\*x, x], R = PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, f\*x, x]}, Simp[(R\*(f\*x)^(m + 1)\*(d + e\*x^2)^(q + 1))/(d\*f\*(m + 1)), x] + Dist[1/(d\*f^2\*(m + 1)), Int[(f\*x)^(m + 2)\*(d + e\*x^2)^q\*ExpandToSum[(d\*f\*(m + 1)\*Qx)/x - e\*R\*(m + 2\*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && LtQ[m, -1]

Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{x^2} dx &= -\frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \sec^{-1}(cx)) - \dots \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \sec^{-1}(cx)) - \dots \\
&= \frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \sec^{-1}(cx)) - \dots \\
&= \frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1 + c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) - \dots \\
&= \frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1 + c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) - \dots \\
&= \frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1 + c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{x} + 2dex (a + b \sec^{-1}(cx)) - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 136, normalized size = 0.84

$$\frac{c^2 \left( 2ac(-3d^2 + 6dex^2 + e^2x^4) + bx\sqrt{1 - \frac{1}{c^2x^2}} (6c^2d^2 - e^2x^2) \right) + 2bc^3 \sec^{-1}(cx) (-3d^2 + 6dex^2 + e^2x^4) - bex(12c^2d^2 + e^2x^4)}{6c^3x}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^2,x]

[Out] (c^2\*(b\*sqrt[1 - 1/(c^2\*x^2)]\*x\*(6\*c^2\*d^2 - e^2\*x^2) + 2\*a\*c\*(-3\*d^2 + 6\*d\*e\*x^2 + e^2\*x^4)) + 2\*b\*c^3\*(-3\*d^2 + 6\*d\*e\*x^2 + e^2\*x^4)\*ArcSec[c\*x] - b\*e\*(12\*c^2\*d + e)\*x\*Log[(1 + sqrt[1 - 1/(c^2\*x^2)])\*x])/(6\*c^3\*x)

**fricas [A]** time = 1.11, size = 230, normalized size = 1.42

$$\frac{2ac^3e^2x^4 + 6bc^4d^2x + 12ac^3dex^2 - 6ac^3d^2 - 4(3bc^3d^2 - 6bc^3de - bc^3e^2)x \arctan(-cx + \sqrt{c^2x^2 - 1}) + (12bc^2d^2 + e^2x^4)}{6c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^2,x, algorithm="fricas")



```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 4*(
3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
(12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x^4
+ 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x
)*arcsec(c*x) + (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

**giac [B]** time = 2.95, size = 6010, normalized size = 37.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(6*b*c^4*d^2*arccos(1/(c*x)))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) + 6*a*c^4*d^2/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8) + 24*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 + 2*
c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2 - 12*b*c
^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)) + 24*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^4 + 2*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 36*b*c^4
*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*b*c^2*d*arccos(1/(c*x))*e/(
c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 12*b*c^2*d*e*lo
g(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1)
- 1/(c*x) - 1))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)
+ 36*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 36*a*c^4*d^2*(1/(c^2*x^2) - 1)^
2/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1
)^4) + 24*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 2*c^4*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c
^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 12*a*c^2*d*e/(c^
4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*b*c^2*d*(1/(c^
```

$$\begin{aligned}
& 2*x^2) - 1)*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 2*c^4* \\
& (1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1) \\
& ^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 24*b*c^2*d \\
& *(1/(c^2*x^2) - 1)*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + \\
& 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c* \\
& x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 36* \\
& b*c^4*d^2*(1/(c^2*x^2) - 1)^2*\text{sqrt}(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2* \\
& x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4 \\
& *(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 24*a*c^4*d^2*(1/(c \\
& ^2*x^2) - 1)^3/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/( \\
& c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\
& (1/(c*x) + 1)^6) + 6*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))/((c^4 + \\
& 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x \\
& ) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 24*b \\
& *c^2*d*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))*e/((c^4 + 2*c^4*(1/(c^2*x^2) - 1) \\
& )/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2 \\
& *x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*b*c^4*d^2*(1/(c^2*x^2) \\
& - 1)^3*\text{sqrt}(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1) \\
& ^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1 \\
& /(c*x) + 1)^8)*(1/(c*x) + 1)^7) + 6*a*c^4*d^2*(1/(c^2*x^2) - 1)^4/((c^4 + 2 \\
& *c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) \\
& + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 2*b*a \\
& \text{rccos}(1/(c*x))*e^2/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*( \\
& 1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^ \\
& 8) + 24*a*c^2*d*(1/(c^2*x^2) - 1)^2*e/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c \\
& *x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - \\
& 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + b*e^2*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + \\
& 1) + 1/(c*x) + 1))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4* \\
& (1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& ^8) - 24*b*c^2*d*(1/(c^2*x^2) - 1)^3*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) + 1/( \\
& c*x) + 1))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2* \\
& x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/( \\
& c*x) + 1)^6) - b*e^2*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + \\
& 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x \\
& ) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*b*c^2*d*(1/(c^2*x^ \\
& 2) - 1)^3*e*\log(\text{abs}(\text{sqrt}(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 2*c^4*(1 \\
& /c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 \\
& - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 2*a*e^2/(c^4 \\
& + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/( \\
& c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 8*b*(1/(c^2*x^2) - \\
& 1)*\arccos(1/(c*x))*e^2/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2 \\
& *c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) \\
& + 1)^8)*(1/(c*x) + 1)^2) - 12*b*c^2*d*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))* \\
& e/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1) \\
& ^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)
\end{aligned}$$



$$\begin{aligned} & \left( \frac{1}{(c^2 x^2) - 1} \right) / \left( \frac{1}{(c x) + 1} \right)^2 - 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^3 / \left( \frac{1}{(c x) + 1} \right)^6 \\ & - c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^4 / \left( \frac{1}{(c x) + 1} \right)^8 \left( \frac{1}{(c x) + 1} \right)^8 + b \left( \frac{1}{(c^2 x^2) - 1} \right)^4 e^2 \log(\text{abs}(\text{sqrt}(-1/(c^2 x^2) + 1) - 1/(c x) - 1)) / ((c^4 + 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right) / \left( \frac{1}{(c x) + 1} \right)^2 - 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^3 / \left( \frac{1}{(c x) + 1} \right)^6 - c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^4 / \left( \frac{1}{(c x) + 1} \right)^8) \left( \frac{1}{(c x) + 1} \right)^8) \\ & + 2 b \left( \frac{1}{(c^2 x^2) - 1} \right)^3 \text{sqrt}(-1/(c^2 x^2) + 1) e^2 / ((c^4 + 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right) / \left( \frac{1}{(c x) + 1} \right)^2 - 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^3 / \left( \frac{1}{(c x) + 1} \right)^6 - c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^4 / \left( \frac{1}{(c x) + 1} \right)^8) \left( \frac{1}{(c x) + 1} \right)^7) \\ & - 2 a \left( \frac{1}{(c^2 x^2) - 1} \right)^4 e^2 / ((c^4 + 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right) / \left( \frac{1}{(c x) + 1} \right)^2 - 2 c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^3 / \left( \frac{1}{(c x) + 1} \right)^6 - c^4 \left( \frac{1}{(c^2 x^2) - 1} \right)^4 / \left( \frac{1}{(c x) + 1} \right)^8) \left( \frac{1}{(c x) + 1} \right)^8) \end{aligned}$$

**maple [A]** time = 0.06, size = 286, normalized size = 1.77

$$\frac{a e^2 x^3}{3} + 2 a e d x - \frac{a d^2}{x} + \frac{b e^2 \text{arcsec}(c x) x^3}{3} + 2 b \text{arcsec}(c x) e d x - \frac{b \text{arcsec}(c x) d^2}{x} + \frac{c b d^2}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b d^2}{c x^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{2 b \sqrt{c^2 x^2 - 1}}{c^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^2,x)

[Out]  $\frac{1}{3} a e^2 x^3 + 2 a e d x - a d^2/x + \frac{1}{3} b e^2 \text{arcsec}(c x) x^3 + 2 b \text{arcsec}(c x) e d x - b \text{arcsec}(c x) d^2/x + c b / ((c^2 x^2 - 1)/c^2/x^2)^{(1/2)} d^2 - b/c/x^2 / ((c^2 x^2 - 1)/c^2/x^2)^{(1/2)} d^2 - 2 b/c^2 (c^2 x^2 - 1)^{(1/2)} / ((c^2 x^2 - 1)/c^2/x^2)^{(1/2)} / x e d \ln(c x + (c^2 x^2 - 1)^{(1/2)}) - 1/6 c b e^2 / ((c^2 x^2 - 1)/c^2/x^2)^{(1/2)} x^2 + 1/6 c^3 b e^2 / ((c^2 x^2 - 1)/c^2/x^2)^{(1/2)} - 1/6 c^4 b e^2 (c^2 x^2 - 1)^{(1/2)} / ((c^2 x^2 - 1)/c^2/x^2)^{(1/2)} / x \ln(c x + (c^2 x^2 - 1)^{(1/2)})$

**maxima [A]** time = 0.34, size = 198, normalized size = 1.22

$$\frac{1}{3} a e^2 x^3 + \left( c \sqrt{-\frac{1}{c^2 x^2} + 1} - \frac{\text{arcsec}(c x)}{x} \right) b d^2 + \frac{1}{12} \left( 4 x^3 \text{arcsec}(c x) - \frac{2 \sqrt{-\frac{1}{c^2 x^2} + 1} + \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c^2 \left(\frac{1}{c^2 x^2} - 1\right) + c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c^2} \right) / c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^2,x, algorithm="maxima")

[Out]  $\frac{1}{3} a e^2 x^3 + (c \text{sqrt}(-1/(c^2 x^2) + 1) - \text{arcsec}(c x)/x) b d^2 + \frac{1}{12} (4 x^3 \text{arcsec}(c x) - (2 \text{sqrt}(-1/(c^2 x^2) + 1)/(c^2 * (1/(c^2 x^2) - 1) + c^2) + \log(\text{sqrt}(-1/(c^2 x^2) + 1) + 1)/c^2 - \log(\text{sqrt}(-1/(c^2 x^2) + 1) - 1)/c^2)$

$/c) * b * e^2 + 2 * a * d * e * x + (2 * c * x * \operatorname{arcsec}(c * x) - \log(\sqrt{-1/(c^2 * x^2) + 1}) + 1) + \log(-\sqrt{-1/(c^2 * x^2) + 1}) * b * d * e / c - a * d^2 / x$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e x^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{c x}\right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2,x)`

[Out] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2, x)`

**sympy** [A] time = 8.56, size = 207, normalized size = 1.28

$$-\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\sqrt{1 - \frac{1}{c^2x^2}} - \frac{bd^2 \operatorname{asec}(cx)}{x} + 2bdex \operatorname{asec}(cx) + \frac{be^2x^3 \operatorname{asec}(cx)}{3} - \frac{2bde \begin{cases} \operatorname{acosh}(cx) & \text{for } |cx| < 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**2,x)`

[Out] `-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*sqrt(1 - 1/(c**2*x**2)) - b*d**2*asec(c*x)/x + 2*b*d*e*x*asec(c*x) + b*e**2*x**3*asec(c*x)/3 - 2*b*d*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)`

$$3.84 \quad \int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=158

$$-\frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) + \frac{bcd^2 \sqrt{c^2 x^2 - 1}}{9x^2 \sqrt{c^2 x^2}} + \frac{2bcd \sqrt{c^2 x^2 - 1} (c^2 d + 9e)}{9 \sqrt{c^2 x^2}}$$

[Out]  $-1/3*d^2*(a+b*\text{arcsec}(c*x))/x^3-2*d*e*(a+b*\text{arcsec}(c*x))/x+e^2*x*(a+b*\text{arcsec}(c*x))-b*e^2*x*\text{arctanh}(c*x/(c^2*x^2-1)^{(1/2)})/(c^2*x^2)^{(1/2)}+2/9*b*c*d*(c^2*d+9*e)*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}+1/9*b*c*d^2*(c^2*x^2-1)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 5238, 12, 1265, 451, 217, 206}

$$-\frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) + \frac{bcd^2 \sqrt{c^2 x^2 - 1}}{9x^2 \sqrt{c^2 x^2}} + \frac{2bcd \sqrt{c^2 x^2 - 1} (c^2 d + 9e)}{9 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcSec}[c*x])/x^4, x]$

[Out]  $(2*b*c*d*(c^2*d + 9*e)*\text{Sqrt}[-1 + c^2*x^2])/(9*\text{Sqrt}[c^2*x^2]) + (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(9*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcSec}[c*x]))/(3*x^3) - (2*d*e*(a + b*\text{ArcSec}[c*x]))/x + e^2*x*(a + b*\text{ArcSec}[c*x]) - (b*e^2*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/\text{Sqrt}[c^2*x^2]$

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 206

$\text{Int}[(a_)+(b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_)+(b_.)*(x_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))
```

Rule 1265

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{x^4} dx &= -\frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) - \frac{(bcx)}{x^2} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) - \frac{(bcx)}{x^2} \\
&= \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) - \frac{(bcx)}{x^2} \\
&= \frac{2bcd (c^2 d + 9e) \sqrt{-1 + c^2 x^2}}{9\sqrt{c^2 x^2}} + \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) - \frac{(bcx)}{x^2} \\
&= \frac{2bcd (c^2 d + 9e) \sqrt{-1 + c^2 x^2}}{9\sqrt{c^2 x^2}} + \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) - \frac{(bcx)}{x^2} \\
&= \frac{2bcd (c^2 d + 9e) \sqrt{-1 + c^2 x^2}}{9\sqrt{c^2 x^2}} + \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{2de (a + b \sec^{-1}(cx))}{x} + e^2 x (a + b \sec^{-1}(cx)) - \frac{(bcx)}{x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 127, normalized size = 0.80

$$\frac{c \left( bcdx \sqrt{1 - \frac{1}{c^2 x^2}} (2c^2 dx^2 + d + 18ex^2) - 3a (d^2 + 6dex^2 - 3e^2 x^4) \right) - 9be^2 x^3 \log \left( x \left( \sqrt{1 - \frac{1}{c^2 x^2}} + 1 \right) \right) - 3bc \sec^{-1}(cx)}{9cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^4,x]

[Out] (c\*(b\*c\*d\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + 2\*c^2\*d\*x^2 + 18\*e\*x^2) - 3\*a\*(d^2 + 6\*d\*e\*x^2 - 3\*e^2\*x^4)) - 3\*b\*c\*(d^2 + 6\*d\*e\*x^2 - 3\*e^2\*x^4)\*ArcSec[c\*x] - 9\*b\*e^2\*x^3\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x])/(9\*c\*x^3)

**fricas [A]** time = 3.06, size = 221, normalized size = 1.40

$$\frac{9ace^2x^4 + 9be^2x^3 \log(-cx + \sqrt{c^2x^2 - 1}) - 18acdex^2 - 6(bcd^2 + 6bcde - 3bce^2)x^3 \arctan(-cx + \sqrt{c^2x^2 - 1}) - 3bc \sec^{-1}(cx)}{9cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^4,x, algorithm="fricas")



```
[Out] 1/9*(9*a*c*e^2*x^4 + 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*e
*x^2 - 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2 -
1)) - 3*a*c*d^2 + 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b
*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arcsec(c*x) +
(b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

**giac [B]** time = 14.58, size = 4960, normalized size = 31.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/9*(3*b*c^4*d^2*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) + 3*a*c^4*d^2/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8) + 12*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^2 - 2*
c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2 - 6*b*c^
4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8)*(1/(c*x) + 1)) + 12*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^2 - 2*c^2
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2 + 18*b*c^4*
d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 18*b*c^2*d*arccos(1/(c*x))*e/(c
^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 2*b*c^4*d^2*(-1/
(c^2*x^2) + 1)^(3/2)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^
2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8)*(1/(c*x) + 1)^3) + 18*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/((c^2 - 2*c^2*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
- c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 12*b*c^4*d^2*
(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 18*a*c^2*d*e/(c^2 - 2*c^2*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 2*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt
(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8)*(1/(c*x) + 1)^5) - 36*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)*e/((c^2 - 2*c^2*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
- c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 12*a*c^4*d^2*(
```

$$\begin{aligned}
& 1/(c^2*x^2) - 1)^3/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2* \\
& (1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& ^8)*(1/(c*x) + 1)^6) + 3*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))/((c^ \\
& 2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/ \\
& (c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8 - \\
& 36*b*c^2*d*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))*e/((c^2 - 2*c^2*(1/(c^2*x^2) \\
& - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/ \\
& (c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 6*b*c^4*d^2*(1/(c^2*x^ \\
& 2) - 1)^3*\sqrt{-1/(c^2*x^2) + 1}/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + \\
& 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4 \\
& /((1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) - 36*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)*e/ \\
& ((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3 \\
& /((1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3 \\
& ) + 3*a*c^4*d^2*(1/(c^2*x^2) - 1)^4/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) \\
& ) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1) \\
& )^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 9*b*\arccos(1/(c*x))*e^2/(c^2 - 2*c^ \\
& 2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + \\
& 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 36*a*c^2*d*(1/(c^2*x^2) - \\
& 1)^2*e/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) \\
& ) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) \\
& ) + 1)^4) + 9*b*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/(c^2 - 2 \\
& *c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) \\
& + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 9*b*e^2*\log(\text{abs}(\sqrt{- \\
& 1/(c^2*x^2) + 1} - 1/(c*x) - 1))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + \\
& 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/ \\
& (1/(c*x) + 1)^8) + 36*b*c^2*d*(1/(c^2*x^2) - 1)^2*\sqrt{-1/(c^2*x^2) + 1}*e/ \\
& ((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3 \\
& /((1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5 \\
& ) - 9*a*e^2/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2* \\
& x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 36 \\
& *b*(1/(c^2*x^2) - 1)*\arccos(1/(c*x))*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/ \\
& (c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) \\
& ) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b*c^2*d*(1/(c^2*x^2) - 1)^4 \\
& *\arccos(1/(c*x))*e/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2* \\
& (1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& ^8)*(1/(c*x) + 1)^8) - 18*b*(1/(c^2*x^2) - 1)*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) \\
& + 1} + 1/(c*x) + 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c \\
& ^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + \\
& 1)^8)*(1/(c*x) + 1)^2) + 18*b*(1/(c^2*x^2) - 1)*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x \\
& ^2) + 1} - 1/(c*x) - 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + \\
& 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) \\
& ) + 1)^8)*(1/(c*x) + 1)^2) - 36*b*c^2*d*(1/(c^2*x^2) - 1)^3*\sqrt{-1/(c^2*x^ \\
& 2) + 1}*e/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x \\
& ^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c \\
& *x) + 1)^7) + 36*a*(1/(c^2*x^2) - 1)*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1
\end{aligned}$$

$$\begin{aligned} & /((c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) \\ & ) - 1)^4/(1/(c*x) + 1)^8*(1/(c*x) + 1)^2 - 54*b*(1/(c^2*x^2) - 1)^2*\arccos \\ & s(1/(c*x))*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^4 + 18*a*c^2*d*(1/(c^2*x^2) - 1)^4*e/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^4) - 54*a*(1/(c^2*x^2) - 1)^2*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^4) + 36*b*(1/(c^2*x^2) - 1)^3*\arccos(1/(c*x))*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^6) + 18*b*(1/(c^2*x^2) - 1)^3*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^6) - 18*b*(1/(c^2*x^2) - 1)^3*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^6) + 36 \\ & *a*(1/(c^2*x^2) - 1)^3*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^6) - 9*b*(1/(c^2*x^2) - 1)^4*\arccos(1/(c*x))*e^2/ \\ & ((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^8) - 9*b*(1/(c^2*x^2) - 1)^4*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} + 1/(c*x) + 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^8) + 9*b*(1/(c^2*x^2) - 1)^4*e^2*\log(\text{abs}(\sqrt{-1/(c^2*x^2) + 1} - 1/(c*x) - 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^8) - 9*a*(1/(c^2*x^2) - 1)^4*e^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)* \\ & (1/(c*x) + 1)^8))*c \end{aligned}$$

**maple [A]** time = 0.06, size = 255, normalized size = 1.61

$$ax e^2 - \frac{a d^2}{3x^3} - \frac{2aed}{x} + b \operatorname{arcsec}(cx) x e^2 - \frac{b \operatorname{arcsec}(cx) d^2}{3x^3} - \frac{2b \operatorname{arcsec}(cx) ed}{x} + \frac{2c^3 b d^2}{9\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{cb d^2}{9x^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{2cb ed}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^4,x)

[Out] a\*x\*e^2-1/3\*a\*d^2/x^3-2\*a\*e\*d/x+b\*arcsec(c\*x)\*x\*e^2-1/3\*b\*arcsec(c\*x)\*d^2/x^3-2\*b\*arcsec(c\*x)\*e\*d/x+2/9\*c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d^2-1/9\*c\*b/

$x^2/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*d^2+2*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*e*d-2/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^2*e*d-1/9*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^4*d^2-1/c^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*e^2*\ln(c*x+(c^2*x^2-1)^{(1/2)})$

**maxima** [A] time = 0.32, size = 159, normalized size = 1.01

$$2\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)bde+ae^2x-\frac{1}{9}bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\operatorname{arcsec}(cx)}{x^3}\right)+\frac{(2cx\operatorname{arcsec}(cx))}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^4,x, algorithm="maxima")

[Out]  $2*(c*\sqrt{-1/(c^2*x^2)}+1)-\operatorname{arcsec}(c*x)/x)*b*d*e+a*e^2*x-1/9*b*d^2*((c^4*(-1/(c^2*x^2)+1)^{(3/2)}-3*c^4*\sqrt{-1/(c^2*x^2)+1}))/c+3*\operatorname{arcsec}(c*x)/x^3)+1/2*(2*c*x*\operatorname{arcsec}(c*x)-\log(\sqrt{-1/(c^2*x^2)+1}+1)+\log(-\sqrt{-1/(c^2*x^2)+1}+1))*b*e^2/c-2*a*d*e/x-1/3*a*d^2/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2+d)^2 \left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))))/x^4,x)

[Out] int(((d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))))/x^4, x)

**sympy** [A] time = 7.99, size = 211, normalized size = 1.34

$$-\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x+2bcde\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\operatorname{asec}(cx)}{3x^3}-\frac{2bde\operatorname{asec}(cx)}{x}+be^2x\operatorname{asec}(cx)+\frac{bd^2\left(\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3}+\frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x))/x\*\*4,x)

[Out]  $-a*d**2/(3*x**3)-2*a*d*e/x+a*e**2*x+2*b*c*d*e*\sqrt{1-1/(c**2*x**2)}-b*d**2*\operatorname{asec}(c*x)/(3*x**3)-2*b*d*e*\operatorname{asec}(c*x)/x+b*e**2*x*\operatorname{asec}(c*x)+b$

```

*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(
3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) - b*e**2*Piecewise((acosh(c*x), A
bs(c**2*x**2) > 1), (-I*asin(c*x), True))/c

```

$$3.85 \quad \int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=183

$$\frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} + \frac{bcd^2 \sqrt{c^2 x^2 - 1}}{25x^4 \sqrt{c^2 x^2}} + \frac{2bcd \sqrt{c^2 x^2 - 1} (6c^2 d + 25e)}{225x^2 \sqrt{c^2 x^2}}$$

[Out]  $-1/5*d^2*(a+b*\text{arcsec}(c*x))/x^5-2/3*d*e*(a+b*\text{arcsec}(c*x))/x^3-e^2*(a+b*\text{arcsec}(c*x))/x+1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}+1/25*b*c*d^2*(c^2*x^2-1)^{(1/2)}/x^4/(c^2*x^2)^{(1/2)}+2/225*b*c*d*(6*c^2*d+25*e)*(c^2*x^2-1)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {270, 5238, 12, 1265, 453, 264}

$$\frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} + \frac{bc \sqrt{c^2 x^2 - 1} (24c^4 d^2 + 100c^2 de + 225e^2)}{225 \sqrt{c^2 x^2}} + \frac{bcd \sqrt{c^2 x^2 - 1} (6c^2 d + 25e)}{225x^2 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcSec}[c*x])/x^6, x]$

[Out]  $(b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(225*\text{Sqrt}[c^2*x^2]) + (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(25*x^4*\text{Sqrt}[c^2*x^2]) + (2*b*c*d*(6*c^2*d + 25*e)*\text{Sqrt}[-1 + c^2*x^2])/(225*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcSec}[c*x]))/(5*x^5) - (2*d*e*(a + b*\text{ArcSec}[c*x]))/(3*x^3) - (e^2*(a + b*\text{ArcSec}[c*x]))/x$

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 264

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[(m+1)/n + p + 1, 0] \&\& \text{NeQ}[m, -1]$

### Rule 270

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Int}[\text{Exp}[\text{and}[\text{Integrand}[(c*x)^m*(a + b*x^n)^p, x], x]] /; \text{FreeQ}[\{a, b, c, m, n\}, x] \&\&$

IGtQ[p, 0]

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrate[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{x^6} dx &= -\frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} - \frac{(bcx)}{x^2} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} - \frac{(bcx)}{x^2} \\
&= \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{25x^4 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} - \frac{(bcx)}{x^2} \\
&= \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{25x^4 \sqrt{c^2 x^2}} + \frac{2bcd (6c^2 d + 25e) \sqrt{-1 + c^2 x^2}}{225x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} - \frac{(bcx)}{x^2} \\
&= \frac{bc (225e^2 + 4c^2 d (6c^2 d + 25e)) \sqrt{-1 + c^2 x^2}}{225 \sqrt{c^2 x^2}} + \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{25x^4 \sqrt{c^2 x^2}} + \frac{2bcd (6c^2 d + 25e) \sqrt{-1 + c^2 x^2}}{225x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{5x^5} - \frac{2de (a + b \sec^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \sec^{-1}(cx))}{x} - \frac{(bcx)}{x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 127, normalized size = 0.69

$$\frac{-15a(3d^2 + 10dex^2 + 15e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(50dex^2(2c^2x^2 + 1) + 3d^2(8c^4x^4 + 4c^2x^2 + 3) + 225e^2x^4) - 15bcx}{225x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^6,x]

[Out] (-15\*a\*(3\*d^2 + 10\*d\*e\*x^2 + 15\*e^2\*x^4) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(225\*e^2\*x^4 + 50\*d\*e\*x^2\*(1 + 2\*c^2\*x^2) + 3\*d^2\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(3\*d^2 + 10\*d\*e\*x^2 + 15\*e^2\*x^4)\*ArcSec[c\*x])/(225\*x^5)

**fricas [A]** time = 1.32, size = 127, normalized size = 0.69

$$\frac{225 a e^2 x^4 + 150 a d e x^2 + 45 a d^2 + 15 (15 b e^2 x^4 + 10 b d e x^2 + 3 b d^2) \operatorname{arcsec}(c x) - ((24 b c^4 d^2 + 100 b c^2 d e + 225 b e^2) x^4 + 9 b d^2 + 2 (6 b c^2 d^2 + 25 b d e) x^2) \sqrt{c^2 x^2 - 1}}{225 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="fricas")

[Out] -1/225\*(225\*a\*e^2\*x^4 + 150\*a\*d\*e\*x^2 + 45\*a\*d^2 + 15\*(15\*b\*e^2\*x^4 + 10\*b\*d\*e\*x^2 + 3\*b\*d^2)\*arcsec(c\*x) - ((24\*b\*c^4\*d^2 + 100\*b\*c^2\*d\*e + 225\*b\*e^2)\*x^4 + 9\*b\*d^2 + 2\*(6\*b\*c^2\*d^2 + 25\*b\*d\*e)\*x^2)\*sqrt(c^2\*x^2 - 1))/x^5



**giac** [A] time = 0.15, size = 223, normalized size = 1.22

$$\frac{1}{225} \left( 24bc^4d^2\sqrt{-\frac{1}{c^2x^2}+1} + 100bc^2d\sqrt{-\frac{1}{c^2x^2}+1}e + \frac{12bc^2d^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2} + 225b\sqrt{-\frac{1}{c^2x^2}+1}e^2 + \frac{50bd\sqrt{-\frac{1}{c^2x^2}+1}}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="giac")

[Out] 1/225\*(24\*b\*c^4\*d^2\*sqrt(-1/(c^2\*x^2) + 1) + 100\*b\*c^2\*d\*sqrt(-1/(c^2\*x^2) + 1)\*e + 12\*b\*c^2\*d^2\*sqrt(-1/(c^2\*x^2) + 1)/x^2 + 225\*b\*sqrt(-1/(c^2\*x^2) + 1)\*e^2 + 50\*b\*d\*sqrt(-1/(c^2\*x^2) + 1)\*e/x^2 - 225\*b\*arccos(1/(c\*x))\*e^2/(c\*x) - 225\*a\*e^2/(c\*x) - 150\*b\*d\*arccos(1/(c\*x))\*e/(c\*x^3) + 9\*b\*d^2\*sqrt(-1/(c^2\*x^2) + 1)/x^4 - 150\*a\*d\*e/(c\*x^3) - 45\*b\*d^2\*arccos(1/(c\*x))/(c\*x^5) - 45\*a\*d^2/(c\*x^5))\*c

**maple** [A] time = 0.06, size = 191, normalized size = 1.04

$$c^5 \left( \frac{a \left( -\frac{2ed}{3cx^3} - \frac{e^2}{cx} - \frac{d^2}{5cx^5} \right)}{c^4} + \frac{b \left( -\frac{2 \operatorname{arcsec}(cx)ed}{3cx^3} - \frac{\operatorname{arcsec}(cx)e^2}{cx} - \frac{\operatorname{arcsec}(cx)d^2}{5cx^5} + \frac{(c^2x^2-1)(24x^4c^8d^2+100c^6edx^4+12x^2c^6d^2+225c^4e^2x^4)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}c^6x^6} \right)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^6,x)

[Out] c^5\*(a/c^4\*(-2/3/c\*e\*d/x^3-e^2/c/x-1/5\*d^2/c/x^5)+b/c^4\*(-2/3\*arcsec(c\*x)/c\*e\*d/x^3-arcsec(c\*x)\*e^2/c/x-1/5\*arcsec(c\*x)\*d^2/c/x^5+1/225\*(c^2\*x^2-1)\*(24\*c^8\*d^2\*x^4+100\*c^6\*d\*e\*x^4+12\*c^6\*d^2\*x^2+225\*c^4\*e^2\*x^4+50\*c^4\*d\*e\*x^2+9\*c^4\*d^2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))

**maxima** [A] time = 0.33, size = 181, normalized size = 0.99

$$\left( c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x} \right) be^2 + \frac{1}{75} bd^2 \left( \frac{3c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 10c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} - \frac{15 \operatorname{arcsec}(cx)}{x^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="maxima")

[Out]  $(c\sqrt{-1/(c^2x^2) + 1} - \operatorname{arcsec}(cx)/x) * b * e^2 + 1/75 * b * d^2 * ((3c^6 * (-1/(c^2x^2) + 1)^{5/2} - 10c^6 * (-1/(c^2x^2) + 1)^{3/2} + 15c^6 * \sqrt{-1/(c^2x^2) + 1})/c - 15 * \operatorname{arcsec}(cx)/x^5) - 2/9 * b * d * e * ((c^4 * (-1/(c^2x^2) + 1)^{3/2} - 3c^4 * \sqrt{-1/(c^2x^2) + 1})/c + 3 * \operatorname{arcsec}(cx)/x^3) - a * e^2/x - 2/3 * a * d * e/x^3 - 1/5 * a * d^2/x^5$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6,x)`

[Out] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6, x)`

**sympy [A]** time = 10.53, size = 333, normalized size = 1.82

$$\frac{ad^2}{5x^5} - \frac{2ade}{3x^3} - \frac{ae^2}{x} + bce^2 \sqrt{1 - \frac{1}{c^2x^2}} - \frac{bd^2 \operatorname{asec}(cx)}{5x^5} - \frac{2bde \operatorname{asec}(cx)}{3x^3} - \frac{be^2 \operatorname{asec}(cx)}{x} + bd^2 \left\{ \begin{array}{l} \frac{8c^5 \sqrt{c^2x^2-1}}{15x} + \frac{4c^3 \sqrt{c^2x^2-1}}{15x^3} + \\ \frac{8ic^5 \sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3 \sqrt{-c^2x^2+1}}{15x^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**6,x)`

[Out]  $-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c*e**2*\sqrt{1 - 1/(c**2*x**2)} - b*d**2*asec(c*x)/(5*x**5) - 2*b*d*e*asec(c*x)/(3*x**3) - b*e**2*asec(c*x)/x + b*d**2*\operatorname{Piecewise}((8*c**5*\sqrt{c**2*x**2 - 1}/(15*x) + 4*c**3*\sqrt{c**2*x**2 - 1}/(15*x**3) + c*\sqrt{c**2*x**2 - 1}/(5*x**5), \operatorname{Abs}(c**2*x**2) > 1), (8*I*c**5*\sqrt{-c**2*x**2 + 1}/(15*x) + 4*I*c**3*\sqrt{-c**2*x**2 + 1}/(15*x**3) + I*c*\sqrt{-c**2*x**2 + 1}/(5*x**5), \operatorname{True}))/ (5*c) + 2*b*d*e*\operatorname{Piecewise}((2*c**3*\sqrt{c**2*x**2 - 1}/(3*x) + c*\sqrt{c**2*x**2 - 1}/(3*x**3), \operatorname{Abs}(c**2*x**2) > 1), (2*I*c**3*\sqrt{-c**2*x**2 + 1}/(3*x) + I*c*\sqrt{-c**2*x**2 + 1}/(3*x**3), \operatorname{True}))/ (3*c)$

$$3.86 \quad \int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=241

$$\frac{d^2 (a + b \sec^{-1}(cx))}{7x^7} - \frac{2de (a + b \sec^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \sec^{-1}(cx))}{3x^3} + \frac{bcd^2 \sqrt{c^2 x^2 - 1}}{49x^6 \sqrt{c^2 x^2}} + \frac{2bcd \sqrt{c^2 x^2 - 1} (15c^2 d + 49e)}{1225x^4 \sqrt{c^2 x^2}}$$

[Out]  $-1/7*d^2*(a+b*\text{arcsec}(c*x))/x^7-2/5*d*e*(a+b*\text{arcsec}(c*x))/x^5-1/3*e^2*(a+b*\text{arcsec}(c*x))/x^3+2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^{(1/2)}/(c^2*x^2)^{(1/2)}+1/49*b*c*d^2*(c^2*x^2-1)^{(1/2)}/x^6/(c^2*x^2)^{(1/2)}+2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^{(1/2)}/x^4/(c^2*x^2)^{(1/2)}+1/11025*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 5238, 12, 1265, 453, 271, 264}

$$\frac{d^2 (a + b \sec^{-1}(cx))}{7x^7} - \frac{2de (a + b \sec^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \sec^{-1}(cx))}{3x^3} + \frac{2bc^3 \sqrt{c^2 x^2 - 1} (360c^4 d^2 + 1176c^2 de + 1225e^2)}{11025 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^8, x]

[Out]  $(2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/((11025*\text{Sqrt}[c^2*x^2]) + (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2]))/(49*x^6*\text{Sqrt}[c^2*x^2]) + (2*b*c*d*(15*c^2*d + 49*e)*\text{Sqrt}[-1 + c^2*x^2])/((1225*x^4*\text{Sqrt}[c^2*x^2]) + (b*c*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 + c^2*x^2]))/(11025*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcSec[c*x]))/(5*x^5) - (e^2*(a + b*ArcSec[c*x]))/(3*x^3)$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 264**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*
(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^(m)*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^2 (a + b \sec^{-1}(cx))}{x^8} dx &= -\frac{d^2 (a + b \sec^{-1}(cx))}{7x^7} - \frac{2de (a + b \sec^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{49x^6 \sqrt{c^2 x^2}} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{7x^7} - \frac{2de (a + b \sec^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \sec^{-1}(cx))}{3x^3} - \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{49x^6 \sqrt{c^2 x^2}} \\
&= \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{49x^6 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{7x^7} - \frac{2de (a + b \sec^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \sec^{-1}(cx))}{3x^3} \\
&= \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{49x^6 \sqrt{c^2 x^2}} + \frac{2bcd (15c^2 d + 49e) \sqrt{-1 + c^2 x^2}}{1225x^4 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{7x^7} \\
&= \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{49x^6 \sqrt{c^2 x^2}} + \frac{2bcd (15c^2 d + 49e) \sqrt{-1 + c^2 x^2}}{1225x^4 \sqrt{c^2 x^2}} + \frac{bc (1225e^2 + 24c^2 d)}{11025x^7} \\
&= \frac{2bc^3 (1225e^2 + 24c^2 d (15c^2 d + 49e)) \sqrt{-1 + c^2 x^2}}{11025 \sqrt{c^2 x^2}} + \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{49x^6 \sqrt{c^2 x^2}} + \frac{2bc (1225e^2 + 24c^2 d)}{11025x^7}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 153, normalized size = 0.63

$$\frac{-105a(15d^2 + 42dex^2 + 35e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(1225e^2x^4(2c^2x^2 + 1) + 294dex^2(8c^4x^4 + 4c^2x^2 + 3) + 45d^2(1225e^2 + 24c^2d))}{11025x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^8,x]

[Out] (-105\*a\*(15\*d^2 + 42\*d\*e\*x^2 + 35\*e^2\*x^4) + b\*c\*sqrt[1 - 1/(c^2\*x^2)]\*x\*(1225\*e^2\*x^4\*(1 + 2\*c^2\*x^2) + 294\*d\*e\*x^2\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4) + 45\*d^2\*(5 + 6\*c^2\*x^2 + 8\*c^4\*x^4 + 16\*c^6\*x^6)) - 105\*b\*(15\*d^2 + 42\*d\*e\*x^2 + 35\*e^2\*x^4)\*ArcSec[c\*x])/(11025\*x^7)

**fricas [A]** time = 1.55, size = 159, normalized size = 0.66

$$\frac{3675ae^2x^4 + 4410adex^2 + 1575ad^2 + 105(35be^2x^4 + 42bdex^2 + 15bd^2)\operatorname{arcsec}(cx) - (2(360bc^6d^2 + 1176bc^4de^2 + 11025c^2d^2e^2))\sqrt{-1 + c^2x^2}}{11025x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="fricas")

[Out]  $-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4 + 42*b*d*e*x^2 + 15*b*d^2)*\operatorname{arcsec}(c*x) - (2*(360*b*c^6*d^2 + 1176*b*c^4*d*e + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x^4 + 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*\sqrt{c^2*x^2 - 1})/x^7$

**giac** [A] time = 0.16, size = 294, normalized size = 1.22

$$\frac{1}{11025} \left( 720 bc^6 d^2 \sqrt{-\frac{1}{c^2 x^2} + 1} + 2352 bc^4 d \sqrt{-\frac{1}{c^2 x^2} + 1} e + \frac{360 bc^4 d^2 \sqrt{-\frac{1}{c^2 x^2} + 1}}{x^2} + 2450 bc^2 \sqrt{-\frac{1}{c^2 x^2} + 1} e^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")`

[Out]  $1/11025*(720*b*c^6*d^2*\sqrt{-1/(c^2*x^2) + 1} + 2352*b*c^4*d*\sqrt{-1/(c^2*x^2) + 1}*e + 360*b*c^4*d^2*\sqrt{-1/(c^2*x^2) + 1}/x^2 + 2450*b*c^2*\sqrt{-1/(c^2*x^2) + 1}*e^2 + 1176*b*c^2*d*\sqrt{-1/(c^2*x^2) + 1}*e/x^2 + 270*b*c^2*d^2*\sqrt{-1/(c^2*x^2) + 1}/x^4 + 1225*b*\sqrt{-1/(c^2*x^2) + 1}*e^2/x^2 + 882*b*d*\sqrt{-1/(c^2*x^2) + 1}*e/x^4 - 3675*b*\arccos(1/(c*x))*e^2/(c*x^3) - 3675*a*e^2/(c*x^3) - 4410*b*d*\arccos(1/(c*x))*e/(c*x^5) + 225*b*d^2*\sqrt{-1/(c^2*x^2) + 1}/x^6 - 4410*a*d*e/(c*x^5) - 1575*b*d^2*\arccos(1/(c*x))/(c*x^7) - 1575*a*d^2/(c*x^7))*c$

**maple** [A] time = 0.06, size = 223, normalized size = 0.93

$$c^7 \left( \frac{a \left( -\frac{d^2}{7c^3x^7} - \frac{e^2}{3c^3x^3} - \frac{2ed}{5c^3x^5} \right)}{c^4} + \frac{b \left( -\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7} - \frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3} - \frac{2\operatorname{arcsec}(cx)ed}{5c^3x^5} + \frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8edx^6+360x^4c^8d^2+2450c^6e^2x^6+1176c^6d^2ex^4+270c^6d^2x^2+1225c^4e^2x^4+882c^4d^2ex^2+225c^4d^2)}{(c^2x^2-1)/c^2/x^2} \right)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x)`

[Out]  $c^7*(a/c^4*(-1/7*d^2/c^3/x^7-1/3*e^2/c^3/x^3-2/5/c^3*e*d/x^5)+b/c^4*(-1/7*arcsec(c*x)*d^2/c^3/x^7-1/3*arcsec(c*x)*e^2/c^3/x^3-2/5*arcsec(c*x)/c^3*e*d/x^5+1/11025*(c^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^8/x^8))$

**maxima** [A] time = 0.33, size = 241, normalized size = 1.00

$$-\frac{1}{245}bd^2 \left( \frac{5c^8 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{7}{2}} - 21c^8 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} + 35c^8 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 35c^8 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} + \frac{35 \operatorname{arcsec}(cx)}{x^7} \right) + \frac{2}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="maxima")

[Out]  $-\frac{1}{245}bd^2 \left( (5c^8 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{7}{2}} - 21c^8 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} + 35c^8 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 35c^8 \sqrt{-\frac{1}{c^2x^2} + 1} \right) / c + 35 \operatorname{arcsec}(cx) / x^7 + \frac{2}{75}bd^2 \left( (3c^6 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} - 10c^6 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 15c^6 \sqrt{-\frac{1}{c^2x^2} + 1} \right) / c - 15 \operatorname{arcsec}(cx) / x^5 - \frac{1}{9}b^2 \left( c^4 \left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2x^2} + 1} \right) / c + 3 \operatorname{arcsec}(cx) / x^3 - \frac{1}{3}a^2 / x^3 - \frac{2}{5}ad^2 / x^5 - \frac{1}{7}ad^2 / x^7$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))))/x^8,x)

[Out] int(((d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))))/x^8, x)

**sympy** [A] time = 57.34, size = 508, normalized size = 2.11

$$\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2 \operatorname{asec}(cx)}{7x^7} - \frac{2bde \operatorname{asec}(cx)}{5x^5} - \frac{be^2 \operatorname{asec}(cx)}{3x^3} + \frac{bd^2 \left( \frac{16c^7 \sqrt{c^2x^2-1}}{35x} + \frac{8c^5 \sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3 \sqrt{c^2x^2-1}}{35x^5} + \frac{16ic^7 \sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5 \sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3 \sqrt{-c^2x^2+1}}{35x^5} \right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x))/x\*\*8,x)

[Out]  $-a*d^2/(7*x^7) - 2*a*d*e/(5*x^5) - a*e^2/(3*x^3) - b*d^2*asec(c*x)/(7*x^7) - 2*b*d*e*asec(c*x)/(5*x^5) - b*e^2*asec(c*x)/(3*x^3) + b*d^2*Pi$   
 $\operatorname{ecwise}\left(\frac{16*c^7*\sqrt{c^2*x^2-1}}{(35*x)} + \frac{8*c^5*\sqrt{c^2*x^2-1}}{(35*x^3)} + \frac{6*c^3*\sqrt{c^2*x^2-1}}{(35*x^5)} + \frac{16*ic^7*\sqrt{-c^2*x^2+1}}{(35*x)} + \frac{8*ic^5*\sqrt{-c^2*x^2+1}}{(35*x^3)} + \frac{6*ic^3*\sqrt{-c^2*x^2+1}}{(35*x^5)}\right)$

$$\begin{aligned}
& 5x^3) + 6c^3\sqrt{c^2x^2 - 1}/(35x^5) + c\sqrt{c^2x^2 - 1}/(7x \\
& **7), \text{Abs}(c^2x^2) > 1), (16Ic^7\sqrt{-c^2x^2 + 1}/(35x) + 8Ic^5 \\
& \sqrt{-c^2x^2 + 1}/(35x^3) + 6Ic^3\sqrt{-c^2x^2 + 1}/(35x^5) \\
& + Ic\sqrt{-c^2x^2 + 1}/(7x^7), \text{True}))/7c) + 2bde\text{Piecewise}((8c^5 \\
& \sqrt{c^2x^2 - 1}/(15x) + 4c^3\sqrt{c^2x^2 - 1}/(15x^3) + c\sqrt{c^2x^2 - 1}/(5x^5), \\
& \text{Abs}(c^2x^2) > 1), (8Ic^5\sqrt{-c^2x^2 + 1}/(15x) + 4Ic^3\sqrt{-c^2x^2 + 1}/(15x^3) \\
& + Ic\sqrt{-c^2x^2 + 1}/(5x^5), \text{True}))/5c) + be^2\text{Piecewise}((2c^3\sqrt{c^2x^2 - 1} \\
& /3x) + c\sqrt{c^2x^2 - 1}/(3x^3), \text{Abs}(c^2x^2) > 1), (2Ic^3\sqrt{-c^2x^2 + 1}/3x \\
& + Ic\sqrt{-c^2x^2 + 1}/(3x^3), \text{True}))/3c)
\end{aligned}$$



### 3.87 $\int x^3 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=242

$$\frac{1}{4}d^2x^4 (a + b \sec^{-1}(cx)) + \frac{1}{3}dex^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}e^2x^8 (a + b \sec^{-1}(cx)) - \frac{bex(c^2x^2 - 1)^{5/2}(8c^2d + 9e)}{120c^7\sqrt{c^2x^2}} - \frac{be^2x}{56}$$

[Out]  $\frac{1}{4}d^2x^4(a+b\operatorname{arcsec}(cx)) + \frac{1}{3}d^2ex^6(a+b\operatorname{arcsec}(cx)) + \frac{1}{8}e^2x^8(a+b\operatorname{arcsec}(cx)) - \frac{1}{72}b(6c^4d^2+16c^2de+9e^2)x(c^2x^2-1)^{3/2}/c^7/(c^2x^2)^{1/2} - \frac{1}{120}b(8c^2d+9e)x(c^2x^2-1)^{5/2}/c^7/(c^2x^2)^{1/2} - \frac{1}{56}be^2x(c^2x^2-1)^{7/2}/c^7/(c^2x^2)^{1/2} - \frac{1}{24}b(6c^4d^2+8c^2de+3e^2)x(c^2x^2-1)^{1/2}/c^7/(c^2x^2)^{1/2}$

**Rubi [A]** time = 0.22, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {266, 43, 5238, 12, 1251, 771}

$$\frac{1}{4}d^2x^4 (a + b \sec^{-1}(cx)) + \frac{1}{3}dex^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}e^2x^8 (a + b \sec^{-1}(cx)) - \frac{bx(c^2x^2 - 1)^{3/2}(6c^4d^2 + 16c^2de + 9e^2)}{72c^7\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3(d + ex^2)^2(a + b\operatorname{ArcSec}[cx]), x]$

[Out]  $-\frac{b(6c^4d^2 + 8c^2de + 3e^2)x\sqrt{-1 + c^2x^2}}{(24c^7\sqrt{c^2x^2})} - \frac{b(6c^4d^2 + 16c^2de + 9e^2)x(-1 + c^2x^2)^{3/2}}{(72c^7\sqrt{c^2x^2})} - \frac{b(8c^2d + 9e)x(-1 + c^2x^2)^{5/2}}{(120c^7\sqrt{c^2x^2})} - \frac{be^2x(-1 + c^2x^2)^{7/2}}{(56c^7\sqrt{c^2x^2})} + \frac{d^2x^4(a + b\operatorname{ArcSec}[cx])}{4} + \frac{d^2ex^6(a + b\operatorname{ArcSec}[cx])}{3} + \frac{e^2x^8(a + b\operatorname{ArcSec}[cx])}{8}$

#### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 43

$\operatorname{Int}[(a_*)(x_*)^{m_*)((c_*) + (d_*)(x_*)^{n_}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + bx)^m(c + dx)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (\operatorname{!IntegerQ}[n] \mid\mid (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7*m + 4*n + 4, 0]) \mid\mid \operatorname{LtQ}[9*m + 5*(n + 1), 0] \mid\mid \operatorname{GtQ}[m + n + 2, 0])$

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 771

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (
c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

### Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_
)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

### Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int x^3 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx &= \frac{1}{4}d^2x^4 (a + b \sec^{-1}(cx)) + \frac{1}{3}dex^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}e^2x^8 (a + b \sec^{-1}(cx)) \\
&= \frac{1}{4}d^2x^4 (a + b \sec^{-1}(cx)) + \frac{1}{3}dex^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}e^2x^8 (a + b \sec^{-1}(cx)) \\
&= \frac{1}{4}d^2x^4 (a + b \sec^{-1}(cx)) + \frac{1}{3}dex^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}e^2x^8 (a + b \sec^{-1}(cx)) \\
&= \frac{1}{4}d^2x^4 (a + b \sec^{-1}(cx)) + \frac{1}{3}dex^6 (a + b \sec^{-1}(cx)) + \frac{1}{8}e^2x^8 (a + b \sec^{-1}(cx)) \\
&= -\frac{b(6c^4d^2 + 8c^2de + 3e^2)x\sqrt{-1 + c^2x^2}}{24c^7\sqrt{c^2x^2}} - \frac{b(6c^4d^2 + 16c^2de + 9e^2)x(-1 + c^2x^2)}{72c^7\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.29, size = 162, normalized size = 0.67

$$\frac{1}{24}ax^4(6d^2 + 8dex^2 + 3e^2x^4) - \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}(3c^6(70d^2x^2 + 56dex^4 + 15e^2x^6) + c^4(420d^2 + 224dex^2 + 54e^2x^4))}{2520c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out] (a\*x^4\*(6\*d^2 + 8\*d\*e\*x^2 + 3\*e^2\*x^4))/24 - (b\*sqrt[1 - 1/(c^2\*x^2)]\*x\*(14\*4\*e^2 + 8\*c^2\*e\*(56\*d + 9\*e\*x^2) + c^4\*(420\*d^2 + 224\*d\*e\*x^2 + 54\*e^2\*x^4) + 3\*c^6\*(70\*d^2\*x^2 + 56\*d\*e\*x^4 + 15\*e^2\*x^6)))/(2520\*c^7) + (b\*x^4\*(6\*d^2 + 8\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcSec[c\*x])/24

**fricas [A]** time = 0.98, size = 187, normalized size = 0.77

$$\frac{315ac^8e^2x^8 + 840ac^8dex^6 + 630ac^8d^2x^4 + 105(3bc^8e^2x^8 + 8bc^8dex^6 + 6bc^8d^2x^4)\operatorname{arcsec}(cx) - (45bc^6e^2x^6 + \dots)}{2520}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] 1/2520\*(315\*a\*c^8\*e^2\*x^8 + 840\*a\*c^8\*d\*e\*x^6 + 630\*a\*c^8\*d^2\*x^4 + 105\*(3\*b\*c^8\*e^2\*x^8 + 8\*b\*c^8\*d\*e\*x^6 + 6\*b\*c^8\*d^2\*x^4)\*arcsec(c\*x) - (45\*b\*c^6\*

$$e^{2x^6} + 420bc^4d^2 + 448b^2c^2de + 6(28b^2c^6de + 9b^2c^4e^2)x^4 + 144b^2e^2 + 2(105b^2c^6d^2 + 112b^2c^4de + 36b^2c^2e^2)x^2 \sqrt{c^2x^2 - 1} / c^8$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] Timed out

**maple** [A] time = 0.05, size = 214, normalized size = 0.88

$$\frac{a\left(\frac{1}{8}e^2c^8x^8 + \frac{1}{3}c^8edx^6 + \frac{1}{4}x^4c^8d^2\right)}{c^4} + \frac{b\left(\frac{\operatorname{arcsec}(cx)e^2c^8x^8}{8} + \frac{\operatorname{arcsec}(cx)c^8edx^6}{3} + \frac{\operatorname{arcsec}(cx)c^8x^4d^2}{4} - \frac{(c^2x^2-1)(45e^2c^6x^6+168c^6edx^4+210x^2c^6d^2+54c^4e^2x^4+224c^4dex^2+420c^4d^2)}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x)

[Out]  $\frac{1}{c^4} \left( \frac{a}{c^4} \left( \frac{1}{8}e^2c^8x^8 + \frac{1}{3}c^8edx^6 + \frac{1}{4}x^4c^8d^2 \right) + b \operatorname{arcsec}(cx) \left( \frac{1}{8}e^2c^8x^8 + \frac{1}{3}c^8edx^6 + \frac{1}{4}x^4c^8d^2 \right) + \frac{1}{2520} \frac{(c^2x^2-1)(45e^2c^6x^6+168c^6edx^4+210c^6d^2x^2+54c^4e^2x^4+224c^4dex^2+420c^4d^2)}{\sqrt{c^2x^2-1}} \right) / \left( \frac{c^2x^2-1}{c^2x^2} \right)^{1/2} / c/x$

**maxima** [A] time = 0.34, size = 256, normalized size = 1.06

$$\frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12} \left( 3x^4 \operatorname{arcsec}(cx) - \frac{c^2x^3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 3x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^3} \right) bd^2 + \frac{1}{45} \left( 15x^6 \operatorname{arcsec}(cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12} \left( 3x^4 \operatorname{arcsec}(cx) - \frac{c^2x^3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 3x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^3} \right) bd^2 + \frac{1}{45} \left( 15x^6 \operatorname{arcsec}(cx) - \frac{3c^4x^5 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} + 10c^2x^3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 15x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^5} \right) bde + \frac{1}{28}$

$0*(35*x^8*\operatorname{arcsec}(c*x) - (5*c^6*x^7*(-1/(c^2*x^2) + 1)^{(7/2)} + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^{(5/2)} + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^{(3/2)} + 35*x*\sqrt{-1/(c^2*x^2) + 1})/c^7)*b*e^2$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (ex^2 + d)^2 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)`

[Out] `int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)`

**sympy [A]** time = 9.01, size = 493, normalized size = 2.04

$$\frac{ad^2x^4}{4} + \frac{adx^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4 \operatorname{asec}(cx)}{4} + \frac{bdex^6 \operatorname{asec}(cx)}{3} + \frac{be^2x^8 \operatorname{asec}(cx)}{8} - \frac{bd^2 \begin{cases} \frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for} \\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{oth} \end{cases}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x**2+d)**2*(a+b*asec(c*x)), x)`

[Out] `a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asec(c*x)/4 + b*d*e*x**6*asec(c*x)/3 + b*e**2*x**8*asec(c*x)/8 - b*d**2*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c) - b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) - b*e**2*Piecewise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)`

### 3.88 $\int x (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=195

$$\frac{(d + ex^2)^3 (a + b \sec^{-1}(cx))}{6e} - \frac{bcd^3 x \tan^{-1}(\sqrt{c^2 x^2 - 1})}{6e\sqrt{c^2 x^2}} - \frac{bex (c^2 x^2 - 1)^{3/2} (3c^2 d + 2e)}{18c^5 \sqrt{c^2 x^2}} - \frac{be^2 x (c^2 x^2 - 1)^{5/2}}{30c^5 \sqrt{c^2 x^2}} - \frac{bx\sqrt{c^2 x^2}}{18c^5 \sqrt{c^2 x^2}}$$

[Out]  $\frac{1}{6} * (e * x^2 + d)^3 * (a + b * \text{arcsec}(c * x)) / e - \frac{1}{18} * b * e * (3 * c^2 * d + 2 * e) * x * (c^2 * x^2 - 1)^{(3/2)} / c^5 / (c^2 * x^2)^{(1/2)} - \frac{1}{30} * b * e^2 * x * (c^2 * x^2 - 1)^{(5/2)} / c^5 / (c^2 * x^2)^{(1/2)} - \frac{1}{6} * b * c * d^3 * x * \arctan((c^2 * x^2 - 1)^{(1/2)}) / e / (c^2 * x^2)^{(1/2)} - \frac{1}{6} * b * (3 * c^4 * d^2 + 3 * c^2 * d * e + e^2) * x * (c^2 * x^2 - 1)^{(1/2)} / c^5 / (c^2 * x^2)^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5236, 446, 88, 63, 205}

$$\frac{(d + ex^2)^3 (a + b \sec^{-1}(cx))}{6e} - \frac{bx\sqrt{c^2 x^2 - 1} (3c^4 d^2 + 3c^2 de + e^2)}{6c^5 \sqrt{c^2 x^2}} - \frac{bcd^3 x \tan^{-1}(\sqrt{c^2 x^2 - 1})}{6e\sqrt{c^2 x^2}} - \frac{bex (c^2 x^2 - 1)^{3/2} (3c^2 d + 2e)}{18c^5 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]),x]

[Out]  $-(b * (3 * c^4 * d^2 + 3 * c^2 * d * e + e^2) * x * \text{Sqrt}[-1 + c^2 * x^2]) / (6 * c^5 * \text{Sqrt}[c^2 * x^2]) - (b * e * (3 * c^2 * d + 2 * e) * x * (-1 + c^2 * x^2)^{(3/2)}) / (18 * c^5 * \text{Sqrt}[c^2 * x^2]) - (b * e^2 * x * (-1 + c^2 * x^2)^{(5/2)}) / (30 * c^5 * \text{Sqrt}[c^2 * x^2]) + ((d + e * x^2)^3 * (a + b * \text{ArcSec}[c * x])) / (6 * e) - (b * c * d^3 * x * \text{ArcTan}[\text{Sqrt}[-1 + c^2 * x^2]]) / (6 * e * \text{Sqrt}[c^2 * x^2])$

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 88

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 5236

Int[((a\_) + ArcSec[(c\_)\*(x\_)])\*(b\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSec[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int x (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx &= \frac{(d + ex^2)^3 (a + b \sec^{-1}(cx))}{6e} - \frac{(bcx) \int \frac{(d+ex^2)^3}{x\sqrt{-1+c^2x^2}} dx}{6e\sqrt{c^2x^2}} \\
 &= \frac{(d + ex^2)^3 (a + b \sec^{-1}(cx))}{6e} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{(d+ex)^3}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{12e\sqrt{c^2x^2}} \\
 &= \frac{(d + ex^2)^3 (a + b \sec^{-1}(cx))}{6e} - \frac{(bcx) \operatorname{Subst}\left(\int \left(\frac{e(3c^4d^2+3c^2de+e^2)}{c^4\sqrt{-1+c^2x}} + \frac{d^3}{x\sqrt{-1+c^2x}}\right) dx, x, x^2\right)}{12e\sqrt{c^2x^2}} \\
 &= -\frac{b(3c^4d^2 + 3c^2de + e^2)x\sqrt{-1 + c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d + 2e)x(-1 + c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} \\
 &= -\frac{b(3c^4d^2 + 3c^2de + e^2)x\sqrt{-1 + c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d + 2e)x(-1 + c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} \\
 &= -\frac{b(3c^4d^2 + 3c^2de + e^2)x\sqrt{-1 + c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d + 2e)x(-1 + c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 125, normalized size = 0.64

$$\frac{1}{90}x \left( 15ax(3d^2 + 3dex^2 + e^2x^4) - \frac{b\sqrt{1 - \frac{1}{c^2x^2}}(3c^4(15d^2 + 5dex^2 + e^2x^4) + 2c^2e(15d + 2ex^2) + 8e^2)}{c^5} + 15bx \operatorname{sec}^{-1}\left(\frac{cx}{c^2x^2 - 1}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out] (x\*(15\*a\*x\*(3\*d^2 + 3\*d\*e\*x^2 + e^2\*x^4) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*(8\*e^2 + 2\*c^2\*e\*(15\*d + 2\*e\*x^2) + 3\*c^4\*(15\*d^2 + 5\*d\*e\*x^2 + e^2\*x^4))))/c^5 + 15\*b\*x\*(3\*d^2 + 3\*d\*e\*x^2 + e^2\*x^4)\*ArcSec[c\*x])/90

**fricas [A]** time = 1.44, size = 153, normalized size = 0.78

$$\frac{15 ac^6 e^2 x^6 + 45 ac^6 dex^4 + 45 ac^6 d^2 x^2 + 15 (bc^6 e^2 x^6 + 3 bc^6 dex^4 + 3 bc^6 d^2 x^2) \operatorname{arcsec}(cx) - (3 bc^4 e^2 x^4 + 45 bc^4 d^2 x^2)}{90 c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] 1/90\*(15\*a\*c^6\*e^2\*x^6 + 45\*a\*c^6\*d\*e\*x^4 + 45\*a\*c^6\*d^2\*x^2 + 15\*(b\*c^6\*e^2\*x^6 + 3\*b\*c^6\*d\*e\*x^4 + 3\*b\*c^6\*d^2\*x^2)\*arcsec(c\*x) - (3\*b\*c^4\*e^2\*x^4 + 45\*b\*c^4\*d^2 + 30\*b\*c^2\*d\*e + 8\*b\*e^2 + (15\*b\*c^4\*d\*e + 4\*b\*c^2\*e^2)\*x^2)\*sqrt(c^2\*x^2 - 1))/c^6

**giac [B]** time = 0.41, size = 11858, normalized size = 60.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)), x, algorithm="giac")

[Out] 1/90\*(45\*b\*c^4\*d^2\*arccos(1/(c\*x))/(c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12) + 45\*a\*c^4\*d^2/(c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12) + 90\*b\*c^4\*d^2\*(1/(c^2\*x^2) - 1)\*arccos(1/(c\*x))/((c^7 + 6\*c^7\*(1/(c^2\*x^2) - 1)/(1/(c\*x) + 1)^2 + 15\*c^7\*(1/(c^2\*x^2) - 1)^2/(1/(c\*x) + 1)^4 + 20\*c^7\*(1/(c^2\*x^2) - 1)^3/(1/(c\*x) + 1)^6 + 15\*c^7\*(1/(c^2\*x^2) - 1)^4/(1/(c\*x) + 1)^8 + 6\*c^7\*(1/(c^2\*x^2) - 1)^5/(1/(c\*x) + 1)^10 + c^7\*(1/(c^2\*x^2) - 1)^6/(1/(c\*x) + 1)^12))



$$\begin{aligned}
& *x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + \\
& 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c \\
& *x) + 1)^{12}*(1/(c*x) + 1)^2) - 90*b*c^4*d^2*\sqrt{-1/(c^2*x^2) + 1}/((c^7 + \\
& 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c \\
& *x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^ \\
& 2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^ \\
& 7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)) + 90*a*c^4*d^2*(1/(c \\
& ^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c \\
& ^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 \\
& + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/( \\
& 1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^2 \\
& ) - 45*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*\arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2* \\
& x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20 \\
& *c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c \\
& *x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) \\
& - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^4) + 45*b*c^2*d*\arccos(1/(c*x))*e/(c \\
& ^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/( \\
& 1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^ \\
& 2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} \\
& + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}) + 450*b*c^4*d^2*(-1/(c^2*x^2) + \\
& 1)^{(3/2)}/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2* \\
& x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + \\
& 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/( \\
& c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^3) - \\
& 45*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) \\
& + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - \\
& 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7* \\
& (1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1 \\
& )^{12}*(1/(c*x) + 1)^4) - 180*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*\arccos(1/(c*x))/ \\
& ((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^ \\
& 2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/ \\
& (c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^ \\
& 10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^6) + 45*a*c^2* \\
& d*e/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - \\
& 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7* \\
& (1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + \\
& 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}) - 90*b*c^2*d*(1/(c^2*x^2) \\
& - 1)*\arccos(1/(c*x))*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 1 \\
& 5*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/( \\
& c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^ \\
& 2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/( \\
& c*x) + 1)^2) - 900*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*\sqrt{-1/(c^2*x^2) + 1}/((c \\
& ^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/( \\
& 1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^ \\
& 2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10}
\end{aligned}$$

$$\begin{aligned}
& + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^5 - 90*b*c^2*d*s \\
& \text{qrt}(-1/(c^2*x^2) + 1)*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 1 \\
& 5*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/( \\
& c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^ \\
& 2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/( \\
& c*x) + 1)) - 180*a*c^4*d^2*(1/(c^2*x^2) - 1)^3/((c^7 + 6*c^7*(1/(c^2*x^2) - \\
& 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*( \\
& 1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + \\
& 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6 \\
& /((1/(c*x) + 1)^{12}*(1/(c*x) + 1)^6) - 45*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*arcc \\
& \text{os}(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^ \\
& 2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 \\
& + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/( \\
& 1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^8) \\
& - 90*a*c^2*d*(1/(c^2*x^2) - 1)*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) \\
& + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - \\
& 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7* \\
& (1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1 \\
& )^{12}*(1/(c*x) + 1)^2) - 45*b*c^2*d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))*e/ \\
& ((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2 \\
& /((1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/( \\
& c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{1 \\
& 0} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^4) - 900*b*c^4* \\
& d^2*(1/(c^2*x^2) - 1)^3*\text{sqrt}(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - \\
& 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*( \\
& 1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + \\
& 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6 \\
& /((1/(c*x) + 1)^{12}*(1/(c*x) + 1)^7) + 330*b*c^2*d*(-1/(c^2*x^2) + 1)^{(3/2)}* \\
& e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1 \\
& )^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*( \\
& 1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1 \\
& )^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^3) - 45*a*c^ \\
& 4*d^2*(1/(c^2*x^2) - 1)^4/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + \\
& 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1 \\
& /((c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2* \\
& x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1 \\
& /((c*x) + 1)^8) + 90*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6 \\
& *c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x \\
& ) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) \\
& - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7* \\
& (1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{10} + 15*b*arccos(1/(c* \\
& x))*e^2/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2 \\
& ) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15* \\
& c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x \\
& ) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}) - 45*a*c^2*d*(1/(c^2*
\end{aligned}$$



$$\begin{aligned}
& )/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/ \\
& (c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& ^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/( \\
& 1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{11} - 30*b*sqrt(-1/(c^2*x^2) + 1)*e^2/((c^7 \\
& + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/( \\
& c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x \\
& ^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c \\
& ^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)) - 540*b*c^2*d*(1/(c \\
& ^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/ \\
& (c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2* \\
& x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + \\
& 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c* \\
& x) + 1)^{12}*(1/(c*x) + 1)^7) + 45*a*c^4*d^2*(1/(c^2*x^2) - 1)^6/((c^7 + 6*c \\
& ^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) \\
& + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - \\
& 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1 \\
& /(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{12} - 90*a*(1/(c^2*x^2) - \\
& 1)*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^ \\
& 2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15 \\
& *c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c* \\
& x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^2) + 2 \\
& 25*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1) \\
& )/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/ \\
& (c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1) \\
& ^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/( \\
& 1/(c*x) + 1)^{12}*(1/(c*x) + 1)^4) - 45*a*c^2*d*(1/(c^2*x^2) - 1)^4*e/((c^7 \\
& + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/( \\
& c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x \\
& ^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c \\
& ^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^8) - 90*b*c^2*d*(1/( \\
& c^2*x^2) - 1)^5*arccos(1/(c*x))*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) \\
& + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - \\
& 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7* \\
& (1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1 \\
& )^{12}*(1/(c*x) + 1)^{10} + 70*b*(-1/(c^2*x^2) + 1)^{(3/2)}*e^2/((c^7 + 6*c^7*( \\
& 1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1) \\
& ^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^ \\
& 4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^ \\
& 2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^3) - 330*b*c^2*d*(1/(c^2*x^2) \\
& - 1)^4*sqrt(-1/(c^2*x^2) + 1)*e/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + \\
& 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - \\
& 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*( \\
& 1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1) \\
& ^{12}*(1/(c*x) + 1)^9) + 225*a*(1/(c^2*x^2) - 1)^2*e^2/((c^7 + 6*c^7*(1/(c^2 \\
& *x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 2
\end{aligned}$$



$x) + 1)^6 + 15c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^8 - 90*b*(1/(c^2*x^2) - 1)^5*\arccos(1/(c*x))*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{10}) - 70*b*(1/(c^2*x^2) - 1)^4*\sqrt{-1/(c^2*x^2) + 1}*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{10}) - 90*a*(1/(c^2*x^2) - 1)^5*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{10}) + 15*b*(1/(c^2*x^2) - 1)^6*\arccos(1/(c*x))*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{12}) - 30*b*(1/(c^2*x^2) - 1)^5*\sqrt{-1/(c^2*x^2) + 1}*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{11}) + 15*a*(1/(c^2*x^2) - 1)^6*e^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^{10} + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^{12}*(1/(c*x) + 1)^{12}))*c$

**maple [A]** time = 0.05, size = 182, normalized size = 0.93

$$\frac{a\left(\frac{1}{6}e^2c^6x^6 + \frac{1}{2}c^6edx^4 + \frac{1}{2}x^2c^6d^2\right)}{c^4} + \frac{b\left(\frac{\arccos(cx)e^2c^6x^6}{6} + \frac{\arccos(cx)c^6x^4de}{2} + \frac{\arccos(cx)c^6x^2d^2}{2} - \frac{(c^2x^2-1)(3c^4e^2x^4+15c^4dex^2+45d^2c^4+4c^2e^2x^2+30c^2ed+8e^2)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}cx}\right)}{c^4}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)`

[Out]  $1/c^2*(a/c^4*(1/6*e^2*c^6*x^6+1/2*c^6*e*d*x^4+1/2*x^2*c^6*d^2)+b/c^4*(1/6*a*\arccos(c*x)*e^2*c^6*x^6+1/2*\arccos(c*x)*c^6*x^4*d*e+1/2*\arccos(c*x)*c^6*x^2*d^2-1/90*(c^2*x^2-1)*(3*c^4*e^2*x^4+15*c^4*d*e*x^2+45*c^4*d^2+4*c^2*e^2*x^2+30*c^2*d*e+8*e^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)$

**maxima** [A] time = 0.33, size = 192, normalized size = 0.98

$$\frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2 \operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c}\right)bd^2 + \frac{1}{6}\left(3x^4 \operatorname{arcsec}(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 3x}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/6\*a\*e^2\*x^6 + 1/2\*a\*d\*e\*x^4 + 1/2\*a\*d^2\*x^2 + 1/2\*(x^2\*arcsec(c\*x) - x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*b\*d^2 + 1/6\*(3\*x^4\*arcsec(c\*x) - (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*d\*e + 1/90\*(15\*x^6\*arcsec(c\*x) - (3\*c^4\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 10\*c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^5)\*b\*e^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x(e x^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{c x}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x\*(d + e\*x^2)^2\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 6.16, size = 352, normalized size = 1.81

$$\frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2 \operatorname{asec}(cx)}{2} + \frac{bdex^4 \operatorname{asec}(cx)}{2} + \frac{be^2x^6 \operatorname{asec}(cx)}{6} - \frac{bd^2 \left\{ \begin{array}{ll} \frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2| > 1 \\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{array} \right\}}{2c} + b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x)),x)

[Out] a\*d\*\*2\*x\*\*2/2 + a\*d\*e\*x\*\*4/2 + a\*e\*\*2\*x\*\*6/6 + b\*d\*\*2\*x\*\*2\*asec(c\*x)/2 + b\*d\*e\*x\*\*4\*asec(c\*x)/2 + b\*e\*\*2\*x\*\*6\*asec(c\*x)/6 - b\*d\*\*2\*Piecewise((sqrt(c\*\*2\*x\*\*2 - 1)/c, Abs(c\*\*2\*x\*\*2) > 1), (I\*sqrt(-c\*\*2\*x\*\*2 + 1)/c, True))/(2\*c) - b\*d\*e\*Piecewise((x\*\*2\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*c) + 2\*sqrt(c\*\*2\*x\*\*2 - 1)/(3\*c\*\*3), Abs(c\*\*2\*x\*\*2) > 1), (I\*x\*\*2\*sqrt(-c\*\*2\*x\*\*2 + 1)/(3\*c) + 2\*I\*sqrt

```

t(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) - b*e**2*Piecewise((x**4*sqrt(c**2
*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2
- 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4
*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5)
, True))/(6*c)

```



$$3.89 \quad \int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x} dx$$

Optimal. Leaf size=186

$$-d^2 \log\left(\frac{1}{x}\right) (a + b \sec^{-1}(cx)) + dex^2 (a + b \sec^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + b \sec^{-1}(cx)) - \frac{be^2 x^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{12c} - \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}}}{6c^3}$$

[Out]  $-1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arcsec(c*x))+1/4*e^2*x^4*(a+b*arcsec(c*x))+b*d^2*arccsc(c*x)*\ln(1-(I/c/x+(1-1/c^2/x^2)^{(1/2)})^2)-b*d^2*arccsc(c*x)*\ln(1/x)-d^2*(a+b*arcsec(c*x))*\ln(1/x)-1/2*I*b*d^2*polylog(2,(I/c/x+(1-1/c^2/x^2)^{(1/2)})^2)-1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^{(1/2)}/c^3-1/12*b*e^2*x^3*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.41, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {5240, 266, 43, 4732, 6742, 453, 264, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{1}{2} i b d^2 \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) - d^2 \log\left(\frac{1}{x}\right) (a + b \sec^{-1}(cx)) + dex^2 (a + b \sec^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + b \sec^{-1}(cx)) - \frac{be^2 x^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{12c} - \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}}}{6c^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x,x]

[Out]  $-(b*e*(6*c^2*d + e)*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(6*c^3) - (b*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3)/(12*c) - (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcSec[c*x]) + (e^2*x^4*(a + b*ArcSec[c*x]))/4 + b*d^2*ArcCsc[c*x]*\text{Log}[1 - E^{((2*I)*ArcCsc[c*x])}] - b*d^2*ArcCsc[c*x]*\text{Log}[x^{(-1)}] - d^2*(a + b*ArcSec[c*x])*\text{Log}[x^{(-1)}] - (I/2)*b*d^2*\text{PolyLog}[2, E^{((2*I)*ArcCsc[c*x])}]$

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symb
ol] := Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

$m \cdot E^{(2 \cdot I \cdot k \cdot \pi)} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))} / (1 + E^{(2 \cdot I \cdot k \cdot \pi)} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))})$ , x],  
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 4732

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCos[c\*x], u, x] + Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2\*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2\*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

#### Rule 5240

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcCos[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x} dx &= -\text{Subst} \left( \int \frac{(e+dx^2)^2 (a+b \cos^{-1}(\frac{x}{c}))}{x^5} dx, x, \frac{1}{x} \right) \\
&= dex^2 (a+b \sec^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a+b \sec^{-1}(cx)) - d^2 (a+b \sec^{-1}(cx)) \log \left( \right) \\
&= dex^2 (a+b \sec^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a+b \sec^{-1}(cx)) - d^2 (a+b \sec^{-1}(cx)) \log \left( \right) \\
&= dex^2 (a+b \sec^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a+b \sec^{-1}(cx)) - d^2 (a+b \sec^{-1}(cx)) \log \left( \right) \\
&= -\frac{be^2 \sqrt{1-\frac{1}{c^2 x^2}} x^3}{12c} + dex^2 (a+b \sec^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a+b \sec^{-1}(cx)) - bd^2 c \\
&= -\frac{be(6c^2 d+e) \sqrt{1-\frac{1}{c^2 x^2}} x}{6c^3} - \frac{be^2 \sqrt{1-\frac{1}{c^2 x^2}} x^3}{12c} + dex^2 (a+b \sec^{-1}(cx)) + \frac{1}{4} e^2 \\
&= -\frac{be(6c^2 d+e) \sqrt{1-\frac{1}{c^2 x^2}} x}{6c^3} - \frac{be^2 \sqrt{1-\frac{1}{c^2 x^2}} x^3}{12c} - \frac{1}{2} ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+ \\
&= -\frac{be(6c^2 d+e) \sqrt{1-\frac{1}{c^2 x^2}} x}{6c^3} - \frac{be^2 \sqrt{1-\frac{1}{c^2 x^2}} x^3}{12c} - \frac{1}{2} ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+ \\
&= -\frac{be(6c^2 d+e) \sqrt{1-\frac{1}{c^2 x^2}} x}{6c^3} - \frac{be^2 \sqrt{1-\frac{1}{c^2 x^2}} x^3}{12c} - \frac{1}{2} ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+ \\
&= -\frac{be(6c^2 d+e) \sqrt{1-\frac{1}{c^2 x^2}} x}{6c^3} - \frac{be^2 \sqrt{1-\frac{1}{c^2 x^2}} x^3}{12c} - \frac{1}{2} ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+
\end{aligned}$$

**Mathematica [A]** time = 0.39, size = 160, normalized size = 0.86

$$ad^2 \log(x) + adex^2 + \frac{1}{4}ae^2x^4 + \frac{bdex \left( cx \sec^{-1}(cx) - \sqrt{1 - \frac{1}{c^2x^2}} \right)}{c} - \frac{be^2x \sqrt{1 - \frac{1}{c^2x^2}} (c^2x^2 + 2)}{12c^3} + \frac{1}{2}ibd^2 \left( \text{Li}_2 \left( -e^{2i \sec^{-1}(cx)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x,x]

[Out] a\*d\*e\*x^2 + (a\*e^2\*x^4)/4 - (b\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(2 + c^2\*x^2))/(12\*c^3) + (b\*e^2\*x^4\*ArcSec[c\*x])/4 + (b\*d\*e\*x\*(-Sqrt[1 - 1/(c^2\*x^2)] + c\*x\*ArcSec[c\*x]))/c + a\*d^2\*Log[x] + (I/2)\*b\*d^2\*(ArcSec[c\*x]\*(ArcSec[c\*x] + (2\*I)\*Log[1 + E^((2\*I)\*ArcSec[c\*x])]) + PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])])

**fricas [F]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \text{arcsec}(cx)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x,x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arcsec(c\*x))/x, x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]Undef/Unsigne d Inf encountered in limitEvaluation time: 0.95Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple [A]** time = 3.52, size = 242, normalized size = 1.30

$$\frac{ax^4e^2}{4} + aedx^2 + ad^2 \ln(cx) + \frac{ib d^2 \text{arcsec}(cx)^2}{2} + \frac{b \text{arcsec}(cx) x^4 e^2}{4} + be \text{arcsec}(cx) x^2 d - \frac{b \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x^3 e^2}{12c} - \frac{b \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x)`

[Out]  $\frac{1}{4}ax^4e^2+ade^2x^2+ad^2\ln(cx)+\frac{1}{2}Ibd^2\operatorname{arcsec}(cx)^2+\frac{1}{4}b\operatorname{arcsec}(cx)x^4e^2+b\operatorname{arcsec}(cx)x^2d-\frac{1}{12}b/c\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}x^3e^2-b/c\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}x*d-e-Ib/c^2*d*e-\frac{1}{6}b/c^3\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}x*e^2-\frac{1}{6}Ib/c^4e^2-b*d^2\operatorname{arcsec}(cx)*\ln\left(1+\frac{1}{c/x+I(1-1/c^2/x^2)^{1/2}}\right)^2+\frac{1}{2}Ibd^2\operatorname{polylog}\left(2,-\frac{1}{c/x+I(1-1/c^2/x^2)^{1/2}}\right)^2)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}ae^2x^4+adex^2+ad^2\log(x)-\frac{-2ibc^4e^2x^4\log(c)-4ibc^4d^2\log(-cx+1)\log(x)-4ibc^4d^2\log(x)^2-4ibc^4d^2\operatorname{Li}_2(cx)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="maxima")`

[Out]  $\frac{1}{4}ae^2x^4 + ade^2x^2 + ad^2\log(x) - \frac{1}{8}(-2Ib^2c^4e^2x^4\log(c) - 4Ib^2c^4d^2\log(-cx+1)\log(x) - 4Ib^2c^4d^2\log(x)^2 - 4Ib^2c^4d^2\operatorname{dilog}(cx) - 4Ib^2c^4d^2\operatorname{dilog}(-cx) + I(b^2e^2(x^2/c^2 + \log(cx+1)/c^4 + \log(cx-1)/c^4) + 4b^2d^2e(\log(cx+1)/c^2 + \log(cx-1)/c^2) + 32b^2d^2\int(1/4\log(x)/(c^2x^3-x), x))c^4 + 8c^4\int(1/4(b^2e^2x^4 + 4b^2d^2e^2x^2 + 4b^2d^2\log(x))*\sqrt{cx+1}\sqrt{cx-1}/(c^2x^3-x), x) + (-8Ib^2c^4d^2e\log(c) - Ib^2c^2e^2)x^2 - 2(b^2c^4e^2x^4 + 4b^2c^4d^2e^2x^2 + 4b^2c^4d^2\log(x))*\arctan(\sqrt{cx+1}\sqrt{cx-1})) + (Ib^2c^4e^2x^4 + 4Ib^2c^4d^2e^2x^2 + 4Ib^2c^4d^2\log(x))*\log(c^2x^2) + (-4Ib^2c^4d^2\log(x) - 4Ib^2c^2d^2e - Ib^2e^2)*\log(cx+1) + (-4Ib^2c^2d^2e - Ib^2e^2)*\log(cx-1) + (-2Ib^2c^4e^2x^4 - 8Ib^2c^4d^2e^2x^2 - 8Ib^2c^4d^2\log(c))*\log(x))/c^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2+d)^2 \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x,x)`

[Out] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx))(d + ex^2)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x))/x,x)

[Out] Integral((a + b\*asec(c\*x))\*(d + e\*x\*\*2)\*\*2/x, x)

$$3.90 \quad \int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=189

$$-\frac{d^2 (a + b \sec^{-1}(cx))}{2x^2} - 2de \log\left(\frac{1}{x}\right) (a + b \sec^{-1}(cx)) + \frac{1}{2}e^2x^2 (a + b \sec^{-1}(cx)) + \frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d^2 \csc^{-1}(cx)$$

[Out]  $-1/4*b*c^2*d^2*arccsc(c*x) - I*b*d*e*arccsc(c*x)^2 - 1/2*d^2*(a+b*arcsec(c*x))/x^2 + 1/2*e^2*x^2*(a+b*arcsec(c*x)) + 2*b*d*e*arccsc(c*x)*\ln(1-(I/c/x+(1-1/c^2/x^2)^{(1/2)})^2) - 2*b*d*e*arccsc(c*x)*\ln(1/x) - 2*d*e*(a+b*arcsec(c*x))*\ln(1/x) - I*b*d*e*polylog(2, (I/c/x+(1-1/c^2/x^2)^{(1/2)})^2) + 1/4*b*c*d^2*(1-1/c^2/x^2)^{(1/2)}/x - 1/2*b*e^2*x*(1-1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.42, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5240, 266, 43, 4732, 12, 6742, 264, 321, 216, 2326, 4625, 3717, 2190, 2279, 2391}

$$-ibde \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) - \frac{d^2 (a + b \sec^{-1}(cx))}{2x^2} - 2de \log\left(\frac{1}{x}\right) (a + b \sec^{-1}(cx)) + \frac{1}{2}e^2x^2 (a + b \sec^{-1}(cx)) + \frac{bcd^2}{4x}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out]  $(b*c*d^2*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) - (b*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(2*c) - (b*c^2*d^2*\text{ArcCsc}[c*x])/4 - I*b*d*e*\text{ArcCsc}[c*x]^2 - (d^2*(a + b*\text{ArcSec}[c*x]))/(2*x^2) + (e^2*x^2*(a + b*\text{ArcSec}[c*x]))/2 + 2*b*d*e*\text{ArcCsc}[c*x]*\text{Log}[1 - E^((2*I)*\text{ArcCsc}[c*x])] - 2*b*d*e*\text{ArcCsc}[c*x]*\text{Log}[x^{(-1)}] - 2*d*e*(a + b*\text{ArcSec}[c*x])*\text{Log}[x^{(-1)}] - I*b*d*e*\text{PolyLog}[2, E^((2*I)*\text{ArcCsc}[c*x])]$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])



Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n]-1)\*(a+b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c+d\*x)^m\*Log[1+(b\*(F^(g\*(e+f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*Log[1+(b\*(F^(g\*(e+f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a+b\*x]/x, x], x, (F^(e\*(c+d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2326

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(ArcSin[(Rt[-e, 2]\*x)/Sqrt[d]]\*(a+b\*Log[c\*x^n])/Rt[-e, 2], x] - Dist[(b\*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]\*x)/Sqrt[d]]/x, x], x] /; Fr

eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m \* E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

### Rule 4732

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCos[c\*x], u, x] + Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2\*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2\*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

### Rule 5240

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcCos[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2 (a+b \sec^{-1}(cx))}{x^3} dx &= -\text{Subst} \left( \int \frac{(e+dx^2)^2 (a+b \cos^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x} \right) \\
&= -\frac{d^2 (a+b \sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a+b \sec^{-1}(cx)) - 2de (a+b \sec^{-1}(cx)) \log \\
&= -\frac{d^2 (a+b \sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a+b \sec^{-1}(cx)) - 2de (a+b \sec^{-1}(cx)) \log \\
&= -\frac{d^2 (a+b \sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a+b \sec^{-1}(cx)) - 2de (a+b \sec^{-1}(cx)) \log \\
&= -\frac{d^2 (a+b \sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a+b \sec^{-1}(cx)) - 2de (a+b \sec^{-1}(cx)) \log \\
&= \frac{bcd^2 \sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x}{2c} - \frac{d^2 (a+b \sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a+b \sec^{-1}(cx)) \\
&= \frac{bcd^2 \sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x}{2c} - \frac{1}{4}bc^2d^2 \csc^{-1}(cx) - \frac{d^2 (a+b \sec^{-1}(cx))}{2x^2} \\
&= \frac{bcd^2 \sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x}{2c} - \frac{1}{4}bc^2d^2 \csc^{-1}(cx) - ibde \csc^{-1}(cx)^2 - \frac{d^2}{2x^2} \\
&= \frac{bcd^2 \sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x}{2c} - \frac{1}{4}bc^2d^2 \csc^{-1}(cx) - ibde \csc^{-1}(cx)^2 - \frac{d^2}{2x^2} \\
&= \frac{bcd^2 \sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x}{2c} - \frac{1}{4}bc^2d^2 \csc^{-1}(cx) - ibde \csc^{-1}(cx)^2 - \frac{d^2}{2x^2} \\
&= \frac{bcd^2 \sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x}{2c} - \frac{1}{4}bc^2d^2 \csc^{-1}(cx) - ibde \csc^{-1}(cx)^2 - \frac{d^2}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.66, size = 187, normalized size = 0.99

$$\frac{1}{4} \left( -\frac{2ad^2}{x^2} + 8ade \log(x) + 2ae^2x^2 + \frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}} \left( \frac{c^2x^2 \tanh^{-1}\left(\sqrt{1 - c^2x^2}\right)}{\sqrt{1 - c^2x^2}} + 1 \right)}{x} + \frac{2be^2x \left( cx \sec^{-1}(cx) - \sqrt{1 - \frac{1}{c^2x^2}} \right)}{c} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out] ((-2\*a\*d^2)/x^2 + 2\*a\*e^2\*x^2 - (2\*b\*d^2\*ArcSec[c\*x])/x^2 + (2\*b\*e^2\*x\*(-Sqrt[1 - 1/(c^2\*x^2)] + c\*x\*ArcSec[c\*x])/c + (b\*c\*d^2\*Sqrt[1 - 1/(c^2\*x^2)]\*(1 + (c^2\*x^2\*ArcTanh[Sqrt[1 - c^2\*x^2]])/Sqrt[1 - c^2\*x^2]))/x + 8\*a\*d\*e\*Log[x] + (4\*I)\*b\*d\*e\*(ArcSec[c\*x]\*(ArcSec[c\*x] + (2\*I)\*Log[1 + E^((2\*I)\*ArcSec[c\*x])])) + PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]))/4

**fricas [F]** time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \operatorname{arcsec}(cx)}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^3, x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arcsec(c\*x))/x^3, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsec}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arcsec(c\*x))/x^3, x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arcsec(c\*x) + a)/x^3, x)

**maple [A]** time = 2.88, size = 218, normalized size = 1.15

$$\frac{ax^2e^2}{2} + 2aed \ln(cx) - \frac{ad^2}{2x^2} + ibed \operatorname{arcsec}(cx)^2 + \frac{cb d^2 \sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x} + \frac{c^2b d^2 \operatorname{arcsec}(cx)}{4} - \frac{b \operatorname{arcsec}(cx) d^2}{2x^2} + \frac{b \operatorname{arcsec}(cx) x^2 e^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x)`

[Out]  $\frac{1}{2} a e^2 x^2 + 2 a e d \ln(c x) - \frac{1}{2} a d^2 / x^2 + I b e d \operatorname{arcsec}(c x)^2 + \frac{1}{4} c b d^2 / x^2 \left( \frac{c^2 x^2 - 1}{c^2 / x^2} \right)^{1/2} + \frac{1}{4} c^2 b d^2 \operatorname{arcsec}(c x) - \frac{1}{2} b \operatorname{arcsec}(c x) d^2 / x^2 + \frac{1}{2} b \operatorname{arcsec}(c x) x^2 e^{-1/2} / c b \left( \frac{c^2 x^2 - 1}{c^2 / x^2} \right)^{1/2} x e^{-2} - \frac{1}{2} I / c^2 b e^{-2} - 2 b e d \operatorname{arcsec}(c x) \ln\left(1 + \left(\frac{1}{c/x} + I \left(\frac{c^2 x^2 - 1}{c^2 / x^2}\right)^{1/2}\right)^2\right) + I b e d \operatorname{polylog}\left(2, -\left(\frac{1}{c/x} + I \left(\frac{c^2 x^2 - 1}{c^2 / x^2}\right)^{1/2}\right)^2\right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a e^2 x^2 - \frac{1}{4} b d^2 \left( \frac{c^4 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^2 x^2 \left(\frac{1}{c^2 x^2} - 1\right) - 1} - c^3 \arctan\left(c x \sqrt{-\frac{1}{c^2 x^2} + 1}\right) \right) + \frac{2 \operatorname{arcsec}(c x)}{x^2} + 2 a d e \log(x) - \frac{a d^2}{2 x^2} - \frac{-2 i b c^2 e^2 x^2 \log}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} a e^2 x^2 - \frac{1}{4} b d^2 \left( \frac{c^4 x \sqrt{-1/(c^2 x^2) + 1}}{c^2 x^2 (1/(c^2 x^2) - 1) - 1} - c^3 \arctan(c x \sqrt{-1/(c^2 x^2) + 1}) \right) / c + 2 \operatorname{arcsec}(c x) / x^2 + 2 a d e \log(x) - \frac{1}{2} a d^2 / x^2 - \frac{1}{4} (-2 I b c^2 e^2 x^2 \log(c) - 4 I b c^2 d e \log(-c x + 1) \log(x) - 4 I b c^2 d e \log(x)^2 - 4 I b c^2 d e \operatorname{dilog}(c x) - 4 I b c^2 d e \operatorname{dilog}(-c x) - I b e^2 \log(c x - 1) + I (b e^2 (\log(c x + 1) / c^2 + \log(c x - 1) / c^2) + 16 b d e \operatorname{integrate}(1/2 \log(x) / (c^2 x^3 - x), x)) c^2 + 4 c^2 \operatorname{integrate}(1/2 (b e^2 x^2 + 4 b d e \log(x)) \sqrt{c x + 1} \sqrt{c x - 1} / (c^2 x^3 - x), x) - 2 (b c^2 e^2 x^2 + 4 b c^2 d e \log(x)) \arctan(\sqrt{c x + 1} \sqrt{c x - 1}) + (I b c^2 e^2 x^2 + 4 I b c^2 d e \log(x)) \log(c^2 x^2) + (-4 I b c^2 d e \log(x) - I b e^2) \log(c x + 1) + (-2 I b c^2 e^2 x^2 - 8 I b c^2 d e \log(c)) \log(x)) / c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e x^2 + d)^2 \left( a + b \operatorname{acos}\left(\frac{1}{c x}\right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3,x)`

[Out] `int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*asec(c\*x))/x\*\*3,x)

[Out] Integral((a + b\*asec(c\*x))\*(d + e\*x\*\*2)\*\*2/x\*\*3, x)

$$3.91 \quad \int \frac{x^2(a+b \sec^{-1}(cx))}{d+ex^2} dx$$

**Optimal.** Leaf size=546

$$\frac{\sqrt{-d} (a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d} (a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx)) \log\left(\frac{\sqrt{e} - \sqrt{c^2 d + e}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{2e^{3/2}}$$

[Out] x\*(a+b\*arcsec(c\*x))/e-b\*arctanh((1-1/c^2/x^2)^(1/2))/c/e+1/2\*(a+b\*arcsec(c\*x))\*ln(1-c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)-(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)-1/2\*(a+b\*arcsec(c\*x))\*ln(1+c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)-(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)+1/2\*(a+b\*arcsec(c\*x))\*ln(1-c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)+(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)-1/2\*(a+b\*arcsec(c\*x))\*ln(1+c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)+(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)+1/2\*I\*b\*polylog(2,-c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)-(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)-1/2\*I\*b\*polylog(2,c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)-(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)+1/2\*I\*b\*polylog(2,-c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)+(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)-1/2\*I\*b\*polylog(2,c\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))\*(-d)^(1/2)/(e^(1/2)+(c^2\*d+e)^(1/2)))\*(-d)^(1/2)/e^(3/2)

**Rubi [A]** time = 1.29, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5240, 4734, 4628, 266, 63, 208, 4668, 4742, 4520, 2190, 2279, 2391}

$$\frac{ib\sqrt{-d} \text{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d} \text{PolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d} \text{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d} \text{PolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

[Out] (x\*(a + b\*ArcSec[c\*x]))/e - (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(c\*e) + (Sqrt[-d]\*(a + b\*ArcSec[c\*x])\*Log[1 - (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e^(3/2)) - (Sqrt[-d]\*(a + b\*ArcSec[c\*x])\*Log[1 + (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e^(3/2)) + (Sqrt[-d]\*(a + b\*ArcSec[c\*x])\*Log[1 - (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e^(3/2)) - (Sqrt[-d]\*(a + b\*ArcSec[c\*x])\*Log[1 + (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e^(3/2)) + ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, -((c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e]))])/e^(3/2) - ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e^(3/2)

$$\frac{\text{Sec}[c*x]}{(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])} / e^{(3/2)} + ((I/2)*b*\text{Sqrt}[-d]*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x]))}/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])))] / e^{(3/2)} - ((I/2)*b*\text{Sqrt}[-d]*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x]))}/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])))] / e^{(3/2)}$$

Rule 63

$$\text{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_)^n), x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 208

$$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b]$$

Rule 266

$$\text{Int}[(x_)^m*((a_) + (b_.)*(x_)^n)^p, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n]-1)}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$$

Rule 2190

$$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))^n*((c_.) + (d_.)*(x_)^m)} / ((a_) + (b_.)*((F_)^{((g_.)*((e_.) + (f_.)*(x_)))^n}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a] / (b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$$

Rule 2279

$$\text{Int}[\text{Log}[(a_) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_)))^n}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$$

Rule 2391

$$\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)*(x_)^n)]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$$

Rule 4520



```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sin[(c_.) + (d_.)*(x_.)]/(Cos[(c_.) + (d_.)
*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))], x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))], x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

### Rule 4668

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

### Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)/((d_) + (e_.)*(x_.)), x_Symbol]
:= -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \sec^{-1}(cx))}{d + ex^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x^2 (e + dx^2)} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{ex^2} - \frac{d (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} + \frac{b \text{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce} + \frac{d \text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e} (\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e} (\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{3/2}} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{3/2}} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} - \frac{d \text{Subst} \left( \int \frac{(a + bx) \sin(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx) \right)}{2e^{3/2}} - \frac{d \text{Subst} \left( \int \frac{(a + bx)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx) \right)}{2e^{3/2}} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} + \frac{(id) \text{Subst} \left( \int \frac{e^{ix(a + bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2 d + e}}{c} - \sqrt{-d} e^{ix}} dx, x, \sec^{-1}(cx) \right)}{2e^{3/2}} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{ix}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^{3/2}} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{ix}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^{3/2}} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{ix}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 1.47, size = 1023, normalized size = 1.87

$$\frac{ax}{e} - \frac{a\sqrt{d} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{3/2}} + b \left( \frac{cx \sec^{-1}(cx) + \log\left(\cos\left(\frac{1}{2} \sec^{-1}(cx)\right) - \sin\left(\frac{1}{2} \sec^{-1}(cx)\right)\right) - \log\left(\cos\left(\frac{1}{2} \sec^{-1}(cx)\right)\right) + \dots}{ce} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

[Out] (a\*x)/e - (a\*Sqrt[d]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/e^(3/2) + b\*((c\*x\*ArcSec[c\*x] + Log[Cos[ArcSec[c\*x]/2] - Sin[ArcSec[c\*x]/2]) - Log[Cos[ArcSec[c\*x]/2] + Sin[ArcSec[c\*x]/2]))/(c\*e) - (Sqrt[d]\*(8\*ArcSin[Sqrt[1 + (I\*Sqrt[e])]/(c\*Sqrt[d])])/Sqrt[2])\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - (4\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])]/(c\*Sqrt[d])]/Sqrt[2]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + (4\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])]/(c\*Sqrt[d])]/Sqrt[2]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + (2\*I)\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - 2\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - 2\*PolyLog[2, (-I)\*(Sqrt[e] + Sqrt[c^2\*d + e))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]/(4\*e^(3/2))] + (Sqrt[d]\*(8\*ArcSin[Sqrt[1 - (I\*Sqrt[e])]/(c\*Sqrt[d])])/Sqrt[2])\*ArcTan[((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - (4\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])]/(c\*Sqrt[d])]/Sqrt[2]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - (2\*I)\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + (4\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])]/(c\*Sqrt[d])]/Sqrt[2]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + (2\*I)\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - 2\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - 2\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]/(4\*e^(3/2)))]

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arcsec}(cx) + ax^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*x^2\*arcsec(c\*x) + a\*x^2)/(e\*x^2 + d), x)

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x, algorithm="giac")

[Out] Timed out

**maple** [C] time = 3.80, size = 374, normalized size = 0.68

$$\frac{ax}{e} - \frac{ad \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b \operatorname{arcsec}(cx)x}{e} + \frac{icbd \left( \sum_{-R1=\operatorname{RootOf}(c^2d_Z^4+(2c^2d+4e)_Z^2+c^2d)} \frac{(-R1^2c^2d+c^2d+4e) \left( \operatorname{arcsec}(cx) \ln\left(\frac{-R1-\frac{1}{cx}-i\sqrt{\dots}}{-R1}\right)}{-R1(-R1^2c^2d+\dots)} \right)}{8e^2} \right)}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x)

[Out] a\*x/e-a\*d/e/(d\*e)^(1/2)\*arctan(e\*x/(d\*e)^(1/2))+b\*arcsec(c\*x)/e\*x+1/8\*I\*c\*b/e^2\*d\*sum((R1^2\*c^2\*d+c^2\*d+4\*e)/R1/(R1^2\*c^2\*d+c^2\*d+2\*e)\*(I\*arcsec(c\*x)\*ln((R1-1/c/x-I\*(1-1/c^2/x^2)^(1/2))/R1)+dilog((R1-1/c/x-I\*(1-1/c^2/x^2)^(1/2))/R1)),R1=RootOf(c^2\*d\*\_Z^4+(2\*c^2\*d+4\*e)\*\_Z^2+c^2\*d))+2\*I/c\*b/e\*arctan(1/c/x+I\*(1-1/c^2/x^2)^(1/2))-1/8\*I\*c\*b/e^2\*d\*sum((R1^2\*c^2\*d+4\*\_R1^2\*e+c^2\*d)/R1/(R1^2\*c^2\*d+c^2\*d+2\*e)\*(I\*arcsec(c\*x)\*ln((R1-1/c/x-I\*(1-1/c^2/x^2)^(1/2))/R1)+dilog((R1-1/c/x-I\*(1-1/c^2/x^2)^(1/2))/R1)),R1=RootOf(c^2\*d\*\_Z^4+(2\*c^2\*d+4\*e)\*\_Z^2+c^2\*d))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e} - \frac{x}{e} \right) + b \int \frac{x^2 \arctan(\sqrt{cx+1}\sqrt{cx-1})}{ex^2+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out]  $-a*(d*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e})*e) - x/e) + b*\int (x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})/(e*x^2 + d), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^2*(a + b*\arccos(1/(c*x))))/(d + e*x^2), x)$

[Out]  $\text{int}((x^2*(a + b*\arccos(1/(c*x))))/(d + e*x^2), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsc}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x**2*(a+b*\operatorname{arccsc}(c*x))/(e*x**2+d), x)$

[Out]  $\text{Integral}(x**2*(a + b*\operatorname{arccsc}(c*x))/(d + e*x**2), x)$

$$3.92 \quad \int \frac{x(a+b \sec^{-1}(cx))}{d+ex^2} dx$$

**Optimal.** Leaf size=487

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$

[Out]  $-(a+b*\text{arcsec}(c*x))*\ln(1+(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2)/e+1/2*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e+1/2*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e+1/2*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e+1/2*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e+1/2*I*b*\text{polylog}(2,-(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2)/e-1/2*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e-1/2*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e-1/2*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e-1/2*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e$

**Rubi [A]** time = 1.15, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {5240, 4734, 4626, 3719, 2190, 2279, 2391, 4742, 4520}

$$\frac{ib\text{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\text{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\text{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} - \frac{ib\text{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

[Out]  $((a + b*\text{ArcSec}[c*x])*Log[1 - (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(2*e) + ((a + b*\text{ArcSec}[c*x])*Log[1 + (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(2*e) + ((a + b*\text{ArcSec}[c*x])*Log[1 - (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(2*e) + ((a + b*\text{ArcSec}[c*x])*Log[1 + (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(2*e) - ((a + b*\text{ArcSec}[c*x])*Log[1 + E^{((2*I)*\text{ArcSec}[c*x])}])/e - ((I/2)*b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))])/e - ((I/2)*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))])/e - ((I/2)*b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))])/e - ((I/2)*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))])/e$

$\text{Sec}[c*x])]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/e + ((I/2)*b*\text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])}]/e$

### Rule 2190

$\text{Int}[(((F\_)^{(g\_)*(e\_)+(f\_)*(x\_))})^{(n\_)*((c\_)+(d\_)*(x\_))^{(m\_)}}/((a\_)+(b\_)*((F\_)^{(g\_)*(e\_)+(f\_)*(x\_))})^{(n\_)}], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))^n})/a)]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x))^n})/a)], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a\_)+(b\_)*((F\_)^{(e\_)*((c\_)+(d\_)*(x\_))})^{(n\_)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_)*((d\_)+(e\_)*(x\_)^{(n\_)})]/(x\_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 3719

$\text{Int}[((c\_)+(d\_)*(x\_))^{(m\_)}*\text{tan}[(e\_)+(f\_)*(x\_)], x\_Symbol] \rightarrow \text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*E^{(2*I*(e + f*x))}/(1 + E^{(2*I*(e + f*x))}), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 4520

$\text{Int}[(((e\_)+(f\_)*(x\_))^{(m\_)}*\text{Sin}[(c\_)+(d\_)*(x_)])/(Cos[(c\_)+(d\_)*(x_)]*(b\_)+(a_)), x\_Symbol] \rightarrow \text{Simp}[(I*(e + f*x)^{(m+1)})/(b*f*(m+1)), x] + (-\text{Dist}[I, \text{Int}[(e + f*x)^m*E^{(I*(c + d*x))}]/(a - \text{Rt}[a^2 - b^2, 2] + b*E^{(I*(c + d*x))}), x], x] - \text{Dist}[I, \text{Int}[(e + f*x)^m*E^{(I*(c + d*x))}]/(a + \text{Rt}[a^2 - b^2, 2] + b*E^{(I*(c + d*x))}), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{PosQ}[a^2 - b^2]$

### Rule 4626

$\text{Int}[((a\_)+\text{ArcCos}[(c\_)*(x_)]*(b\_))^{(n\_)}]/(x_), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b*x)^n/\text{Cot}[x], x], x, \text{ArcCos}[c*x]] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0]$

### Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^ (n_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{x(a + b \sec^{-1}(cx))}{d + ex^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x(e + dx^2)} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{ex} - \frac{dx(a + b \cos^{-1} \left( \frac{x}{c} \right))}{e(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e} + \frac{d \text{Subst} \left( \int \frac{x(a + b \cos^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{\text{Subst} \left( \int (a + bx) \tan(x) dx, x, \sec^{-1}(cx) \right)}{e} + \frac{d \text{Subst} \left( \int \left( -\frac{\sqrt{-d}(a + b \cos^{-1} \left( \frac{x}{c} \right))}{2d(\sqrt{e} - \sqrt{-d}x)} + \frac{\sqrt{-d}(a + b \cos^{-1} \left( \frac{x}{c} \right))}{2d(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{i(a + b \sec^{-1}(cx))^2}{2be} - \frac{(2i) \text{Subst} \left( \int \frac{e^{2ix}(a + bx)}{1 + e^{2ix}} dx, x, \sec^{-1}(cx) \right)}{e} - \frac{\sqrt{-d} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{i(a + b \sec^{-1}(cx))^2}{2be} - \frac{(a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)})}{e} + \frac{b \text{Subst} \left( \int \log(1 + e^{2ix}) dx, x, \sec^{-1}(cx) \right)}{e} \\
&= -\frac{(a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)})}{e} - \frac{(ib) \text{Subst} \left( \int \frac{\log(1+x)}{x} dx, x, e^{2i \sec^{-1}(cx)} \right)}{2e} \\
&= \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2e} + \frac{(a + b \sec^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2e} \\
&= \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2e} + \frac{(a + b \sec^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2e} \\
&= \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2e} + \frac{(a + b \sec^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2e}
\end{aligned}$$

**Mathematica [A]** time = 0.46, size = 891, normalized size = 1.83

$$4ib \sin^{-1} \left( \frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}} \right) \tan^{-1} \left( \frac{(\sqrt{e} - ic\sqrt{d}) \tan\left(\frac{1}{2} \sec^{-1}(cx)\right)}{\sqrt{dc^2 + e}} \right) + 4ib \sin^{-1} \left( \frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}} \right) \tan^{-1} \left( \frac{(i\sqrt{d}c + \sqrt{e}) \tan\left(\frac{1}{2} \sec^{-1}(cx)\right)}{\sqrt{dc^2 + e}} \right) + b \sec^{-1} \left( \frac{cx}{\sqrt{d}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

[Out] ((4\*I)\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[(((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] + (4\*I)\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] + b\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + 2\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + b\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + 2\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + b\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - 2\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + b\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - 2\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - 2\*b\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] + a\*Log[d + e\*x^2] - I\*b\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - I\*b\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - I\*b\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] - I\*b\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])] + I\*b\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]/(2\*e))

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{bx \operatorname{arcsec}(cx) + ax}{ex^2 + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d), x, algorithm="fricas")

[Out] integral((b\*x\*arcsec(c\*x) + a\*x)/(e\*x^2 + d), x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")`

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Va  
 lue

**maple** [C] time = 1.04, size = 453, normalized size = 0.93

$$\frac{a \ln(c^2 e x^2 + c^2 d)}{2e} - \frac{ic^2 b \left( \sum_{-R1=\text{RootOf}(c^2 d - Z^4 + (2c^2 d + 4e)Z^2 + c^2 d)} \left( \frac{(-R1^2 + 1) \left( \text{arcsec}(cx) \ln \left( \frac{-R1 - \frac{1}{cx} - i \sqrt{1 - \frac{1}{c^2 x^2}}}{-R1} \right) + \text{dilog} \left( \frac{-R1 - \frac{1}{cx} - i \sqrt{1 - \frac{1}{c^2 x^2}}}{-R1} \right)}{-R1^2 c^2 d + c^2 d + 2e} \right) \right)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arcsec(c*x))/(e*x^2+d),x)`

[Out]  $\frac{1}{2} * a / e * \ln(c^2 * e * x^2 + c^2 * d) - 1/4 * I * c^2 * b * \text{sum}((\_R1^2 + 1) / (\_R1^2 * c^2 * d + c^2 * d + 2 * e) * (I * \text{arcsec}(c * x) * \ln((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1) + \text{dilog}((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1)), \_R1 = \text{RootOf}(c^2 * d * \_Z^4 + (2 * c^2 * d + 4 * e) * \_Z^2 + c^2 * d)) * d / e - b / e * \text{arcsec}(c * x) * \ln(1 + I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) - b / e * \text{arcsec}(c * x) * \ln(1 - I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) + I * b / e * \text{dilog}(1 + I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) + I * b / e * \text{dilog}(1 - I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) - 1/4 * I * b * \text{sum}((\_R1^2 * c^2 * d + c^2 * d + 4 * e) / (\_R1^2 * c^2 * d + c^2 * d + 2 * e) * (I * \text{arcsec}(c * x) * \ln((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1) + \text{dilog}((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1)), \_R1 = \text{RootOf}(c^2 * d * \_Z^4 + (2 * c^2 * d + 4 * e) * \_Z^2 + c^2 * d)) / e$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{x \arctan(\sqrt{cx+1} \sqrt{cx-1})}{ex^2+d} dx + \frac{a \log(ex^2+d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")`

[Out]  $b * \text{integrate}(x * \arctan(\text{sqrt}(c * x + 1) * \text{sqrt}(c * x - 1)) / (e * x^2 + d), x) + 1/2 * a * \log(e * x^2 + d) / e$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(a + b*acos(1/(c*x))))/(d + e*x^2), x)`

[Out] `int((x*(a + b*acos(1/(c*x))))/(d + e*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + b \operatorname{asec}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*asec(c*x))/(e*x**2+d), x)`

[Out] `Integral(x*(a + b*asec(c*x))/(d + e*x**2), x)`

$$3.93 \quad \int \frac{a+b \sec^{-1}(cx)}{d+ex^2} dx$$

**Optimal.** Leaf size=509

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

[Out]  $\frac{1}{2}*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}$

**Rubi [A]** time = 0.87, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {5230, 4668, 4742, 4520, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ibPolyLog\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ibPolyLog\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ibPolyLog\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x^2), x]

[Out]  $((a+b*\text{ArcSec}[c*x])*Log[1-(c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e]-\text{Sqrt}[c^2*d+e]))/(2*\text{Sqrt}[-d]*\text{Sqrt}[e]) - ((a+b*\text{ArcSec}[c*x])*Log[1+(c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e]-\text{Sqrt}[c^2*d+e]))/(2*\text{Sqrt}[-d]*\text{Sqrt}[e]) + ((a+b*\text{ArcSec}[c*x])*Log[1-(c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e]+\text{Sqrt}[c^2*d+e]))/(2*\text{Sqrt}[-d]*\text{Sqrt}[e]) - ((a+b*\text{ArcSec}[c*x])*Log[1+(c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e]+\text{Sqrt}[c^2*d+e]))/(2*\text{Sqrt}[-d]*\text{Sqrt}[e]) + ((I/2)*b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e]-\text{Sqrt}[c^2*d+e]))]/(\text{Sqrt}[-d]*\text{Sqrt}[e]) - ((I/2)*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e]-\text{Sqrt}[c^2*d+e]))]/(\text{Sqrt}[-d]*\text{Sqrt}[e]) + ((I/2)*b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e]+\text{Sqrt}[c^2*d+e]))]/(\text{Sqrt}[-d]*\text{Sqrt}[e]) - ((I/2)*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e]+\text{Sqrt}[c^2*d+e]))]/(\text{Sqrt}[-d]*\text{Sqrt}[e]))$

$d \sqrt{e} - ((I/2) * b * \text{PolyLog}[2, (c \sqrt{-d} * E^{(I * \text{ArcSec}[c * x])}) / (\sqrt{e} + \sqrt{c^2 * d + e})]) / (\sqrt{-d} * \sqrt{e})$

#### Rule 2190

$\text{Int}[(((F\_)^{(g\_)*((e\_)+(f\_)*(x\_)))^{(n\_)*((c\_)+(d\_)*(x\_))^{(m\_)}})/((a\_)+(b\_)*((F\_)^{(g\_)*((e\_)+(f\_)*(x\_)))^{(n\_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a)] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist}[(d*m) / (b*f*g*n * \text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a)], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n * \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

#### Rule 4520

$\text{Int}[(((e_)+(f_)*(x_))^{(m_)*\text{Sin}[(c_)+(d_)*(x_)]})/(\text{Cos}[(c_)+(d_)*(x_)]*(b_)+(a_)), x\_Symbol] \rightarrow \text{Simp}[(I*(e + f*x)^{(m+1)}) / (b*f*(m+1)), x] + (-\text{Dist}[I, \text{Int}[(e + f*x)^m * E^{(I*(c + d*x))}] / (a - \text{Rt}[a^2 - b^2, 2] + b * E^{(I*(c + d*x))}), x], x] - \text{Dist}[I, \text{Int}[(e + f*x)^m * E^{(I*(c + d*x))}] / (a + \text{Rt}[a^2 - b^2, 2] + b * E^{(I*(c + d*x))}), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{PosQ}[a^2 - b^2]$

#### Rule 4668

$\text{Int}[((a_)+\text{ArcCos}[(c_)*(x_)]*(b_))^{(n_)*((d_)+(e_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b * \text{ArcCos}[c * x])^n, (d + e * x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{NeQ}[c^2 * d + e, 0] \&\& \text{IntegerQ}[p] \&\& (\text{GtQ}[p, 0] \parallel \text{IGtQ}[n, 0])$

#### Rule 4742

$\text{Int}[((a_)+\text{ArcCos}[(c_)*(x_)]*(b_))^{(n_)} / ((d_)+(e_)*(x_)), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b*x)^n * \text{Sin}[x] / (c*d + e * \text{Cos}[x]), x], x, \text{ArcCos}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{IGtQ}[n, 0]$

#### Rule 5230

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(2*(p + 1))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{d + ex^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e} (\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e} (\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} - \frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} \\
&= \frac{\text{Subst} \left( \int \frac{\frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx)}{2\sqrt{e}} \right)}{2\sqrt{e}} + \frac{\text{Subst} \left( \int \frac{\frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx)}{2\sqrt{e}} \right)}{2\sqrt{e}} \\
&= -\frac{i \text{Subst} \left( \int \frac{\frac{e^{ix(a+bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2d+e}}{c} - \sqrt{-d} e^{ix}} dx, x, \sec^{-1}(cx)}{2\sqrt{e}} \right)}{2\sqrt{e}} - \frac{i \text{Subst} \left( \int \frac{\frac{e^{ix(a+bx)}}{\frac{\sqrt{e}}{c} + \frac{\sqrt{c^2d+e}}{c} - \sqrt{-d} e^{ix}} dx, x, \sec^{-1}(cx)}{2\sqrt{e}} \right)}{2\sqrt{e}} \\
&= \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} + \dots \\
&= \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} + \dots \\
&= \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.40, size = 871, normalized size = 1.71

$$2a \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 4b \sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right) \tan^{-1}\left(\frac{(\sqrt{e}-ic\sqrt{d})\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{dc^2+e}}\right) + 4b \sin^{-1}\left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}}+1}}{\sqrt{2}}\right) \tan^{-1}\left(\frac{(i\sqrt{d}c+\sqrt{e})\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{dc^2+e}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x^2), x]

[Out] (2\*a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]] - 4\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*ArcTan[(((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] + 4\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*ArcTan[(((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]) - I\*b\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - (2\*I)\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + I\*b\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + (2\*I)\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + I\*b\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - (2\*I)\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - I\*b\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + (2\*I)\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + b\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - b\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - b\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + b\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])]/(2\*Sqrt[d]\*Sqrt[e])

**fricas [F]** time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d), x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e\*x^2 + d), x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")`

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Va  
 lue

**maple** [C] time = 1.89, size = 272, normalized size = 0.53

$$\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{icb \left( \sum_{R1=\text{RootOf}(c^2d_Z^4+(2c^2d+4e)_Z^2+c^2d)} \frac{-R1 \left( i \arccsc(cx) \ln\left(\frac{-R1 - \frac{1}{cx} - i \sqrt{1 - \frac{1}{c^2x^2}}}{-R1}\right) + \text{dilog}\left(\frac{-R1 - \frac{1}{cx} - i \sqrt{1 - \frac{1}{c^2x^2}}}{-R1}\right)\right)}{-R1^2 c^2 d + c^2 d + 2e} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/(e*x^2+d),x)`

[Out]  $a/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)})+1/2*I*c*b*\sum(_R1/(_R1^2*c^2*d+c^2*d+2*e))*(I*\arccsc(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)+\text{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\text{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*I*c*b*\sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e))*(I*\arccsc(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)+\text{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\text{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{ex^2+d} dx + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")`

[Out]  $b*\text{integrate}(\arctan(\text{sqrt}(c*x+1)*\text{sqrt}(c*x-1))/(e*x^2+d),x)+a*\arctan(e*x/\text{sqrt}(d*e))/\text{sqrt}(d*e)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{ex^2+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asec(c*x))/(d + e*x**2), x)
```

$$3.94 \quad \int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)} dx$$

Optimal. Leaf size=459

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2d} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2d} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{2d}$$

```
[Out] 1/2*I*(a+b*arcsec(c*x))^2/b/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

**Rubi [A]** time = 0.90, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5240, 4734, 4742, 4520, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2d} + \frac{ibPolyLog\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2d} + \frac{ibPolyLog\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d+e}+\sqrt{e}}\right)}{2d} + \frac{ibPolyLog\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d+e}+\sqrt{e}}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)), x]

```
[Out] ((I/2)*(a + b*ArcSec[c*x])^2)/(b*d) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))]/d + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/d + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))]/d + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/d
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4520

```
Int[(((e_) + (f_)*(x_))^(m_)*Sin[(c_) + (d_)*(x_)])/(Cos[(c_) + (d_)
*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4734

```
Int[(((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_))*((f_)*(x_))^(m_)*((d_) + (e_
)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4742

```
Int[(((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_))/((d_) + (e_)*(x_)), x_Symbol]
:= -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5240

```
Int[(((a_) + ArcSec[(c_)*(x_)]*(b_))^(n_))*((d_) + (e_)*(x_)
^2)^(p_), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^
```

$(m + 2*(p + 1)), x], x, 1/x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[n, 0]$   
 $\&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p]$

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \sec^{-1}(cx)}{x(d + ex^2)} dx &= -\text{Subst} \left( \int \frac{x \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
 &= -\text{Subst} \left( \int \left( -\frac{\sqrt{-d} \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} - \sqrt{-d}x)} + \frac{\sqrt{-d} \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right) \\
 &= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} + \frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} \\
 &= \frac{\text{Subst} \left( \int \frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx) \right)}{2\sqrt{-d}} - \frac{\text{Subst} \left( \int \frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx) \right)}{2\sqrt{-d}} \\
 &= \frac{i(a + b \sec^{-1}(cx))^2}{2bd} - \frac{i \text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2d+e}}{c} - \sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx) \right)}{2\sqrt{-d}} - \frac{i \text{Subst} \left( \int \frac{e^{ix}}{\frac{\sqrt{e}}{c} + \frac{\sqrt{c^2d+e}}{c}} dx, x, \sec^{-1}(cx) \right)}{2\sqrt{-d}} \\
 &= \frac{i(a + b \sec^{-1}(cx))^2}{2bd} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} \\
 &= \frac{i(a + b \sec^{-1}(cx))^2}{2bd} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} \\
 &= \frac{i(a + b \sec^{-1}(cx))^2}{2bd} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d}
 \end{aligned}$$

**Mathematica [A]** time = 0.93, size = 402, normalized size = 0.88

$$-2a \log(d + ex^2) + 4a \log(x) + ib \left( \operatorname{Li}_2 \left( -\frac{(dc^2 + 2e - 2\sqrt{e(dc^2 + e)})e^{2i \operatorname{sec}^{-1}(cx)}}{c^2 d} \right) + \operatorname{Li}_2 \left( -\frac{(dc^2 + 2(e + \sqrt{e(dc^2 + e)}))e^{2i \operatorname{sec}^{-1}(cx)}}{c^2 d} \right) \right) - 4 \operatorname{Si}(\dots)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)), x]

[Out] (4\*a\*Log[x] - 2\*a\*Log[d + e\*x^2] + I\*b\*(2\*ArcSec[c\*x]^2 - 4\*ArcSin[Sqrt[1 + e/(c^2\*d)]]\*ArcTan[(c\*e\*Sqrt[1 - 1/(c^2\*x^2)]]\*x)/Sqrt[e\*(c^2\*d + e)] + (2\*I)\*ArcSec[c\*x]\*Log[1 + ((c^2\*d + 2\*e - 2\*Sqrt[e\*(c^2\*d + e)])\*E^((2\*I)\*ArcSec[c\*x]))/(c^2\*d)] + (2\*I)\*ArcSin[Sqrt[1 + e/(c^2\*d)]]\*Log[1 + ((c^2\*d + 2\*e - 2\*Sqrt[e\*(c^2\*d + e)])\*E^((2\*I)\*ArcSec[c\*x]))/(c^2\*d)] + (2\*I)\*ArcSec[c\*x]\*Log[1 + ((c^2\*d + 2\*(e + Sqrt[e\*(c^2\*d + e)]))E^((2\*I)\*ArcSec[c\*x]))/(c^2\*d)] - (2\*I)\*ArcSin[Sqrt[1 + e/(c^2\*d)]]\*Log[1 + ((c^2\*d + 2\*(e + Sqrt[e\*(c^2\*d + e)]))E^((2\*I)\*ArcSec[c\*x]))/(c^2\*d)] + PolyLog[2, -(((c^2\*d + 2\*e - 2\*Sqrt[e\*(c^2\*d + e)])\*E^((2\*I)\*ArcSec[c\*x]))/(c^2\*d))] + PolyLog[2, -(((c^2\*d + 2\*(e + Sqrt[e\*(c^2\*d + e)]))E^((2\*I)\*ArcSec[c\*x]))/(c^2\*d)))])/(4\*d)

**fricas [F]** time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \operatorname{arcsec}(cx) + a}{ex^3 + dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d), x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e\*x^3 + d\*x), x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d), x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

maple [C] time = 1.03, size = 2933, normalized size = 6.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arcsec}(c*x))/x/(e*x^2+d), x)$

[Out] 
$$I*b/c^4*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}-2*e))^{(1/2)}/d/(c^2*d+e)*\text{arcsec}(c*x)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}+5/2*b*e/d/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)+I*b*(e*(c^2*d+e))^{(1/2)}/d/(c^2*d+e)*\text{arcsec}(c*x)^2+3/4*I*b*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/d/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}-5/4*I*b*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/d/(c^2*d+e)*e-1/4*I*b*(e*(c^2*d+e))^{(1/2)}/d/(c^2*d+e)*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}-1/2*I*b/c^2*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/d^2*(e*(c^2*d+e))^{(1/2)}-I*b/c^2*\text{arcsec}(c*x)^2/d^2*(e*(c^2*d+e))^{(1/2)}+2*I*b/c^2*\text{arcsec}(c*x)^2*e/d^2+2*I*b/c^4*\text{arcsec}(c*x)^2*e^2/d^3-5/2*I*b*\text{arcsec}(c*x)^2*e/d/(c^2*d+e)-3/2*b/d/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)*(e*(c^2*d+e))^{(1/2)}+b/c^2/d^2*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)*(e*(c^2*d+e))^{(1/2)}-2*b/c^2/d^2*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)*e-2*b/c^4/d^3*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)*e^2+I*b/c^2*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/d^2*e-1/2*I*b*c^2*\text{arcsec}(c*x)^2/(c^2*d+e)-1/4*I*b*c^2*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/(c^2*d+e)+4*b/c^2*e^2/d^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)-1/4*b*c^2/e/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*\text{arcsec}(c*x)*(e*(c^2*d+e))^{(1/2)}-4*I*b/c^2*e^2*\text{arcsec}(c*x)^2/d^2/(c^2*d+e)+1/8*I*b*c^2*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/e/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}-2*I*b/c^4*\text{arcsec}(c*x)^2*e/d^3*(e*(c^2*d+e))^{(1/2)}-2*I*b/c^4*e^3*\text{arcsec}(c*x)^2/d^3/(c^2*d+e)-2*I*b/c^2*e^2*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/d^2/(c^2*d+e)-I*b/c^4*e^3*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}/d^3/(c^2*d+e)-I*b/c^4*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}*e/d^3*(e*(c^2*d+e))^{(1/2)}-1/8*I*b*c^2*(e*(c^2*d+e))^{(1/2)}/e/(c^2*d+e)*\text{polylog}(2, c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}+2*b/c^4*e^3/d^3/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))^{(1/2)}$$

$$2*d+e))^{(1/2)-2*e))*\text{arcsec}(c*x)+2*b/c^4/d^3*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\text{arcsec}(c*x)*e*(e*(c^2*d+e))^{(1/2)+1/4*b*c^2*(e*(c^2*d+e))^{(1/2)}/e/(c^2*d+e)*\text{arcsec}(c*x)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e})))+1/2*b*c^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\text{arcsec}(c*x)+1/4*I*b*\text{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))/d+1/2*I*b/d*\text{sum}((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*\text{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1))+\text{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\text{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*b/d*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\text{arcsec}(c*x)+I*b*\text{arcsec}(c*x)^2/d-1/2*a/d*\ln(c^2*e*x^2+c^2*d)+a/d*\ln(c*x)-2*b/c^4*e^2/d^3/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\text{arcsec}(c*x)*(e*(c^2*d+e))^{(1/2)+3*I*b/c^2*e*\text{arcsec}(c*x)^2/d^2/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)+I*b/c^4*e^2*\text{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))/d^3/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)+2*I*b/c^4*e^2*\text{arcsec}(c*x)^2/d^3/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)+3/2*I*b/c^2*e*\text{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))/d^2/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)-3*b/c^2*e/d^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\text{arcsec}(c*x)*(e*(c^2*d+e))^{(1/2)}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{\log(ex^2+d)}{d}-\frac{2\log(x)}{d}\right)+b\int\frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{ex^3+dx}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d),x, algorithm="maxima")

[Out] -1/2\*a\*(log(e\*x^2 + d)/d - 2\*log(x)/d) + b\*integrate(arctan(sqrt(c\*x + 1))\*sqrt(c\*x - 1))/(e\*x^3 + d\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\cos\left(\frac{1}{cx}\right)}{x(ex^2+d)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x\*(d + e\*x^2)),x)

[Out] int((a + b\*acos(1/(c\*x)))/(x\*(d + e\*x^2)), x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x/(e\*x\*\*2+d),x)

[Out] Integral((a + b\*asec(c\*x))/(x\*(d + e\*x\*\*2)), x)

$$3.95 \quad \int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)} dx$$

**Optimal.** Leaf size=551

$$\frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} + \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{2(-d)^{3/2}}$$

[Out]  $-a/d/x - b \operatorname{arcsec}(c*x)/d/x + 1/2*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} - 1/2*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} + 1/2*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} - 1/2*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} + 1/2*I*b*\operatorname{polylog}(2, -c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} - 1/2*I*b*\operatorname{polylog}(2, c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} + 1/2*I*b*\operatorname{polylog}(2, -c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} - 1/2*I*b*\operatorname{polylog}(2, c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(3/2)} + b*c*(1-1/c^2/x^2)^{(1/2)}/d$

**Rubi [A]** time = 1.10, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5240, 4734, 4620, 261, 4668, 4742, 4520, 2190, 2279, 2391}

$$\frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{2(-d)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(x^2\*(d + e\*x^2)), x]

[Out]  $(b*c*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]/d - a/(d*x) - (b*\operatorname{ArcSec}[c*x])/(d*x) + (\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcSec}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e]))/(2*(-d)^{(3/2)}) - (\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcSec}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e]))/(2*(-d)^{(3/2)}) + (\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcSec}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e]))/(2*(-d)^{(3/2)}) - (\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcSec}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e]))/(2*(-d)^{(3/2)}) + ((I/2)*b*\operatorname{Sqrt}[e]*\operatorname{PolyLog}[2, -(c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])) - ((I/2)*b*\operatorname{Sqrt}[e]*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])) + ((I/2)*b*\operatorname{Sqrt}[e]*\operatorname{PolyLog}[2, -(c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) - ((I/2)*b*\operatorname{Sqrt}[e]*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])})]/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e]))$

$$\frac{\text{rt}[c^2*d + e])]}{(-d)^{3/2}} - \left(\frac{(I/2)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{(-d)^{3/2}} + \frac{(I/2)*b*\text{Sqrt}[e]*\text{PolyLog}[2, -(c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{(-d)^{3/2}} - \frac{(I/2)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{(-d)^{3/2}}\right)$$

### Rule 261

$$\text{Int}[(x_)^{(m_*)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)}/(b*n*(p+1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{EqQ}[m, n-1] \&\& \text{NeQ}[p, -1]$$

### Rule 2190

$$\text{Int}[\frac{((F_)^{((g_)*(e_) + (f_)*(x_)))^{(n_*)}*((c_) + (d_)*(x_))^{(m_*)}}{((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_)))^{(n_*)}}), x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m * \text{Log}[1 + (b*(F^{(g*(e + f*x))))^n]/a]}{b*f*g*n*\text{Log}[F]}, x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x))))^n]/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$$

### Rule 2279

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))^{(n_*)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$$

### Rule 2391

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_*)})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$$

### Rule 4520

$$\text{Int}[\frac{((e_) + (f_)*(x_))^{(m_*)}*\text{Sin}[(c_) + (d_)*(x_)]}{(\text{Cos}[(c_) + (d_)*(x_)]*(b_) + (a_)), x\_Symbol] \rightarrow \text{Simp}[\frac{I*(e + f*x)^{(m+1)}}{b*f*(m+1)}, x] + (-\text{Dist}[I, \text{Int}[\frac{(e + f*x)^m * E^{(I*(c + d*x))}}{a - \text{Rt}[a^2 - b^2, 2] + b * E^{(I*(c + d*x))}}, x], x] - \text{Dist}[I, \text{Int}[\frac{(e + f*x)^m * E^{(I*(c + d*x))}}{a + \text{Rt}[a^2 - b^2, 2] + b * E^{(I*(c + d*x))}}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{PosQ}[a^2 - b^2]$$

### Rule 4620

$$\text{Int}[\frac{((a_) + \text{ArcCos}[(c_)*(x_)]*(b_))^{(n_*)}}{x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCos}[c*x])^n, x] + \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcCos}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{GtQ}[n, 0]$$

Rule 4668

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || IGtQ[n, 0])
```

Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_)^m)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]]
/; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x]
/; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^2(d + ex^2)} dx &= -\text{Subst} \left( \int \frac{x^2 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{d} - \frac{e \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \text{Subst} \left( \int \cos^{-1} \left( \frac{x}{c} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \sec^{-1}(cx)}{dx} - \frac{b \text{Subst} \left( \int \frac{x}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2d} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2d} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \sec^{-1}(cx)}{dx} - \frac{\sqrt{e} \text{Subst} \left( \int \frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx) \right)}{2d} - \frac{\sqrt{e} \text{Subst} \left( \int \frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \cos(x)} dx, x, \sec^{-1}(cx) \right)}{2d} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \sec^{-1}(cx)}{dx} + \frac{(i\sqrt{e}) \text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} - \sqrt{c^2d+e} - \sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx) \right)}{2d} + \frac{(-i\sqrt{e}) \text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} + \sqrt{c^2d+e} - \sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx) \right)}{2d} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \sec^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2(-d)^{3/2}} - \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right)}{2(-d)^{3/2}} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \sec^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2(-d)^{3/2}} - \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right)}{2(-d)^{3/2}} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \sec^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2(-d)^{3/2}} - \frac{\sqrt{e} (a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right)}{2(-d)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 1.50, size = 997, normalized size = 1.81

$$-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) a}{d^{3/2}} - \frac{a}{dx} + b \left( \frac{c \sqrt{1 - \frac{1}{c^2 x^2}} - \frac{\sec^{-1}(cx)}{x}}{d} - \frac{\sqrt{e} \left( 8 \sin^{-1}\left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}}\right) \tan^{-1}\left(\frac{(i\sqrt{d}c + \sqrt{e}) \tan\left(\frac{1}{2} \sec^{-1}(cx)\right)}{\sqrt{dc^2 + e}}\right)}{\sqrt{dc^2 + e}} \right) - 2i s}{d} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^2\*(d + e\*x^2)), x]

[Out]  $-(a/(d*x)) - (a*\text{Sqrt}[e]*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/d^{3/2} + b*((c*\text{Sqrt}[1 - 1/(c^2*x^2)] - \text{ArcSec}[c*x])/x)/d - (\text{Sqrt}[e]*(8*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTan}[(I*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tan}[\text{ArcSec}[c*x]/2])/(\text{Sqrt}[c^2*d + e]) - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) - (4*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) + (4*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) + (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + E^{((2*I)*\text{ArcSec}[c*x])}] - 2*\text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) - 2*\text{PolyLog}[2, ((-I)*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) + \text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])})]/(4*d^{3/2}) + (\text{Sqrt}[e]*(8*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTan}[(I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) - (4*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) + (4*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) + (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + E^{((2*I)*\text{ArcSec}[c*x])}] - 2*\text{PolyLog}[2, ((-I)*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) - 2*\text{PolyLog}[2, (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})]/(c*\text{Sqrt}[d]) + \text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])})]/(4*d^{3/2}))$

**fricas [F]** time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e\*x^4 + d\*x^2), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym  
m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [C] time = 3.67, size = 331, normalized size = 0.60

$$\frac{ae \arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}} - \frac{a}{dx} + \frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d} - \frac{b \operatorname{arcsec}(cx)}{dx} - \frac{icbe \left( \sum_{R1=\operatorname{RootOf}(c^2d_Z^4+(2c^2d+4e)_Z^2+c^2d)} \operatorname{arcsec}(cx) \ln\left(\frac{R1-\frac{1}{cx}}{\dots}\right) \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d),x)

[Out]  $-a*e/d/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)})-a/d/x+c*b/d*((c^2*x^2-1)/c^2/x^2)^{(1/2)}-b*\operatorname{arcsec}(c*x)/d/x-1/2*I*c*b*e/d*\sum(R1/((R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/R1)+\operatorname{dilog}((R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/R1)),R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*I*c*b*e/d*\sum(1/R1/((R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/R1)+\operatorname{dilog}((R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/R1)),R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d} + \frac{1}{dx} \right) + b \int \frac{\arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{ex^4+dx^2}\right)}{ex^4+dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d),x, algorithm="maxima")

[Out] -a\*(e\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d) + 1/(d\*x)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x^4 + d\*x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 (ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x^2\*(d + e\*x^2)),x)

[Out] int((a + b\*acos(1/(c\*x)))/(x^2\*(d + e\*x^2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x^2 (d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*2/(e\*x\*\*2+d),x)

[Out] Integral((a + b\*asec(c\*x))/(x\*\*2\*(d + e\*x\*\*2)), x)



$$3.96 \quad \int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=608

$$\frac{d(a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{e^3} - \frac{d(a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{e^3} - \frac{d(a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{e^3}$$

[Out]  $\frac{1}{2} d (a + b \operatorname{arcsec}(c x)) / e^{2 / (e + d / x^2)} + \frac{1}{2} x^2 (a + b \operatorname{arcsec}(c x)) / e^{2 + 2 d (a + b \operatorname{arcsec}(c x)) \ln(1 + (1/c/x + I*(1 - 1/c^2/x^2)^{1/2})^2) / e^3 - d (a + b \operatorname{arcsec}(c x)) \ln(1 - c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^3 - d (a + b \operatorname{arcsec}(c x)) \ln(1 + c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^3 - d (a + b \operatorname{arcsec}(c x)) \ln(1 - c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (c^2*d + e)^{1/2})) / e^3 - d (a + b \operatorname{arcsec}(c x)) \ln(1 + c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (c^2*d + e)^{1/2})) / e^3 - I*b*d*\operatorname{polylog}(2, -(1/c/x + I*(1 - 1/c^2/x^2)^{1/2})^2) / e^3 + I*b*d*\operatorname{polylog}(2, -c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^3 + I*b*d*\operatorname{polylog}(2, c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^3 + I*b*d*\operatorname{polylog}(2, -c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (c^2*d + e)^{1/2})) / e^3 + I*b*d*\operatorname{polylog}(2, c*(1/c/x + I*(1 - 1/c^2/x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (c^2*d + e)^{1/2})) / e^3 + \frac{1}{2} b*d*\operatorname{arctan}((c^2*d + e)^{1/2} / c/x/e^{1/2} / (1 - 1/c^2/x^2)^{1/2}) / e^{5/2} / (c^2*d + e)^{1/2} - \frac{1}{2} b*x*(1 - 1/c^2/x^2)^{1/2} / c/e^2$

**Rubi [A]** time = 1.31, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5240, 4734, 4628, 264, 4626, 3719, 2190, 2279, 2391, 4730, 377, 205, 4742, 4520}

$$\frac{\operatorname{ibDPolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{e^3} + \frac{\operatorname{ibDPolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{e^3} + \frac{\operatorname{ibDPolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{e^3} + \frac{\operatorname{ibDPolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^5*(a + b*\operatorname{ArcSec}[c*x]))/(d + e*x^2)^2, x]$

[Out]  $-(b*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*\operatorname{ArcSec}[c*x]))/(2*e^2*(e + d/x^2)) + (x^2*(a + b*\operatorname{ArcSec}[c*x]))/(2*e^2) + (b*d*\operatorname{ArcTan}[\operatorname{Sqrt}[c^2*d + e]$

$$\begin{aligned} & /((c*\sqrt{e}*\sqrt{1 - 1/(c^2*x^2)}*x)))/(2*e^{(5/2)}*\sqrt{c^2*d + e}) - (d*(a \\ & + b*\text{ArcSec}[c*x])*\text{Log}[1 - (c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} - \sqrt{c^2*d + e}))]/e^3 - (d*(a + b*\text{ArcSec}[c*x])*\text{Log}[1 + (c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} - \sqrt{c^2*d + e}))]/e^3 - (d*(a + b*\text{ArcSec}[c*x])*\text{Log}[1 - (c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} + \sqrt{c^2*d + e}))]/e^3 - (d*(a + b*\text{ArcSec}[c*x])*\text{Log}[1 + (c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} + \sqrt{c^2*d + e}))]/e^3 + (2*d*(a + b*\text{ArcSec}[c*x])*\text{Log}[1 + E^{((2*I)*\text{ArcSec}[c*x])}])/e^3 + (I*b*d*\text{PolyLog}[2, -((c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} - \sqrt{c^2*d + e}))])/e^3 + (I*b*d*\text{PolyLog}[2, (c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} - \sqrt{c^2*d + e}))])/e^3 + (I*b*d*\text{PolyLog}[2, -((c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} + \sqrt{c^2*d + e}))])/e^3 + (I*b*d*\text{PolyLog}[2, (c*\sqrt{-d}*E^{(I*\text{ArcSec}[c*x]))}/(\sqrt{e} + \sqrt{c^2*d + e}))])/e^3 - (I*b*d*\text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])}])/e^3 \end{aligned}$$

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 377

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3719

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 4520

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]/(Cos[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.)), x\_Symbol] := Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (-Dist[I, Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a - Rt[a^2 - b^2, 2] + b\*E^(I\*(c + d\*x))), x], x] - Dist[I, Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a + Rt[a^2 - b^2, 2] + b\*E^(I\*(c + d\*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

Rule 4626

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := -Subst[Int[(a + b\*x)^n/Cot[x], x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 4628

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(d\*(m + 1)), x] + Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4730

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCos[c\*x]))/(2\*e\*(p + 1)), x] + Dist[(b\*c)/(2\*e\*(p + 1)), Int[(d + e\*x^2)^(p + 1)/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 4734

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCos[c\*x])^n, (

$f*x)^m*(d + e*x^2)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2\*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 4742

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Subst[Int[((a + b\*x)^n\*Sin[x])/(c\*d + e\*Cos[x]), x], x, ArcCos[c\*x]] / ; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 5240

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcCos[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x^3 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e^2 x^3} - \frac{2d (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e^3 x} + \frac{d^2 x (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)^2} + \frac{2d^2 x}{e^2 (e + dx^2)^2} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(2d) \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} - \frac{(2d^2) \text{Subst} \left( \int \frac{x (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} - \frac{\text{Subst} \left( \int \frac{2d^2 x}{e^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} - \frac{(2d) \text{Subst} \left( \int (a + bx) \tan(x) dx, x, \sec^{-1} \left( \frac{cx}{b} \right) \right)}{e^3} \\
&= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} - \frac{id (a + b \sec^{-1}(cx))^2}{be^3} \\
&= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} - \frac{id (a + b \sec^{-1}(cx))^2}{be^3} \\
&= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} + \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} + \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} + \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \sec^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \sec^{-1}(cx))}{2e^2} + \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}}
\end{aligned}$$

**Mathematica [B]** time = 4.25, size = 1255, normalized size = 2.06

$$\frac{2ad^2}{ex^2+d} + 4a \log(ex^2 + d)d - 2aex^2 + b \left( -2e \sec^{-1}(cx)x^2 + \frac{2e \sqrt{1 - \frac{1}{c^2 x^2}} x}{c} + \frac{d^{3/2} \sec^{-1}(cx)}{\sqrt{d} - i\sqrt{e}x} + \frac{d^{3/2} \sec^{-1}(cx)}{i\sqrt{e}x + \sqrt{d}} + 2d \sin^{-1}\left(\frac{1}{cx}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] 
$$\begin{aligned} & -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*\text{Log}[d + e*x^2] + b*((2*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/c - 2*e*x^2*\text{ArcSec}[c*x] + (d^{3/2}*\text{ArcSec}[c*x])/(\text{Sqrt}[d] - I*\text{Sqrt}[e]*x) + (d^{3/2}*\text{ArcSec}[c*x])/(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x) + 2*d*\text{ArcSin}[1/(c*x)] + (16*I)*d*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTan}[((( -I)*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tan}[\text{ArcSec}[c*x]/2])/\text{Sqrt}[c^2*d + e]] + (16*I)*d*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTan}[(I*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tan}[\text{ArcSec}[c*x]/2])/\text{Sqrt}[c^2*d + e]] + 4*d*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] + 8*d*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] + 4*d*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] + 8*d*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] + 4*d*\text{ArcSec}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] - 8*d*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] + 4*d*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] - 8*d*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] - 8*d*\text{ArcSec}[c*x]*\text{Log}[1 + E^{((2*I)*\text{ArcSec}[c*x])}] - (d*\text{Sqrt}[e]*\text{Log}[(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(\text{Sqrt}[e] + c*(I*c*\text{Sqrt}[d] - \text{Sqrt}[-(c^2*d) - e])*\text{Sqrt}[1 - 1/(c^2*x^2)])*x)/(\text{Sqrt}[-(c^2*d) - e]*(\text{Sqrt}[d] - I*\text{Sqrt}[e]*x)))]/\text{Sqrt}[-(c^2*d) - e] - (d*\text{Sqrt}[e]*\text{Log}[(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(-\text{Sqrt}[e] + c*(I*c*\text{Sqrt}[d] + \text{Sqrt}[-(c^2*d) - e])*\text{Sqrt}[1 - 1/(c^2*x^2)])*x)/(\text{Sqrt}[-(c^2*d) - e]*(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x)))]/\text{Sqrt}[-(c^2*d) - e] - (4*I)*d*\text{PolyLog}[2, ((-I)*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] - (4*I)*d*\text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] - (4*I)*d*\text{PolyLog}[2, ((-I)*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] - (4*I)*d*\text{PolyLog}[2, (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]] + (4*I)*d*\text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])}]]/e^3 \end{aligned}$$

**fricas [F]** time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5 \operatorname{arcsec}(cx) + ax^5}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^5\*arcsec(c\*x) + a\*x^5)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] Timed out

maple [C] time = 1.67, size = 783, normalized size = 1.29

$$\frac{ax^2}{2e^2} - \frac{ad \ln(c^2ex^2 + c^2d)}{e^3} - \frac{c^2ad^2}{2e^3(c^2ex^2 + c^2d)} + \frac{c^2b \operatorname{arcsec}(cx)x^4}{2(c^2ex^2 + c^2d)e} + \frac{c^2b \operatorname{arcsec}(cx)dx^2}{(c^2ex^2 + c^2d)e^2} - \frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}x^3}{2(c^2ex^2 + c^2d)e} - \frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(c^2ex^2 + c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

[Out]  $\frac{1}{2}ax^2/e^2 - a/e^3d \ln(c^2ex^2 + c^2d) - 1/2c^2a/e^3d^2/(c^2ex^2 + c^2d) + 1/2c^2b/(c^2ex^2 + c^2d)/e \operatorname{arcsec}(cx) * x^4 + c^2b/(c^2ex^2 + c^2d)/e^2 \operatorname{arcsec}(cx) * dx^2 - 1/2c^2b/(c^2ex^2 + c^2d)/e * ((c^2x^2 - 1)/c^2/x^2)^{(1/2)} * x^3 - 1/2c^2b/(c^2ex^2 + c^2d)/e^2 * ((c^2x^2 - 1)/c^2/x^2)^{(1/2)} * xd - 1/2I*b/(c^2ex^2 + c^2d)/e^2d - 2I*b/e^3d \operatorname{dilog}(1 + I*(1/c/x + I*(1 - 1/c^2/x^2)^{(1/2)})) + 1/2I*c^2b/e^3d^2 \operatorname{sum}((\_R1^2 + 1)/(\_R1^2 * c^2d + c^2d + 2e) * (I \operatorname{arcsec}(cx) * \ln((\_R1 - 1/c/x - I*(1 - 1/c^2/x^2)^{(1/2)})/\_R1) + \operatorname{dilog}((\_R1 - 1/c/x - I*(1 - 1/c^2/x^2)^{(1/2)})/\_R1)), \_R1 = \operatorname{RootOf}(c^2d * \_Z^4 + (2c^2d + 4e) * \_Z^2 + c^2d)) + 1/2I*b/e^3d \operatorname{sum}((\_R1^2 * c^2d + c^2d + 4e)/(\_R1^2 * c^2d + c^2d + 2e) * (I \operatorname{arcsec}(cx) * \ln((\_R1 - 1/c/x - I*(1 - 1/c^2/x^2)^{(1/2)})/\_R1) + \operatorname{dilog}((\_R1 - 1/c/x - I*(1 - 1/c^2/x^2)^{(1/2)})/\_R1)), \_R1 = \operatorname{RootOf}(c^2d * \_Z^4 + (2c^2d + 4e) * \_Z^2 + c^2d)) + 2b/e^3d \operatorname{arcsec}(cx) * \ln(1 + I*(1/c/x + I*(1 - 1/c^2/x^2)^{(1/2)})) + 2b/e^3d \operatorname{arcsec}(cx) * \ln(1 - I*(1/c/x + I*(1 - 1/c^2/x^2)^{(1/2)})) - 1/2I*b/(c^2ex^2 + c^2d)/e * x^2 - 1/2I*b*(e*(c^2d + e))^{(1/2)}/(c^2d + e)/e^3 \operatorname{arctanh}(1/4 * (2c^2d * (1/c/x + I*(1 - 1/c^2/x^2)^{(1/2)})^2 + 2c^2d + 4e)/(c^2d * e + e^2))^{(1/2)} * d - 2I*b/e^3d \operatorname{dilog}(1 - I*(1/c/x + I*(1 - 1/c^2/x^2)^{(1/2)}))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{d^2}{e^4x^2 + de^3} - \frac{x^2}{e^2} + \frac{2d \log(ex^2 + d)}{e^3}\right) + b \int \frac{x^5 \arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^2x^4 + 2dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2\*a\*(d^2/(e^4\*x^2 + d\*e^3) - x^2/e^2 + 2\*d\*log(e\*x^2 + d)/e^3) + b\*integrate(x^5\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^5\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out



$$3.97 \quad \int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=570

$$\frac{(a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2 d + e}}}\right)}{2e^2} + \frac{(a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2 d + e}}}\right)}{2e^2} + \frac{(a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2 d + e}}}\right)}{2e^2}$$

[Out]  $\frac{1}{2}(-a - b \operatorname{arcsec}(cx)) / (e + d/x^2) - (a + b \operatorname{arcsec}(cx)) \ln(1 + (1/c/x + I(1 - 1/c^2/x^2)^{1/2})^2 / e^{2+1/2} (a + b \operatorname{arcsec}(cx)) \ln(1 - c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^{2+1/2} (a + b \operatorname{arcsec}(cx)) \ln(1 + c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^{2+1/2} (a + b \operatorname{arcsec}(cx)) \ln(1 - c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^{2+1/2} (a + b \operatorname{arcsec}(cx)) \ln(1 + c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^{2+1/2} I * b * \operatorname{polylog}(2, -(1/c/x + I(1 - 1/c^2/x^2)^{1/2})^2) / e^{2-1/2} I * b * \operatorname{polylog}(2, -c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^{2-1/2} I * b * \operatorname{polylog}(2, c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2*d + e)^{1/2})) / e^{2-1/2} I * b * \operatorname{polylog}(2, -c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (c^2*d + e)^{1/2})) / e^{2-1/2} I * b * \operatorname{polylog}(2, c(1/c/x + I(1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (c^2*d + e)^{1/2})) / e^{2-1/2} b * \arctan((c^2*d + e)^{1/2} / c/x / e^{1/2} / (1 - 1/c^2/x^2)^{1/2}) / e^{3/2} / (c^2*d + e)^{1/2}$

**Rubi [A]** time = 1.22, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5240, 4734, 4626, 3719, 2190, 2279, 2391, 4730, 377, 205, 4742, 4520}

$$\frac{ibPolyLog\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2 d + e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2 d + e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{2e^2} - \frac{ibPolyLog\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2, x]

[Out]  $-(a + b \operatorname{ArcSec}[c*x]) / (2*e*(e + d/x^2)) - (b \operatorname{ArcTan}[\operatorname{Sqrt}[c^2*d + e] / (c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]]*x)) / (2*e^{3/2}*\operatorname{Sqrt}[c^2*d + e]) + ((a + b \operatorname{ArcSec}[c*x]) * \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcSec}[c*x])}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])]) /$

$$\begin{aligned} & (2e^2) + ((a + b \operatorname{ArcSec}[c*x]) \operatorname{Log}[1 + (c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])})] / (\sqrt{e} - \sqrt{c^2*d + e})) / (2e^2) + ((a + b \operatorname{ArcSec}[c*x]) \operatorname{Log}[1 - (c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])})] / (\sqrt{e} + \sqrt{c^2*d + e})) / (2e^2) + ((a + b \operatorname{ArcSec}[c*x]) \operatorname{Log}[1 + (c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])})] / (\sqrt{e} + \sqrt{c^2*d + e})) / (2e^2) - ((a + b \operatorname{ArcSec}[c*x]) \operatorname{Log}[1 + E^{(2I) \operatorname{ArcSec}[c*x]}] / e^2 - ((I/2) * b * \operatorname{PolyLog}[2, -((c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])}) / (\sqrt{e} - \sqrt{c^2*d + e}))]) / e^2 - ((I/2) * b * \operatorname{PolyLog}[2, (c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])}) / (\sqrt{e} - \sqrt{c^2*d + e}))]) / e^2 - ((I/2) * b * \operatorname{PolyLog}[2, -((c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])}) / (\sqrt{e} + \sqrt{c^2*d + e}))]) / e^2 - ((I/2) * b * \operatorname{PolyLog}[2, (c \sqrt{-d} * E^{(I \operatorname{ArcSec}[c*x])}) / (\sqrt{e} + \sqrt{c^2*d + e}))]) / e^2 + ((I/2) * b * \operatorname{PolyLog}[2, -E^{(2I) \operatorname{ArcSec}[c*x]}] / e^2 \end{aligned}$$

### Rule 205

$$\operatorname{Int}[(a + (b \cdot x^2)^{-1}), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2] * \operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]])/a, x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$$

### Rule 377

$$\operatorname{Int}[(a + (b \cdot x^{n_1})^{p_1}) / ((c + (d \cdot x^{n_2}))^{m_1}), x_{\text{Symbol}}] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/(a + b \cdot x^n)^{1/n}] \text{ ; FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \operatorname{EqQ}[n \cdot p + 1, 0] \ \&\& \ \operatorname{IntegerQ}[n]$$

### Rule 2190

$$\operatorname{Int}[(F^{(g \cdot (e + f \cdot x))})^{n_1} \cdot ((c + (d \cdot x))^{m_1}) / ((a + (b \cdot F^{(g \cdot (e + f \cdot x))})^{n_1}) / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F])), x] - \operatorname{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(c + d \cdot x)^{m-1} \cdot \operatorname{Log}[1 + (b \cdot F^{(g \cdot (e + f \cdot x))})^n] / a], x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \ \operatorname{IGtQ}[m, 0]$$

### Rule 2279

$$\operatorname{Int}[\operatorname{Log}[(a + (b \cdot F^{(e \cdot (c + d \cdot x))})^n)], x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/(d \cdot e \cdot n \cdot \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b \cdot x]/x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] \text{ ; FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \operatorname{GtQ}[a, 0]$$

### Rule 2391

$$\operatorname{Int}[\operatorname{Log}[(c + (d \cdot e^{(x^n)}) / (x)], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c \cdot e^{x^n})] / n, x] \text{ ; FreeQ}\{c, d, e, n, x\} \ \&\& \ \operatorname{EqQ}[c \cdot d, 1]$$

### Rule 3719

$$\operatorname{Int}[(c + (d \cdot x))^{m_1} \cdot \tan[(e + f \cdot x)], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(I \cdot (c + d \cdot x)^{m+1}) / (d \cdot (m+1)), x] - \operatorname{Dist}[2 \cdot I, \operatorname{Int}[(c + d \cdot x)^m \cdot E^{(2I \cdot (e + f \cdot x))}], x]$$

$$\int \frac{(f(x))^{m+1}}{(1 + E^{2I*(e + f*x)})} dx$$
 ; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

### Rule 4520

$$\int \frac{((e_.) + (f_.)*(x_))^{(m_.)} \sin[(c_.) + (d_.)*(x_)]}{(\cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))} dx$$
 :> Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (-Dist[I, Int[((e + f\*x)^m \* E^(I\*(c + d\*x)))/(a - Rt[a^2 - b^2, 2] + b \* E^(I\*(c + d\*x))], x], x] - Dist[I, Int[((e + f\*x)^m \* E^(I\*(c + d\*x)))/(a + Rt[a^2 - b^2, 2] + b \* E^(I\*(c + d\*x))], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

### Rule 4626

$$\int ((a_.) + \arccos[(c_.)*(x_)]*(b_))^{(n_.)} / (x_.) dx$$
 :> -Subst[Int[(a + b\*x)^n / Cot[x], x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

### Rule 4730

$$\int ((a_.) + \arccos[(c_.)*(x_)]*(b_)) * (x_.) * ((d_.) + (e_.)*(x_.)^2)^{(p_.)} dx$$
 :> Simp[((d + e\*x^2)^(p + 1) \* (a + b \* ArcCos[c\*x])) / (2 \* e \* (p + 1)), x] + Dist[(b\*c) / (2 \* e \* (p + 1)), Int[(d + e\*x^2)^(p + 1) / Sqrt[1 - c^2 \* x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2 \* d + e, 0] && NeQ[p, -1]

### Rule 4734

$$\int ((a_.) + \arccos[(c_.)*(x_)]*(b_))^{(n_.)} * ((f_.)*(x_))^{(m_.)} * ((d_.) + (e_.)*(x_.)^2)^{(p_.)} dx$$
 :> Int[ExpandIntegrand[(a + b \* ArcCos[c\*x])^n, (f\*x)^m \* (d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2 \* d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

### Rule 4742

$$\int ((a_.) + \arccos[(c_.)*(x_)]*(b_))^{(n_.)} / ((d_.) + (e_.)*(x_)) dx$$
 :> -Subst[Int[(a + b\*x)^n \* Sin[x] / (c\*d + e \* Cos[x]), x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

### Rule 5240

$$\int ((a_.) + \operatorname{ArcSec}[(c_.)*(x_)]*(b_))^{(n_.)} * (x_.)^{(m_.)} * ((d_.) + (e_.)*(x_.)^2)^{(p_.)} dx$$
 :> -Subst[Int[((e + d\*x^2)^p \* (a + b \* ArcCos[x/c])^n) / x^(m + 2 \* (p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e^2 x} - \frac{dx (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{dx (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x, \right. \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \text{Subst} \left( \int \frac{x (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \text{Subst} \left( \int \left( -\frac{\sqrt{-d}}{2d} \right)}{e^2} \right. \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{\text{Subst} \left( \int (a + bx) \tan(x) dx, x, \sec^{-1}(cx) \right)}{e^2} + \frac{d \text{Subst} \left( \int \left( -\frac{\sqrt{-d}}{2d} \right)}{e^2} \right. \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \sec^{-1}(cx))^2}{2be^2} - \frac{(2i) \text{Subst} \left( \int \frac{e^{2ix} (a + bx)}{1 + e^{2ix}} dx, x, \sec^{-1}(cx) \right)}{e^2} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \sec^{-1}(cx))^2}{2be^2} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} - \frac{(a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)})}{e^2} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} - \frac{(a + b \sec^{-1}(cx)) \log(1 + e^{2i \sec^{-1}(cx)})}{e^2} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c \sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^2} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c \sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^2} \\
&= -\frac{a + b \sec^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \sec^{-1}(cx)) \log \left( 1 - \frac{c \sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^2}
\end{aligned}$$

**Mathematica [B]** time = 1.32, size = 1213, normalized size = 2.13

$$\frac{2ad}{ex^2+d} + \frac{b \sec^{-1}(cx)\sqrt{d}}{\sqrt{d}-i\sqrt{e}x} + \frac{b \sec^{-1}(cx)\sqrt{d}}{i\sqrt{e}x+\sqrt{d}} + 2b \sin^{-1}\left(\frac{1}{cx}\right) + 8ib \sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right) \tan^{-1}\left(\frac{(\sqrt{e}-ic\sqrt{d})\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{dc^2+e}}\right) + 8ib \sin$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] ((2\*a\*d)/(d + e\*x^2) + (b\*Sqrt[d]\*ArcSec[c\*x])/(Sqrt[d] - I\*Sqrt[e]\*x) + (b\*Sqrt[d]\*ArcSec[c\*x])/(Sqrt[d] + I\*Sqrt[e]\*x) + 2\*b\*ArcSin[1/(c\*x)] + (8\*I)\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] + (8\*I)\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] + 2\*b\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 4\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 2\*b\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 4\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 2\*b\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - 4\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 2\*b\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - 4\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - 4\*b\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - (b\*Sqrt[e]\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)]/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))]/Sqrt[-(c^2\*d) - e] - (b\*Sqrt[e]\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)]/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))]/Sqrt[-(c^2\*d) - e] + 2\*a\*Log[d + e\*x^2] - (2\*I)\*b\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (2\*I)\*b\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (2\*I)\*b\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (2\*I)\*b\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (2\*I)\*b\*PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])]]/(4\*e^2)

**fricas [F]** time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3 \operatorname{arcsec}(cx) + ax^3}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^3\*arcsec(c\*x) + a\*x^3)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] Timed out

**maple** [C] time = 1.37, size = 594, normalized size = 1.04

$$\frac{a \ln(c^2 e x^2 + c^2 d)}{2e^2} + \frac{c^2 a d}{2e^2 (c^2 e x^2 + c^2 d)} - \frac{c^2 b x^2 \operatorname{arcsec}(c x)}{2e (c^2 e x^2 + c^2 d)} + \frac{i b \sqrt{e (c^2 d + e)} \operatorname{arctanh}\left(\frac{2c^2 d \left(\frac{1}{c x} + i \sqrt{1 - \frac{1}{c^2 x^2}}\right)^2 + 2c^2 d + 4e}{4\sqrt{c^2 e d + e^2}}\right)}{2 (c^2 d + e) e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

[Out]  $\frac{1}{2} a / e^2 * \ln(c^2 * e * x^2 + c^2 * d) + 1/2 * c^2 * a / e^2 * d / (c^2 * e * x^2 + c^2 * d) - 1/2 * c^2 * b * x^2 * \operatorname{arcsec}(c * x) / e / (c^2 * e * x^2 + c^2 * d) + 1/2 * I * b * (e * (c^2 * d + e))^{(1/2)} / (c^2 * d + e) / e^2 * \operatorname{arctanh}(1/4 * (2 * c^2 * d * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})^2 + 2 * c^2 * d + 4 * e) / (c^2 * d * e + e^2)^{(1/2)}) - 1/4 * I * b / e^2 * \sum((\_R1^2 * c^2 * d + c^2 * d + 4 * e) / (\_R1^2 * c^2 * d + c^2 * d + 2 * e) * (I * \operatorname{arcsec}(c * x) * \ln((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1) + \operatorname{dilog}((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1)), \_R1 = \operatorname{RootOf}(c^2 * d * \_Z^4 + (2 * c^2 * d + 4 * e) * \_Z^2 + c^2 * d)) - b / e^2 * \operatorname{arcsec}(c * x) * \ln(1 + I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) - b / e^2 * \operatorname{arcsec}(c * x) * \ln(1 - I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) + I * b / e^2 * \operatorname{dilog}(1 + I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) + I * b / e^2 * \operatorname{dilog}(1 - I * (1/c/x + I * (1 - 1/c^2/x^2)^{(1/2)})) - 1/4 * I * c^2 * b / e^2 * \sum((\_R1^2 + 1) / (\_R1^2 * c^2 * d + c^2 * d + 2 * e) * (I * \operatorname{arcsec}(c * x) * \ln((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1) + \operatorname{dilog}((\_R1 - 1/c/x - I * (1 - 1/c^2/x^2)^{(1/2)}) / \_R1)), \_R1 = \operatorname{RootOf}(c^2 * d * \_Z^4 + (2 * c^2 * d + 4 * e) * \_Z^2 + c^2 * d)) * d$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{d}{e^3 x^2 + d e^2} + \frac{\log(e x^2 + d)}{e^2} \right) + b \int \frac{x^3 \arctan(\sqrt{c x + 1} \sqrt{c x - 1})}{e^2 x^4 + 2 d e x^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(d/(e^3\*x^2 + d\*e^2) + log(e\*x^2 + d)/e^2) + b\*integrate(x^3\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^3\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out



$$3.98 \quad \int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$$

Optimal. Leaf size=131

$$-\frac{a+b \sec^{-1}(cx)}{2e(d+ex^2)} + \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

[Out]  $1/2*(-a-b*\text{arcsec}(c*x))/e/(e*x^2+d)+1/2*b*c*x*\text{arctan}((c^2*x^2-1)^{(1/2)})/d/e/(c^2*x^2)^{(1/2)}-1/2*b*c*x*\text{arctan}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)})/(c^2*d+e)^{(1/2)}/d/e^{(1/2)}/(c^2*d+e)^{(1/2)}/(c^2*x^2)^{(1/2)}$

**Rubi** [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5236, 446, 86, 63, 205}

$$-\frac{a+b \sec^{-1}(cx)}{2e(d+ex^2)} + \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out]  $-(a+b*\text{ArcSec}[c*x])/(2*e*(d+e*x^2))+(b*c*x*\text{ArcTan}[\text{Sqrt}[-1+c^2*x^2]])/(2*d*e*\text{Sqrt}[c^2*x^2])-(b*c*x*\text{ArcTan}[(\text{Sqrt}[e]*\text{Sqrt}[-1+c^2*x^2])/\text{Sqrt}[c^2*d+e]])/(2*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d+e]*\text{Sqrt}[c^2*x^2])$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-(a\*d)/b+(d\*x^p)/b)^n, x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 86

Int[((e\_.) + (f\_.)\*(x\_))^(p\_)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 5236

Int[((a\_) + ArcSec[(c\_)\*(x\_)]\*(b\_))\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSec[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x(a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx &= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex^2)} + \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex)} dx}{2e\sqrt{c^2x^2}} \\
 &= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex^2)} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{4e\sqrt{c^2x^2}} \\
 &= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex^2)} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{4d\sqrt{c^2x^2}} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{4de\sqrt{c^2x^2}} \\
 &= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex^2)} - \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}} dx, x, \sqrt{-1+c^2x^2}\right)}{2cd\sqrt{c^2x^2}} + \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}} dx, x, \sqrt{-1+c^2x^2}\right)}{2cde\sqrt{c^2x^2}} \\
 &= -\frac{a + b \sec^{-1}(cx)}{2e(d + ex^2)} + \frac{bcx \tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.64, size = 286, normalized size = 2.18

$$\frac{-\frac{2a}{d+ex^2} + \frac{b\sqrt{e} \log\left(\frac{4cd\sqrt{e}x\left(c\sqrt{d-i}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)+4ide}{b\sqrt{c^2(-d)-e}(\sqrt{d+i}\sqrt{e}x)}\right)}{d\sqrt{c^2(-d)-e}} + \frac{b\sqrt{e} \log\left(\frac{4cd\sqrt{e}x\left(c\sqrt{d+i}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)-4ide}{b\sqrt{c^2(-d)-e}(\sqrt{d-i}\sqrt{e}x)}\right)}{d\sqrt{c^2(-d)-e}} - \frac{2b \sec^{-1}(cx)}{d+ex^2} - \frac{2b \sin^{-1}\left(\frac{1}{cx}\right)}{d}}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] ((-2\*a)/(d + e\*x^2) - (2\*b\*ArcSec[c\*x]))/(d + e\*x^2) - (2\*b\*ArcSin[1/(c\*x)])  
/d + (b\*Sqrt[e]\*Log[((4\*I)\*d\*e + 4\*c\*d\*Sqrt[e]\*(c\*Sqrt[d] - I\*Sqrt[-(c^2\*d)  
- e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x]/(b\*Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*  
x)))/(d\*Sqrt[-(c^2\*d) - e]) + (b\*Sqrt[e]\*Log[((-4\*I)\*d\*e + 4\*c\*d\*Sqrt[e]\*(  
c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x]/(b\*Sqrt[-(c^2\*d)  
- e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))/(d\*Sqrt[-(c^2\*d) - e])/(4\*e)

**fricas [A]** time = 0.61, size = 384, normalized size = 2.93

$$\left[ \frac{2ac^2d^2 + 2ade + \sqrt{-c^2de - e^2}(bex^2 + bd) \log\left(\frac{c^2ex^2 - c^2d + 2\sqrt{-c^2de - e^2}\sqrt{c^2x^2 - 1} - 2e}{ex^2 + d}\right) + 2(bc^2d^2 + bde) \operatorname{arcsec}(cx) - 4(c^2d^3e + d^2e^2 + (c^2d^2e^2 + de^3)x^2)}{4(c^2d^3e + d^2e^2 + (c^2d^2e^2 + de^3)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] [-1/4\*(2\*a\*c^2\*d^2 + 2\*a\*d\*e + sqrt(-c^2\*d\*e - e^2)\*(b\*e\*x^2 + b\*d)\*log((c^2  
2\*e\*x^2 - c^2\*d + 2\*sqrt(-c^2\*d\*e - e^2)\*sqrt(c^2\*x^2 - 1) - 2e)/(e\*x^2 +  
d)) + 2\*(b\*c^2\*d^2 + b\*d\*e)\*arcsec(c\*x) - 4\*(b\*c^2\*d^2 + b\*d\*e + (b\*c^2\*d\*e  
+ b\*e^2)\*x^2)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)))/(c^2\*d^3\*e + d^2\*e^2 + (c^2  
2\*d^2\*e^2 + d\*e^3)\*x^2), -1/2\*(a\*c^2\*d^2 + a\*d\*e + sqrt(c^2\*d\*e + e^2)\*(b\*e  
\*x^2 + b\*d)\*arctan(sqrt(c^2\*d\*e + e^2)\*sqrt(c^2\*x^2 - 1)/(c^2\*d + e)) + (b\*  
c^2\*d^2 + b\*d\*e)\*arcsec(c\*x) - 2\*(b\*c^2\*d^2 + b\*d\*e + (b\*c^2\*d\*e + b\*e^2)\*x  
^2)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)))/(c^2\*d^3\*e + d^2\*e^2 + (c^2\*d^2\*e^2 +  
d\*e^3)\*x^2)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Va  
 lue

**maple [B]** time = 0.08, size = 354, normalized size = 2.70

$$\frac{c^2 a}{2e(c^2 e x^2 + c^2 d)} - \frac{c^2 b \operatorname{arcsec}(cx)}{2e(c^2 e x^2 + c^2 d)} - \frac{b\sqrt{c^2 x^2 - 1} \operatorname{arctan}\left(\frac{1}{\sqrt{c^2 x^2 - 1}}\right)}{2ce\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x d} + \frac{b\sqrt{c^2 x^2 - 1} \ln\left(\frac{2\sqrt{-\frac{c^2 d + e}{e}} \sqrt{c^2 x^2 - 1} e^{-2\sqrt{-c^2 d} cx}}{cex + \sqrt{-c^2 d}}\right)}{4ce\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} x d \sqrt{-\frac{c^2 d + e}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

[Out]  $-1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*\operatorname{arcsec}(c*x)-1/2/c*b/e*(c^2*x^2-1)^{(1/2)/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d*\operatorname{arctan}(1/(c^2*x^2-1)^{(1/2))}+1/4/c*b/e*(c^2*x^2-1)^{(1/2)/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(-(c^2*d+e)/e)^{(1/2)*\ln(2*((-(c^2*d+e)/e)^{(1/2)*(c^2*x^2-1)^{(1/2)*e-(-(c^2*e*d)^{(1/2)*c*x-e)/(c*e*x+(-(c^2*e*d)^{(1/2))}))+1/4/c*b/e*(c^2*x^2-1)^{(1/2)/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(-(c^2*d+e)/e)^{(1/2)*\ln(-2*((-(c^2*d+e)/e)^{(1/2)*(c^2*x^2-1)^{(1/2)*e+(-(c^2*e*d)^{(1/2)*c*x-e)/(-c*e*x+(-(c^2*e*d)^{(1/2))})}$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left( (c^2 e^2 x^2 + c^2 d e) \int \frac{x e^{\left(\frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1)\right)}}{c^2 e^2 x^4 + (c^2 e^2 x^4 + (c^2 d e - e^2) x^2 - d e)(cx+1)(cx-1) + (c^2 d e - e^2) x^2 - d e} dx - \operatorname{arctan}\left(\sqrt{cx+1} \sqrt{cx-1}\right) \right) b}{2(e^2 x^2 + d e)} - \frac{a}{2(e^2 x^2 + d e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out]  $1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*\operatorname{integrate}(1/2*x*e^{(1/2*\log(c*x + 1) + 1/2*\log(c*x - 1))}/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)*e^{(\log(c*x + 1) + \log(c*x - 1))}, x) - \operatorname{arctan}(\sqrt{cx + 1}*\sqrt{cx - 1}))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{asec}(cx))}{(d + ex^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

$$3.99 \quad \int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^2} dx$$

**Optimal.** Leaf size=546

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

[Out]  $-1/2 * e * (a + b * \text{arcsec}(c * x)) / d^2 / (e + d/x^2) + 1/2 * I * (a + b * \text{arcsec}(c * x))^2 / b / d^2 - 1/2 * (a + b * \text{arcsec}(c * x)) * \ln(1 - c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2 * d + e)^{1/2}) / d^2 - 1/2 * (a + b * \text{arcsec}(c * x)) * \ln(1 + c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2 * d + e)^{1/2}) / d^2 - 1/2 * (a + b * \text{arcsec}(c * x)) * \ln(1 - c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (c^2 * d + e)^{1/2}) / d^2 - 1/2 * (a + b * \text{arcsec}(c * x)) * \ln(1 + c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (c^2 * d + e)^{1/2}) / d^2 + 1/2 * I * b * \text{polylog}(2, -c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2 * d + e)^{1/2}) / d^2 + 1/2 * I * b * \text{polylog}(2, c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (c^2 * d + e)^{1/2}) / d^2 + 1/2 * I * b * \text{polylog}(2, -c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (c^2 * d + e)^{1/2}) / d^2 + 1/2 * I * b * \text{polylog}(2, c * (1/c/x + I * (1 - 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (c^2 * d + e)^{1/2}) / d^2 - 1/2 * b * \arctan((c^2 * d + e)^{1/2} / c / x / e^{1/2} / (1 - 1/c^2/x^2)^{1/2}) * e^{1/2} / d^2 / (c^2 * d + e)^{1/2}$

**Rubi [A]** time = 1.14, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5240, 4734, 4730, 377, 205, 4742, 4520, 2190, 2279, 2391}

$$\frac{ib \text{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib \text{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib \text{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} + \frac{ib \text{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^2), x]

[Out]  $-(e * (a + b * \text{ArcSec}[c * x])) / (2 * d^2 * (e + d/x^2)) + ((I/2) * (a + b * \text{ArcSec}[c * x])^2) / (b * d^2) - (b * \text{Sqrt}[e] * \text{ArcTan}[\text{Sqrt}[c^2 * d + e] / (c * \text{Sqrt}[e] * \text{Sqrt}[1 - 1/(c^2 * x^2)]] * x)) / (2 * d^2 * \text{Sqrt}[c^2 * d + e]) - ((a + b * \text{ArcSec}[c * x]) * \text{Log}[1 - (c * \text{Sqrt}[-d] * E^{I * \text{ArcSec}[c * x]})] / (\text{Sqrt}[e] - \text{Sqrt}[c^2 * d + e])) / (2 * d^2) - ((a + b * \text{ArcSec}[$

$$c*x)]*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))]/(2*d^2) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))]/(2*d^2) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))]/(2*d^2) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/d^2$$
Rule 205

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]]/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$$
Rule 377

$$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}/((c_ + (d_)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$$
Rule 2190

$$\text{Int}[(F_)^{((g_)*((e_ + (f_)*(x_)))^{(n_)*((c_ + (d_)*(x_))^{(m_)})))/((a_ + (b_)*((F_)^{((g_)*((e_ + (f_)*(x_)))^{(n_)}))^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*Log[F]), \text{Int}[(c + d*x)^{(m-1)}*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$$
Rule 2279

$$\text{Int}[\text{Log}[(a_ + (b_)*((F_)^{((e_)*((c_ + (d_)*(x_)))^{(n_)}))], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*Log[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$
Rule 2391

$$\text{Int}[\text{Log}[(c_)*((d_ + (e_)*(x_)^{(n_)}))]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$
Rule 4520

$$\text{Int}[(e_ + (f_)*(x_))^{(m_)*\text{Sin}[(c_ + (d_)*(x_))]/(\text{Cos}[(c_ + (d_)*(x_))*b_ + a_]), x\_Symbol] \rightarrow \text{Simp}[(I*(e + f*x)^{(m+1)})/(b*f*(m+1)), x] + (-\text{Dist}[I, \text{Int}[(e + f*x)^m*E^(I*(c + d*x))]/(a - \text{Rt}[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x] - \text{Dist}[I, \text{Int}[(e + f*x)^m*E^(I*(c + d*x))]/(a +$$

$\text{Rt}[a^2 - b^2, 2] + b \cdot E^{(I \cdot (c + d \cdot x))}, x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

### Rule 4730

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) \cdot (x_)] \cdot (b_.)] \cdot (x_.) \cdot ((d_.) + (e_.) \cdot (x_.)^2)^{(p_.)}, x\_ \text{Symbol}]$  :>  $\text{Simp}[(d + e \cdot x^2)^{(p + 1)} \cdot (a + b \cdot \text{ArcCos}[c \cdot x]) / (2 \cdot e \cdot (p + 1)), x] + \text{Dist}[(b \cdot c) / (2 \cdot e \cdot (p + 1)), \text{Int}[(d + e \cdot x^2)^{(p + 1)} / \text{Sqrt}[1 - c^2 \cdot x^2], x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2 \cdot d + e, 0] && NeQ[p, -1]

### Rule 4734

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) \cdot (x_)] \cdot (b_.)]^{(n_.)} \cdot ((f_.) \cdot (x_.)^m)^{(m_.)} \cdot ((d_.) + (e_.) \cdot (x_.)^2)^{(p_.)}, x\_ \text{Symbol}]$  :>  $\text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcCos}[c \cdot x])^n, (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2 \cdot d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

### Rule 4742

$\text{Int}[(a_.) + \text{ArcCos}[(c_.) \cdot (x_)] \cdot (b_.)]^{(n_.)} / ((d_.) + (e_.) \cdot (x_)), x\_ \text{Symbol}]$  :>  $-\text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Sin}[x] / (c \cdot d + e \cdot \text{Cos}[x]), x], x, \text{ArcCos}[c \cdot x]] /;$  FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

### Rule 5240

$\text{Int}[(a_.) + \text{ArcSec}[(c_.) \cdot (x_)] \cdot (b_.)]^{(n_.)} \cdot (x_.)^m \cdot ((d_.) + (e_.) \cdot (x_.)^2)^{(p_.)}, x\_ \text{Symbol}]$  :>  $-\text{Subst}[\text{Int}[(e + d \cdot x^2)^p \cdot (a + b \cdot \text{ArcCos}[x/c])^n / x^{(m + 2 \cdot (p + 1))}, x], x, 1/x] /;$  FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

### Rubi steps



$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{x^3 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{ex \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)^2} + \frac{x \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{x^{a+b \cos^{-1} \left( \frac{x}{c} \right)}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{x^{a+b \cos^{-1} \left( \frac{x}{c} \right)}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int \left( -\frac{\sqrt{-d} \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} - \sqrt{-d}x)} + \frac{\sqrt{-d} \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d} \quad (be) \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\text{Subst} \left( \int \frac{a+b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} - \frac{\text{Subst} \left( \int \frac{a+b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} - \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2d+e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}x}} \right)}{2d^2 \sqrt{c^2d+e}} - \frac{\text{Subst} \left( \int \frac{(a+bx) \sin(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cos(x)} dx, x, \sec^{-1} \left( \frac{x}{c} \right) \right)}{2(-d)^{3/2}} \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i(a + b \sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2d+e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}x}} \right)}{2d^2 \sqrt{c^2d+e}} + \frac{i \text{Subst} \left( \int \frac{\sqrt{e}}{c} dx, x, \sec^{-1} \left( \frac{x}{c} \right) \right)}{2(-d)^{3/2}} \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i(a + b \sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2d+e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}x}} \right)}{2d^2 \sqrt{c^2d+e}} - \frac{(a + b \sec^{-1}(cx))}{2(-d)^{3/2}} \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i(a + b \sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2d+e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}x}} \right)}{2d^2 \sqrt{c^2d+e}} - \frac{(a + b \sec^{-1}(cx))}{2(-d)^{3/2}} \\
&= -\frac{e(a + b \sec^{-1}(cx))}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i(a + b \sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2d+e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}x}} \right)}{2d^2 \sqrt{c^2d+e}} - \frac{(a + b \sec^{-1}(cx))}{2(-d)^{3/2}}
\end{aligned}$$

**Mathematica** [F] time = 37.05, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^2), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^2), x]

**fricas** [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{e^2x^5 + 2dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [C] time = 2.68, size = 3095, normalized size = 5.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^2,x)

[Out]  $-1/2*a/d^2*\ln(c^2*e*x^2+c^2*d)+a/d^2*\ln(c*x)+1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-$   
 $1/2*I*b/c^2*polylog(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^(1/2)/(-c^2*d-2*(e*(c$   
 $^2*d+e))^(1/2)-2*e))/d^3*(e*(c^2*d+e))^(1/2)-1/4*I*b*c^2*polylog(2,c^2*d*(1$

$$\begin{aligned}
& /c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))/d/(c^2*d+e)-2*b/c^4/d^4*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)*e^{2+1/2*b*c^2/d/(c^2*d+e)}*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)+I*b*\operatorname{arcsec}(c*x)^2/d^2-1/2*b/d^2*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)+1/4*I*b*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))/d^2+1/2*I*b/d^2*\sum((\_R1^2*c^2*d+2*c^2*d+4*e)/(\_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/\_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/\_R1)),\_R1=\operatorname{RootOf}(c^2*d*\_Z^4+(2*c^2*d+4*e)*\_Z^2+c^2*d))-5/4*I*b*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))/d^2/(c^2*d+e)*e^{-1/4*I*b*(e*(c^2*d+e))^{(1/2)}/d^2/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e))-5/2*I*b*\operatorname{arcsec}(c*x)^2/d^2/(c^2*d+e)*e+I*b/c^4*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*e^{2/d^4+2*I*b/c^4*\operatorname{arcsec}(c*x)^2*e^{2/d^4+I*b/c^2*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*e/d^3+1/2*I*b*(e*(c^2*d+e))^{(1/2)}/d^2/(c^2*d+e)*\operatorname{arctanh}(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^{(1/2)}+3/4*I*b*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))/d^2/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}+4*b/c^2*e^{2/(c^2*d+e)}/d^3*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)+2*b/c^4*e^3/(c^2*d+e)/d^4*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)+2*b/c^4/d^4*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)*e*(e*(c^2*d+e))^{(1/2)}-2*I*b/c^2*e^2*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e)))/(c^2*d+e)/d^3-I*b/c^4*e^3*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e)))/(c^2*d+e)/d^4-3/2*b/d^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)*e*(e*(c^2*d+e))^{(1/2)}+1/2*b*(e*(c^2*d+e))^{(1/2)}/d^2/(c^2*d+e)*\operatorname{arcsec}(c*x)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e))*e^{(1/2)}-2*e))/5/2*b/d^2*e/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)+b/c^2/d^3*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)*e*(e*(c^2*d+e))^{(1/2)}-2*b/c^2/d^3*\ln(1-c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*\operatorname{arcsec}(c*x)*e^{-1/2*I*b*c^2*\operatorname{arcsec}(c*x)^2/d/(c^2*d+e)-2*I*b/c^4*\operatorname{arcsec}(c*x)^2*e/d^4*(e*(c^2*d+e))^{(1/2)}-2*I*b/c^4*e^3*\operatorname{arcsec}(c*x)^2/(c^2*d+e)/d^4-I*b/c^4*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e))*e/d^4*(e*(c^2*d+e))^{(1/2)}-4*I*b/c^2*\operatorname{arcsec}(c*x)^2/d^3/(c^2*d+e)*e^{-1/2*b*c^2*x^2*\operatorname{arcsec}(c*x)*e/(c^2*d+e)^2+c^2*d)/d^2+2*I*b/c^2*\operatorname{arcsec}(c*x)^2*e/d^3-I*b/c^2*\operatorname{arcsec}(c*x)^2/d^3*(e*(c^2*d+e))^{(1/2)}+I*b*(e*(c^2*d+e))^{(1/2)}/d^2/(c^2*d+e)*\operatorname{arcsec}(c*x)^2+2*I*b/c^4*e^2*\operatorname{arcsec}(c*x)^2/(c^2*d+e)/d^4*(e*(c^2*d+e))^{(1/2)}+1/8*I*b*c^2*\operatorname{polylog}(2,c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)}-2*e)))/d/e/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}-3*b/c^2*e/(c^2*d+e)/d^3*\ln(1-c^2*d*(1/c/x
\end{aligned}$$

+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d-2\*(e\*(c^2\*d+e))^(1/2)-2\*e))\*arcsec(c\*x)\*(e\*(c^2\*d+e))^(1/2)-1/8\*I\*b\*c^2\*(e\*(c^2\*d+e))^(1/2)/e/(c^2\*d+e)/d\*polylog(2, c^2\*d\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d+2\*(e\*(c^2\*d+e))^(1/2)-2\*e))+I\*b/c^4\*e^2\*polylog(2, c^2\*d\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d-2\*(e\*(c^2\*d+e))^(1/2)-2\*e))/(c^2\*d+e)/d^4\*(e\*(c^2\*d+e))^(1/2)-2\*b/c^4\*e^2/(c^2\*d+e)/d^4\*ln(1-c^2\*d\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d-2\*(e\*(c^2\*d+e))^(1/2)-2\*e))\*arcsec(c\*x)\*(e\*(c^2\*d+e))^(1/2)-1/4\*b\*c^2/d/e/(c^2\*d+e)\*ln(1-c^2\*d\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d-2\*(e\*(c^2\*d+e))^(1/2)-2\*e))\*arcsec(c\*x)\*(e\*(c^2\*d+e))^(1/2)+1/4\*b\*c^2\*(e\*(c^2\*d+e))^(1/2)/e/(c^2\*d+e)/d\*arcsec(c\*x)\*ln(1-c^2\*d\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d+2\*(e\*(c^2\*d+e))^(1/2)-2\*e))+3\*I\*b/c^2\*arcsec(c\*x)^2/d^3/(c^2\*d+e)\*(e\*(c^2\*d+e))^(1/2)\*e+3/2\*I\*b/c^2\*e\*polylog(2, c^2\*d\*(1/c/x+I\*(1-1/c^2/x^2)^(1/2))^2/(-c^2\*d-2\*(e\*(c^2\*d+e))^(1/2)-2\*e))/(c^2\*d+e)/d^3\*(e\*(c^2\*d+e))^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{1}{dex^2+d^2}-\frac{\log(ex^2+d)}{d^2}+\frac{2\log(x)}{d^2}\right)+b\int\frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^2x^5+2dex^3+d^2x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(1/(d\*e\*x^2+d^2)-log(e\*x^2+d)/d^2+2\*log(x)/d^2)+b\*integrate(arctan(sqrt(c\*x+1)\*sqrt(c\*x-1))/(e^2\*x^5+2\*d\*e\*x^3+d^2\*x),x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x(e x^2+d)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*acos(1/(c\*x)))/(x\*(d+e\*x^2)^2),x)

[Out] int((a+b\*acos(1/(c\*x)))/(x\*(d+e\*x^2)^2),x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.100 \quad \int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=784

$$\frac{3\sqrt{-d} (a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d} (a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{4e^{5/2}} + \frac{3\sqrt{-d} (a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{4e^{5/2}}$$

[Out]  $x*(a+b*\text{arcsec}(c*x))/e^2 - b*\text{arctanh}((1-1/c^2/x^2)^{(1/2)})/c/e^2 + 3/4*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} - 3/4*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} + 3/4*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} - 3/4*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} + 3/4*I*b*\text{polylog}(2, -c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} - 3/4*I*b*\text{polylog}(2, c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} + 3/4*I*b*\text{polylog}(2, -c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} - 3/4*I*b*\text{polylog}(2, c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)} - 1/4*d*(a+b*\text{arcsec}(c*x))/e^2/(-d/x+(-d)^{(1/2)}*e^{(1/2)}) + 1/4*d*(a+b*\text{arcsec}(c*x))/e^2/(d/x+(-d)^{(1/2)}*e^{(1/2)}) - 1/4*b*\text{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}*d^{(1/2)}/e^2/(c^2*d+e)^{(1/2)} - 1/4*b*\text{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}*d^{(1/2)}/e^2/(c^2*d+e)^{(1/2)}$

**Rubi [A]** time = 2.35, antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5240, 4734, 4628, 266, 63, 208, 4668, 4744, 725, 206, 4742, 4520, 2190, 2279, 2391}

$$\frac{3ib\sqrt{-d} \text{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{-d} \text{PolyLog}\left(2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d} \text{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}}\right)}{4e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2, x]

```
[Out] -(d*(a + b*ArcSec[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcSec[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSec[c*x]))/e^2 - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) - (b*Sqrt[d]*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])))/(4*e^2*Sqrt[c^2*d + e]) - (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])))/(4*e^2*Sqrt[c^2*d + e]) + (3*Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, -(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, -(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4520

```
Int[(((e_) + (f_)*(x_))^(m_))*Sin[(c_) + (d_)*(x_)]/(Cos[(c_) + (d_)
*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 4628

```
Int[(((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_))*((d_)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

### Rule 4668

```
Int[(((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_))*((d_) + (e_)*(x_)^2)^(p_), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
```

tQ[p, 0] || IGtQ[n, 0])

#### Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] + Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

#### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e^2 x^2} - \frac{d (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{d (a + b \cos^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e^2} + \frac{b \text{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce^2} + \frac{d \text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= \frac{x (a + b \sec^{-1}(cx))}{e^2} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{5/2}} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{5/2}} \\
&= -\frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \sec^{-1}(cx))}{e^2} - \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{ce^2} \\
&= -\frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \sec^{-1}(cx))}{e^2} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{x^2}{c^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \sec^{-1}(cx))}{e^2} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{x^2}{c^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \sec^{-1}(cx))}{e^2} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{x^2}{c^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \sec^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \sec^{-1}(cx))}{e^2} - \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{x^2}{c^2}} \right)}{ce^2}
\end{aligned}$$

**Mathematica [A]** time = 2.54, size = 1331, normalized size = 1.70

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] (4\*a\*Sqrt[e]\*x + (2\*a\*d\*Sqrt[e]\*x)/(d + e\*x^2) - 6\*a\*Sqrt[d]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]] + b\*(4\*Sqrt[e]\*x\*ArcSec[c\*x] + (d\*ArcSec[c\*x])/((-I)\*Sqrt[d] + Sqrt[e]\*x) + (d\*ArcSec[c\*x])/(I\*Sqrt[d] + Sqrt[e]\*x) + 12\*Sqrt[d]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[(((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - 12\*Sqrt[d]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] + (3\*I)\*Sqrt[d]\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) + (6\*I)\*Sqrt[d]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) - (3\*I)\*Sqrt[d]\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) - (6\*I)\*Sqrt[d]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) - (3\*I)\*Sqrt[d]\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) + (6\*I)\*Sqrt[d]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) + (3\*I)\*Sqrt[d]\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) - (6\*I)\*Sqrt[d]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) + (I\*Sqrt[d]\*Sqrt[e]\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)]/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e] - (I\*Sqrt[d]\*Sqrt[e]\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)]/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e] + (4\*Sqrt[e]\*Log[Cos[ArcSec[c\*x]/2] - Sin[ArcSec[c\*x]/2])/c - (4\*Sqrt[e]\*Log[Cos[ArcSec[c\*x]/2] + Sin[ArcSec[c\*x]/2])/c - 3\*Sqrt[d]\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) + 3\*Sqrt[d]\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) + 3\*Sqrt[d]\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d]) - 3\*Sqrt[d]\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e]))\*E^(I\*ArcSec[c\*x])]/(c\*Sqrt[d])))/(4\*e^(5/2))

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{bx^4 \operatorname{arcsec}(cx) + ax^4}{e^2x^4 + 2dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



$$\frac{1}{2} a \left( \frac{dx}{e^3 x^2 + d e^2} - \frac{3 d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{2x}{e^2} \right) + b \int \frac{x^4 \arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{e^2 x^4 + 2 dex^2 + d^2}\right) dx}{e^2 x^4 + 2 dex^2 + d^2}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{dx}{e^3 x^2 + d e^2} - \frac{3 d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{2x}{e^2} \right) + b \int \frac{x^4 \arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{e^2 x^4 + 2 dex^2 + d^2}\right) dx}{e^2 x^4 + 2 dex^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(d\*x/(e^3\*x^2 + d\*e^2) - 3\*d\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e^2) + 2\*x/e^2) + b\*integrate(x^4\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^4\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

$$3.101 \quad \int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^2} dx$$

Optimal. Leaf size=745

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

[Out]  $\frac{1}{4}(a+b \operatorname{arcsec}(c x)) \ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}-1/4*(a+b \operatorname{arcsec}(c x)) \ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}+1/4*(a+b \operatorname{arcsec}(c x)) \ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}-1/4*(a+b \operatorname{arcsec}(c x)) \ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}+1/4*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}-1/4*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}+1/4*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}-1/4*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(3/2)}/(-d)^{(1/2)}+1/4*(a+b \operatorname{arcsec}(c x))/e/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/4*(-a-b \operatorname{arcsec}(c x))/e/(d/x+(-d)^{(1/2)}*e^{(1/2)})+1/4*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}/e/d^{(1/2)}/(c^2*d+e)^{(1/2)}+1/4*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}/e/d^{(1/2)}/(c^2*d+e)^{(1/2)}$

**Rubi [A]** time = 1.22, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5240, 4668, 4744, 725, 206, 4742, 4520, 2190, 2279, 2391}

$$\frac{ib \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

```
[Out] (a + b*ArcSec[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSec[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d +
e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) + ((a + b*ArcSec[c*
x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4
*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c
*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcSec[c
*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(
4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLo
g[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))]/(Sqrt[
-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]
*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))]/(Sqrt[-d]*e^(3/2)) - ((I
/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]
)]/(Sqrt[-d]*e^(3/2))
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)), x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 4668

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || IGtQ[n, 0])
```

### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

### Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] + Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{d(a + b \cos^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \cos^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} - \frac{d(a + b \cos^{-1} \left( \frac{x}{c} \right))}{2e(-de - d^2x^2)} \right) dx \right) \\
&= \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{-de - d^2x^2} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \text{Subst} \left( \int \frac{1}{(\sqrt{-d}\sqrt{e} - dx)\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} - \frac{b \text{Subst} \left( \int \frac{1}{(\sqrt{-d}\sqrt{e} + dx)\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} - \frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b \tanh^{-1} \left( \frac{c^2d + \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b \tanh^{-1} \left( \frac{c^2d + \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b \tanh^{-1} \left( \frac{c^2d + \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b \tanh^{-1} \left( \frac{c^2d + \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&= \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{a + b \sec^{-1}(cx)}{4e(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b \tanh^{-1} \left( \frac{c^2d + \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}e\sqrt{c^2d+e}}
\end{aligned}$$

**Mathematica [A]** time = 1.80, size = 1245, normalized size = 1.67

$$-\frac{2a\sqrt{e}x}{ex^2+d} + \frac{2a \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + b \left[ \frac{i \log\left(\frac{ie^{i \sec^{-1}(cx)}(\sqrt{e}-\sqrt{dc^2+e})}{c\sqrt{d}}+1\right) \sec^{-1}(cx)}{\sqrt{d}} + \frac{i \log\left(\frac{ie^{i \sec^{-1}(cx)}(\sqrt{dc^2+e}-\sqrt{e})}{c\sqrt{d}}+1\right) \sec^{-1}(cx)}{\sqrt{d}} + \frac{i \log\left(1-\frac{i(\sqrt{e}+\sqrt{dc^2+e})}{c\sqrt{d}}\right) \sec^{-1}(cx)}{\sqrt{d}} \right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] 
$$\begin{aligned} &((-2*a*\text{Sqrt}[e]*x)/(d + e*x^2) + (2*a*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/\text{Sqrt}[d] + \\ &b*(\text{ArcSec}[c*x]/(I*\text{Sqrt}[d] - \text{Sqrt}[e]*x) - \text{ArcSec}[c*x]/(I*\text{Sqrt}[d] + \text{Sqrt}[e]* \\ &x) - (4*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{ArcTan}[\frac{((-I)*\text{c}*\text{S} \\ &\text{qrt}[d] + \text{Sqrt}[e])*\text{Tan}[\text{ArcSec}[c*x]/2}]{\text{Sqrt}[c^2*d + e]})/\text{Sqrt}[d] + (4*\text{ArcSin} \\ &[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{ArcTan}[\frac{(I*\text{c}*\text{Sqrt}[d] + \text{Sqrt}[e]) \\ &*\text{Tan}[\text{ArcSec}[c*x]/2}]{\text{Sqrt}[c^2*d + e]})/\text{Sqrt}[d] - (I*\text{ArcSec}[c*x]*\text{Log}[1 + (I* \\ &(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])})/\text{Sqrt}[d] - ((2* \\ &I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \\ &\text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])])/\text{Sqrt}[d] + (I*\text{ArcSec}[c*x]* \\ &\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])])/\text{S} \\ &\text{qrt}[d] + ((2*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 + (I \\ &*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])])/\text{Sqrt}[d] + (I \\ &*\text{ArcSec}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{S} \\ &\text{qrt}[d])])/\text{Sqrt}[d] - ((2*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[2] \\ &]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])])/\text{S} \\ &\text{qrt}[d] - (I*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[ \\ &c*x])})/(\text{c}*\text{Sqrt}[d])])/\text{Sqrt}[d] + ((2*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{c}*\text{Sqrt}[d] \\ &)]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{S} \\ &\text{qrt}[d])])/\text{Sqrt}[d] - (I*\text{Sqrt}[e]*\text{Log}[(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(\text{Sqrt}[e] + \text{c}*(I*\text{c}*\text{Sqr} \\ &\text{t}[d] - \text{Sqrt}[-(c^2*d) - e]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x)/(\text{Sqrt}[-(c^2*d) - e]*(\text{S} \\ &\text{qrt}[d] - I*\text{Sqrt}[e]*x)))/(\text{Sqrt}[d]*\text{Sqrt}[-(c^2*d) - e]) + (I*\text{Sqrt}[e]*\text{Log}[(2*\text{S} \\ &\text{qrt}[d]*\text{Sqrt}[e]*(-\text{Sqrt}[e] + \text{c}*(I*\text{c}*\text{Sqrt}[d] + \text{Sqrt}[-(c^2*d) - e]*\text{Sqrt}[1 - 1/( \\ &c^2*x^2)])*x)/(\text{Sqrt}[-(c^2*d) - e]*(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x)))/(\text{Sqrt}[d]*\text{Sqrt} \\ &[-(c^2*d) - e]) + \text{PolyLog}[2, ((-I)*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec} \\ &[c*x])})/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[d] - \text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E \\ &^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[d] - \text{PolyLog}[2, ((-I)*(\text{Sqrt}[e] + \text{Sqrt}[c \\ &^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[d] + \text{PolyLog}[2, (I*(\text{Sqrt}[e] \\ &+ \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(\text{c}*\text{Sqrt}[d])]/\text{Sqrt}[d]))/(4*e^{(3/2)}) \end{aligned}$$

**fricas** [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arcsec}(cx) + ax^2}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

[Out] `integral((b*x^2*arcsec(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Va  
 lue

**maple** [C] time = 3.78, size = 1756, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)`

[Out] `-1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*c^2*b*arcsec(c*x)/e*x/(c^2*e*x^2+c^2*d)+I/c^4*b*((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)-1/2*I/c^2*b*((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+I/c^4*b*((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2))*e/(c^2*d+e)/d^3+I/c^4*b*(-(c^2*d-2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/((-c^2*d+2*(e*(c^2*d+e)))^(1/2)-2*e)*d^(1/2))/((c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)-1/2*I/c^2*b*((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2))/e/d^2-I/c^4*b*((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/((c^2*d+2*(e*(c^2*d+e)))^(1/2)+2*e)*d^(1/2))`

$$\begin{aligned} & *d+e)^{(1/2)+2*e}*d)^{(1/2))/d^3+I/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e} \\ & *d)^{(1/2)*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)))/((-c^2*d+2*(e*(c^2*d+e) \\ & )^{(1/2)-2*e}*d)^{(1/2)))/(c^2*d+e)/d^2-I/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2) \\ & +2*e}*d)^{(1/2)*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)))/((-c^2*d+2*(e*(c^2 \\ & *d+e))^{(1/2)-2*e}*d)^{(1/2)))/e/d^3*(e*(c^2*d+e))^{(1/2)-1/4}*I*c*b/e*\operatorname{sum}(1/_R1 \\ & /(_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2) \\ & )/_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2))/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+ \\ & (2*c^2*d+4*e)*_Z^2+c^2*d))-I/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1 \\ & /2)*\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+ \\ & 2*e}*d)^{(1/2)))/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)+1/2}*I/c^2*b*(-(c^2*d-2*(e \\ & (c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)))/(( \\ & -c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^{( \\ & 1/2)+1/4}*I*c*b/e*\operatorname{sum}(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c \\ & /x-I*(1-1/c^2/x^2)^{(1/2))/_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2))/_R1) \\ & ),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*I/c^2*b*(-(c^2*d-2*( \\ & e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)))/ \\ & ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)))/e/d^2+I/c^4*b*(-(c^2*d-2*(e*( \\ & c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)))/((- \\ & c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2))*e/(c^2*d+e)/d^3+I/c^2*b*((c^2*d+ \\ & 2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2) \\ & ))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)))/(c^2*d+e)/d^2-I/c^4*b*(-(c^2 \\ & *d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{( \\ & 1/2)))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)))/d^3 \end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{x}{e^2x^2+de}-\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e}\right)+b\int\frac{x^2\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^2x^4+2dex^2+d^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2\*a\*(x/(e^2\*x^2 + d\*e) - arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e)) + b\*integrate(x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(ex^2+d)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^2,x)

```
[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

$$3.102 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=739

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

[Out]  $-1/4*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}+1/4*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}-1/4*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}+1/4*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}-1/4*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}+1/4*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}-1/4*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}+1/4*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(1/2)}+1/4*(-a-b*\text{arcsec}(c*x))/d/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/4*(a+b*\text{arcsec}(c*x))/d/(d/x+(-d)^{(1/2)}*e^{(1/2)})-1/4*b*\text{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}/d^{(3/2)}/(c^2*d+e)^{(1/2)}-1/4*b*\text{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}/d^{(3/2)}/(c^2*d+e)^{(1/2)}$

**Rubi [A]** time = 2.21, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5230, 4734, 4668, 4744, 725, 206, 4742, 4520, 2190, 2279, 2391}

$$\frac{ib\text{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\text{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\text{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\text{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^2, x]

```
[Out] -(a + b*ArcSec[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSec[c*x])/(
4*d*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(
c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])))/(4*d^(3/2)*Sqrt[c^2*d +
e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*
Sqrt[1 - 1/(c^2*x^2)])))/(4*d^(3/2)*Sqrt[c^2*d + e]) - ((a + b*ArcSec[c*x])
*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-
d)^(3/2)*Sqrt[e]) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*
x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSec[
c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(
4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcS
ec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4)*b*P
olyLog[2, -(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(
(-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -(c
*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqr
t[e]) + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4520

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)])/(Cos[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.)), x\_Symbol] := Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (-Dist[I, Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a - Rt[a^2 - b^2, 2] + b\*E^(I\*(c + d\*x))), x], x] - Dist[I, Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a + Rt[a^2 - b^2, 2] + b\*E^(I\*(c + d\*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

#### Rule 4668

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCos[c\*x])^n, (d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2\*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || IGtQ[n, 0])

#### Rule 4734

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCos[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2\*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 4742

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Subst[Int[((a + b\*x)^n\*Sin[x])/(c\*d + e\*Cos[x]), x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 5230

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcCos[x/c])^n)/x^(2\*(p + 1))



, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{x^2 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{e \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)^2} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \left( -\frac{d(a + b \cos^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \cos^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\left( \frac{1}{4} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right) \right) - \frac{1}{4} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right) - \frac{1}{2} \\
&= -\frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{1}{2} \text{Subst} \left( \int \left( -\frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2d\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} - \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2d\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \text{Subst} \left( \int \frac{1}{d^2 + \frac{de}{c^2} - x^2} dx, x, \frac{-d + \frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4cd} - \frac{b \text{Subst} \left( \int \frac{1}{d^2 + \frac{de}{c^2} - x^2} dx, x, \frac{-d + \frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4cd} \\
&= -\frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4d^{3/2}\sqrt{c^2d+e}} \\
&= -\frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4d^{3/2}\sqrt{c^2d+e}} \\
&= -\frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \sec^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4d^{3/2}\sqrt{c^2d+e}}
\end{aligned}$$

**Mathematica [A]** time = 2.28, size = 1239, normalized size = 1.68

$$\frac{1}{2} \left( \frac{ax}{d^2 + ex^2d} + \frac{a \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}} + b \left( \frac{i \log\left(\frac{ie^{i \sec^{-1}(cx)}(\sqrt{e} - \sqrt{dc^2 + e})}{c\sqrt{d}} + 1\right) \sec^{-1}(cx)}{\sqrt{e}} + \frac{i \log\left(\frac{ie^{i \sec^{-1}(cx)}(\sqrt{dc^2 + e} - \sqrt{e})}{c\sqrt{d}} + 1\right) \sec^{-1}(cx)}{\sqrt{e}} + \dots \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^2, x]

[Out] ((a\*x)/(d^2 + d\*e\*x^2) + (a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(d^(3/2)\*Sqrt[e]) + (b\*((Sqrt[d]\*ArcSec[c\*x])/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x) + (Sqrt[d]\*ArcSec[c\*x])/(I\*Sqrt[d]\*Sqrt[e] + e\*x) - (4\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(((I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]]/Sqrt[e] + (4\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(((I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]]/Sqrt[e] - (I\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] - ((2\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] + (I\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] + ((2\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] + (I\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] - ((2\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] - (I\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] + ((2\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])/Sqrt[e] + (I\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e] - (I\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e] + PolyLog[2, ((I)\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]/Sqrt[e] - PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*Arc

Sec[c\*x]))/(c\*Sqrt[d])/Sqrt[e] - PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])/Sqrt[e] + PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])/Sqrt[e])]/(2\*d^(3/2)))/2

**fricas [F]** time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [C]** time = 4.34, size = 1748, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

[Out]  $\frac{1}{2}c^2ax/d/(c^2ex^2+c^2d)+\frac{1}{2}a/d/(de)^{1/2}\arctan(ex/(de)^{1/2})+\frac{1}{2}c^2b\operatorname{arcsec}(cx)*x/d/(c^2ex^2+c^2d)-I/c^2b*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))/((-c^2d+2*(e*(c^2d+e))^{1/2}-2e)*d)^{1/2})e/(c^2d+e)/d^3-I/c^4b*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))/((-c^2d+2*(e*(c^2d+e))^{1/2}-2e)*d)^{1/2})/d^4/(c^2d+e)*e^2+1/4*I*c*b/d*\operatorname{sum}(\_R1/(\_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{1/2}))/\_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{1/2}))/\_R1), \_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+\frac{1}{2}I/c^2b*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))/((-c^2d+2*(e*(c^2d+e))^{1/2}-2e)*d)^{1/2})$

$$\frac{d+e)^{(1/2)-2*e)*d)^{(1/2)}}{d^3+1/2*I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)})/d^3/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}-I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)})*e/(c^2*d+e)/d^3+I/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)})/d^4*e-I/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)*e-1/4*I*c*b/d*\sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x))*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*I/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e)*d)^{(1/2)})/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)+I/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e)*d)^{(1/2)})/d^4*(e*(c^2*d+e))^{(1/2)}-I/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*e^2+I/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)})/d^4*(e*(c^2*d+e))^{(1/2)+I/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e)*d)^{(1/2)})/d^4*e+1/2*I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2))}/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e)*d)^{(1/2)})/d^3$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{x}{dex^2 + d^2} + \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d} \right) + b \int \frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^2x^4 + 2dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(x/(d\*e\*x^2 + d^2) + arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

$$3.103 \quad \int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)^2} dx$$

**Optimal.** Leaf size=785

$$\frac{3\sqrt{e} \left( a + b \sec^{-1}(cx) \right) \log \left( 1 - \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{4(-d)^{5/2}} + \frac{3\sqrt{e} \left( a + b \sec^{-1}(cx) \right) \log \left( 1 + \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{4(-d)^{5/2}} - \frac{3\sqrt{e} \left( a + b \sec^{-1}(cx) \right)}{4(-d)^{5/2}}$$

[Out]  $-a/d^2/x - b \operatorname{arcsec}(c*x)/d^2/x - 3/4*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} - 3/4*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} - 3/4*I*b*\operatorname{polylog}(2, -c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*I*b*\operatorname{polylog}(2, c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} - 3/4*I*b*\operatorname{polylog}(2, -c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*I*b*\operatorname{polylog}(2, c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 1/4*e*(a+b*\operatorname{arcsec}(c*x))/d^2/(-d/x+(-d)^{(1/2)})*e^{(1/2)} - 1/4*e*(a+b*\operatorname{arcsec}(c*x))/d^2/(d/x+(-d)^{(1/2)})*e^{(1/2)} + 1/4*b*e*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)})*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}/d^{(5/2)}/(c^2*d+e)^{(1/2)} + 1/4*b*e*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)})*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}/d^{(5/2)}/(c^2*d+e)^{(1/2)} + b*c*(1-1/c^2/x^2)^{(1/2)}/d^2$

**Rubi [A]** time = 2.31, antiderivative size = 785, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {5240, 4734, 4620, 261, 4668, 4744, 725, 206, 4742, 4520, 2190, 2279, 2391}

$$\frac{3ib\sqrt{e} \operatorname{PolyLog} \left( 2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e} \operatorname{PolyLog} \left( 2, \frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e} \operatorname{PolyLog} \left( 2, -\frac{c\sqrt{-d} e^{i \sec^{-1}(cx)}}{\sqrt{c^2 d + e} + \sqrt{e}} \right)}{4(-d)^{5/2}} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSec}[c*x])/(x^2*(d + e*x^2)^2), x]$

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)]/d^2 - a/(d^2*x) - (b*ArcSec[c*x])/(d^2*x) + (e*(a + b*ArcSec[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcSec[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]/(4*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]/(4*d^(5/2)*Sqrt[c^2*d + e]) - (3*Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(4*(-d)^(5/2)) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(-d)^(5/2))
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

### Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```



Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2,
-(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4520

```
Int[(((e_.) + (f_.)*(x_)^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))], x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))], x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4668

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
:= -Subst[Int[(a + b*x)^n*Sin[x]/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
```

```
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] + Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

#### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*(a + b*ArcCos[x/c])^n]/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^2 (d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{x^4 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{d^2} + \frac{e^2 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} - \frac{2e \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \text{Subst} \left( \int \frac{1}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{a}{d^2 x} - \frac{b \text{Subst} \left( \int \cos^{-1} \left( \frac{x}{c} \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{a}{d^2 x} - \frac{b \sec^{-1}(cx)}{d^2 x} - \frac{b \text{Subst} \left( \int \frac{x}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd^2} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \sec^{-1}(cx)}{d^2 x} + \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\sqrt{e} \text{Subst} \left( \int \frac{1}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{1}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \sec^{-1}(cx)}{d^2 x} + \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{(i\sqrt{e}) \text{Subst} \left( \int \frac{1}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} - \frac{(i\sqrt{e}) \text{Subst} \left( \int \frac{1}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \sec^{-1}(cx)}{d^2 x} + \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{be \tanh^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{-d} \sqrt{e} + \frac{d}{x}} \right)}{d^2} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \sec^{-1}(cx)}{d^2 x} + \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{be \tanh^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{-d} \sqrt{e} + \frac{d}{x}} \right)}{d^2} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \sec^{-1}(cx)}{d^2 x} + \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e \left( a + b \sec^{-1}(cx) \right)}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{be \tanh^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{-d} \sqrt{e} + \frac{d}{x}} \right)}{d^2}
\end{aligned}$$

**Mathematica** [A] time = 2.25, size = 1291, normalized size = 1.64

$$-6\sqrt{e} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)a - \frac{4\sqrt{d}a}{x} - \frac{2\sqrt{d}exa}{ex^2+d} + b \left( 4\sqrt{d} \sqrt{1 - \frac{1}{c^2x^2}} c - \frac{4\sqrt{d} \sec^{-1}(cx)}{x} - \frac{\sqrt{d}e \sec^{-1}(cx)}{ex-i\sqrt{d}\sqrt{e}} - \frac{\sqrt{d}e \sec^{-1}(cx)}{ex+i\sqrt{d}\sqrt{e}} + 12\sqrt{e} \sin \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^2\*(d + e\*x^2)^2), x]

[Out] ((-4\*a\*Sqrt[d])/x - (2\*a\*Sqrt[d]\*e\*x)/(d + e\*x^2) - 6\*a\*Sqrt[e]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]] + b\*(4\*c\*Sqrt[d]\*Sqrt[1 - 1/(c^2\*x^2)] - (4\*Sqrt[d]\*ArcSec[c\*x])/x - (Sqrt[d]\*e\*ArcSec[c\*x])/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x) - (Sqrt[d]\*e\*ArcSec[c\*x])/(I\*Sqrt[d]\*Sqrt[e] + e\*x) + 12\*Sqrt[e]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2]]/Sqrt[c^2\*d + e] - 12\*Sqrt[e]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2]]/Sqrt[c^2\*d + e] + (3\*I)\*Sqrt[e]\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (6\*I)\*Sqrt[e]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (3\*I)\*Sqrt[e]\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (6\*I)\*Sqrt[e]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (3\*I)\*Sqrt[e]\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (6\*I)\*Sqrt[e]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (3\*I)\*Sqrt[e]\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (6\*I)\*Sqrt[e]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (I\*e\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x))]/Sqrt[-(c^2\*d) - e] + (I\*e\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x)/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x))]/Sqrt[-(c^2\*d) - e] - 3\*Sqrt[e]\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 3\*Sqrt[e]\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + 3\*Sqrt[e]\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - 3\*Sqrt[e]\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d]))])/ (4\*d^(5/2))

**fricas** [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{e^2x^6 + 2dex^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e^2\*x^6 + 2\*d\*e\*x^4 + d^2\*x^2), x)

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="giac")

[Out] Timed out

**maple** [C] time = 15.84, size = 1817, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^2,x)

[Out] 
$$\begin{aligned} & -1/2*a/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)-3/2*a/d^2*e/(d*e)^{(1/2)}*\arctan(e*x/(d* \\ & e)^{(1/2)})-a/d^2/x-I*b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\arctan \\ & \operatorname{anh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d \\ & )^{(1/2)})*e^2/d^5+I*b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^3*\ar \\ & \operatorname{ctan}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d \\ & )^{(1/2)})/d^5/(c^2*d+e)+3/4*I*c*b/d^2*e*\operatorname{sum}(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I \\ & *arcsec(c*x)*\ln((_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-I*( \\ & 1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))- \\ & I*b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^2*\arctan(c*d*(1/c/x+I \\ & *(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5/(c^2 \\ & *d+e)*(e*(c^2*d+e))^{(1/2)}-I*b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/ \\ & 2)}*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2 \\ & *e)*d)^{(1/2)})*e^2/d^5-1/2*I*b/c^2*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/ \\ & 2)}*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2 \\ & *e)*d)^{(1/2)})/d^4/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}*e+I*b/c^4*(-(c^2*d-2*(e*(c^ \\ & 2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^2*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/ \\ & (-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5/(c^2*d+e)*(e*(c^2*d+e))^{(1 \\ & /2)}-I*b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x \end{aligned}$$

$$+I*(1-1/c^2/x^2)^{(1/2)} / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} * e/d^5 * (e*(c^2*d+e))^{(1/2)} + c*b/d^2 * ((c^2*x^2-1)/c^2/x^2)^{(1/2)} + I*b/c^4 * ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * \arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e/d^5 * (e*(c^2*d+e))^{(1/2)} - 3/4 * I*c*b/d^2 * e * \sum(\_R1 / (\_R1^2 * c^2*d + c^2*d+2*e) * (I*\operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)}) / \_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)}) / \_R1)), \_R1 = \operatorname{RootOf}(c^2*d*_Z^4 + (2*c^2*d+4*e)*_Z^2 + c^2*d)) + I*b/c^2 * ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^2 * \arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}) / d^4 / (c^2*d+e) + I*b/c^4 * (- (c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^3 * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}) / d^5 / (c^2*d+e) - 1/2 * I*b/c^2 * ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * \arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}) / d^4 * e - 1/2 * I*b/c^2 * (- (c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}) / d^4 * e + I*b/c^2 * (- (c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^2 * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}) / d^4 / (c^2*d+e) + 1/2 * I*b/c^2 * (- (c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}) / d^4 / (c^2*d+e) * (e*(c^2*d+e))^{(1/2)} - 1/2 * b * \operatorname{arcsec}(c*x) / d^2 * e * c^2 * x / (c^2 * e * x^2 + c^2 * d) - b * \operatorname{arcsec}(c*x) / d^2 / x$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a \left( \frac{3ex^2 + 2d}{d^2ex^3 + d^3x} + \frac{3e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d^2} \right) + b \int \frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^2x^6 + 2dex^4 + d^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2\*a\*((3\*e\*x^2 + 2\*d)/(d^2\*e\*x^3 + d^3\*x) + 3\*e\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^2)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^6 + 2\*d\*e\*x^4 + d^2\*x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 (ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x^2\*(d + e\*x^2)^2), x)

```
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

$$3.104 \quad \int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=707

$$\frac{(a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2d+e}}}\right)}{2e^3} + \frac{(a + b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2d+e}}}\right)}{2e^3} + \frac{(a + b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{d}e^{i\sec^{-1}(cx)}}{\sqrt{e + \sqrt{c^2d+e}}}\right)}{2e^3}$$

[Out]  $\frac{1}{4}(-a-b\operatorname{arcsec}(c*x))/e/(e+d/x^2)^2+1/2*(-a-b\operatorname{arcsec}(c*x))/e^2/(e+d/x^2)-1/8*b*(c^2*d+2*e)*\arctan((c^2*d+e)^{(1/2)}/c/x/e^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/e^{(5/2)}/(c^2*d+e)^{(3/2)}-(a+b\operatorname{arcsec}(c*x))*\ln(1+(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2)/e^{3+1/2}*(a+b\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{3+1/2}*(a+b\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{3+1/2}*(a+b\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{3+1/2}*(a+b\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{3+1/2}*I*b*\operatorname{polylog}(2,-(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})^2)/e^{3-1/2}*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{3-1/2}*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{3-1/2}*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{3-1/2}*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{3-1/2}*b*\arctan((c^2*d+e)^{(1/2)}/c/x/e^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/e^{(5/2)}/(c^2*d+e)^{(1/2)}-1/8*b*c*d*(1-1/c^2/x^2)^{(1/2)}/e^2/(c^2*d+e)/(e+d/x^2)/x$

**Rubi [A]** time = 1.40, antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {5240, 4734, 4626, 3719, 2190, 2279, 2391, 4730, 382, 377, 205, 4742, 4520}

$$\frac{ib\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2d+e}}}\right)}{2e^3} - \frac{ib\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e - \sqrt{c^2d+e}}}\right)}{2e^3} - \frac{ib\operatorname{PolyLog}\left(2, -\frac{c\sqrt{d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e} + \sqrt{e}}\right)}{2e^3} - \frac{ib\operatorname{PolyLog}\left(2, \frac{c\sqrt{d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e} + \sqrt{e}}\right)}{2e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3, x]



```
[Out] -(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(8*e^2*(c^2*d + e)*(e + d/x^2)*x) - (a + b*ArcSec[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcSec[c*x])/(2*e^2*(e + d/x^2)) - (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))]/(Sqrt[e] - Sqrt[c^2*d + e]))/(2*e^3) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))]/(Sqrt[e] - Sqrt[c^2*d + e]))/(2*e^3) + ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))]/(Sqrt[e] + Sqrt[c^2*d + e]))/(2*e^3) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))]/(Sqrt[e] + Sqrt[c^2*d + e]))/(2*e^3) - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])])/e^3 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])])/e^3
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

### Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4520

```
Int[((e_.) + (f_.)*(x_)^(m_.))*Sin[(c_.) + (d_.)*(x_)]/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] := -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x]))/(2*e*(p + 1)), x]
+ Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
```

$e, 0] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IntegerQ}[p] \ \&\& \text{IntegerQ}[m]$

### Rule 4742

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)(x_.)](b_.)]^{(n_.)}/((d_.) + (e_.)(x_.)), x\_Symbol]$   
 $:\> -\text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]/(c*d + e*\text{Cos}[x]), x], x, \text{ArcCos}[c*x]] /$   
 $;\text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \text{IGtQ}[n, 0]$

### Rule 5240

$\text{Int}[(a_.) + \text{ArcSec}[(c_.)(x_.)](b_.)]^{(n_.)}(x_.)^{(m_.)}((d_.) + (e_.)(x_.)^2)^{(p_.)}, x\_Symbol]$   
 $:\> -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcCos}[x/c])^n/x^{(m + 2*(p + 1))}, x], x, 1/x] /;$   
 $\text{FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \text{IGtQ}[n, 0]$   
 $\ \&\& \text{IntegerQ}[m] \ \&\& \text{IntegerQ}[p]$

### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{x (e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{e^3 x} - \frac{dx (a + b \cos^{-1}\left(\frac{x}{c}\right))}{e (e + dx^2)^3} - \frac{dx (a + b \cos^{-1}\left(\frac{x}{c}\right))}{e^2 (e + dx^2)^2} - \frac{dx (a + b \cos^{-1}\left(\frac{x}{c}\right))}{e^3} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \text{Subst} \left( \int \frac{x (a + b \cos^{-1}\left(\frac{x}{c}\right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \text{Subst} \left( \int \frac{x^2 (a + b \cos^{-1}\left(\frac{x}{c}\right))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= -\frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} + \frac{\text{Subst} \left( \int (a + bx) \tan(x) dx, x, \sec^{-1}(cx) \right)}{e^3} + \frac{d \text{Subst} \left( \int \frac{x^2 (a + b \sec^{-1}(cx))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} + \frac{i (a + b \sec^{-1}(cx))^2}{2be^3} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} + \frac{i (a + b \sec^{-1}(cx))^2}{2be^3} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} \\
&= -\frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \sec^{-1}(cx)}{4e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \sec^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2e^{5/2} \sqrt{c^2 d + e}}
\end{aligned}$$

**Mathematica [B]** time = 8.01, size = 1805, normalized size = 2.55

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$\begin{aligned} & -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*\text{Log}[d + e*x^2])/(2*e^3) + b*((( (-7*I)/16)*\text{Sqrt}[d]*(-\text{ArcSec}[c*x]/(I*\text{Sqrt}[d]*\text{Sqrt}[e] + e*x)) + (I*(\text{ArcSin}[1/(c*x)]/\text{Sqrt}[e] - \text{Log}[(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(\text{Sqrt}[e] + c*(I*c*\text{Sqrt}[d] - \text{Sqrt}[-(c^2*d) - e]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x])/(\text{Sqrt}[-(c^2*d) - e]*(\text{Sqrt}[d] - I*\text{Sqrt}[e]*x)))/\text{Sqrt}[-(c^2*d) - e]))/\text{Sqrt}[d]))/e^{5/2} + (((7*I)/16)*\text{Sqrt}[d]*(-\text{ArcSec}[c*x]/((-I)*\text{Sqrt}[d]*\text{Sqrt}[e] + e*x)) - (I*(\text{ArcSin}[1/(c*x)]/\text{Sqrt}[e] - \text{Log}[(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(-\text{Sqrt}[e] + c*(I*c*\text{Sqrt}[d] + \text{Sqrt}[-(c^2*d) - e]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x])/(\text{Sqrt}[-(c^2*d) - e]*(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x)))/\text{Sqrt}[-(c^2*d) - e]))/\text{Sqrt}[d]))/e^{5/2} - (d*(-\text{ArcSec}[c*x]/(\text{Sqrt}[e]*((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2)) + (\text{ArcSin}[1/(c*x)]/\text{Sqrt}[e] - I*((c*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x)/((c^2*d + e)*((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)) + ((2*c^2*d + e)*\text{Log}[(-4*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*(I*\text{Sqrt}[e] + c*(c*\text{Sqrt}[d] - \text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)))]/(c^2*d + e)^{3/2}))/d)/(16*e^{5/2}) - (d*((I*c*\text{Sqrt}[e]*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(\text{Sqrt}[d]*(c^2*d + e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)) - \text{ArcSec}[c*x]/(\text{Sqrt}[e]*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) + \text{ArcSin}[1/(c*x)]/(d*\text{Sqrt}[e])) - (I*(2*c^2*d + e)*\text{Log}[(4*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*((-I)*\text{Sqrt}[e] + c*(c*\text{Sqrt}[d] + \text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)))]/(d*(c^2*d + e)^{3/2}))/d)/(16*e^{5/2}) + ((I/4)*(8*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{ArcTan}[(I*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tan}[\text{ArcSec}[c*x]/2]/\text{Sqrt}[c^2*d + e]] - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) - (4*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) + (4*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) + (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + E^{((2*I)*\text{ArcSec}[c*x])})] - 2*\text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) - 2*\text{PolyLog}[2, ((-I)*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) + \text{PolyLog}[2, -E^{((2*I)*\text{ArcSec}[c*x])})])/e^3 + ((I/4)*(8*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{ArcTan}[( (-I)*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tan}[\text{ArcSec}[c*x]/2]/\text{Sqrt}[c^2*d + e]] - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) - (4*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) - (2*I)*\text{ArcSec}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])*E^{(I*\text{ArcSec}[c*x])})/(c*\text{Sqrt}[d])]) + (4*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])$$

) $E^{(I \operatorname{ArcSec}[c*x])} / (c \sqrt{d})] + (2*I) \operatorname{ArcSec}[c*x] * \operatorname{Log}[1 + E^{((2*I) \operatorname{ArcSec}[c*x])}] - 2 * \operatorname{PolyLog}[2, ((-I) * (-\sqrt{e} + \sqrt{c^2*d + e})) * E^{(I \operatorname{ArcSec}[c*x])}] / (c \sqrt{d})] - 2 * \operatorname{PolyLog}[2, (I * (\sqrt{e} + \sqrt{c^2*d + e})) * E^{(I \operatorname{ArcSec}[c*x])}] / (c \sqrt{d})] + \operatorname{PolyLog}[2, -E^{((2*I) \operatorname{ArcSec}[c*x])})] / e^3$

**fricas** [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bx^5 \operatorname{arcsec}(cx) + ax^5}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")`

[Out] `integral((b*x^5*arcsec(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

[Out] Timed out

**maple** [C] time = 1.96, size = 1626, normalized size = 2.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x)`

[Out]  $c^2*a/e^3*d/(c^2*e*x^2+c^2*d)-1/4*I*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^2*d+5/8*I*c^2*b*(e*(c^2*d+e))^{1/2}/e^3/(c^2*d+e)^2*\operatorname{arctanh}(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))^2+2*c^2*d+4e)/(c^2*d*e+e^2)^{1/2})*d-1/2*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arcsec}(c*x)*d*x^2-1/8*c^5*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{1/2}*x^3*d-1/8*c^5*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{1/2}*x*d^2-3/4*c^6*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arcsec}(c*x)*d*x^4-1/2*c^6*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arcsec}(c*x)*d^2*x^2-c^2*b/e^3/(c^2*d+e)*d*\operatorname{arcsec}(c*x)*\ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))-c^2*b/e^3/(c^2*d+e)*d*\operatorname{arcsec}(c*x)*\ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))+I*c^2*b/e^3/(c^2*d+e)*d*\operatorname{dilog}(1+I*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))+I*c^2*b/e^3/(c^2*d+e)*d*\operatorname{dilog}(1-I*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))-1/8*I*c^4*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d^2-1/4*I*c^2*b/e^2/(c^2*$

```

d+e)*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(
1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=
RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))*d-1/4*I*c^2*b/e^3/(c^2*d+e)*d*
sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_
R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I*c^4*b/e^3/(c^2*
d+e)*d^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x
-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*a/e^3*ln(c^2*e*x^2+c^2
*d)-1/4*I*b/e^2/(c^2*d+e)*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*
e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/
x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2
*d))-1/4*c^4*a*d^2/e^3/(c^2*e*x^2+c^2*d)^2-3/4*c^4*b/(c^2*e*x^2+c^2*d)^2/(c
^2*d+e)*arcsec(c*x)*x^4+3/4*I*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)^2*arctanh
(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1
/2))-1/8*I*c^4*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^4+I*b/e^2/(c^2*d+e)*dilog(
1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e^2/(c^2*d+e)*dilog(1-I*(1/c/x+I*(1-
1/c^2/x^2)^(1/2)))-b/e^2/(c^2*d+e)*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2
)^(1/2)))-b/e^2/(c^2*d+e)*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))

```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a\left(\frac{4dex^2 + 3d^2}{e^5x^4 + 2de^4x^2 + d^2e^3} + \frac{2\log(ex^2 + d)}{e^3}\right) + b\int\frac{x^5\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4\*a\*((4\*d\*e\*x^2 + 3\*d^2)/(e^5\*x^4 + 2\*d\*e^4\*x^2 + d^2\*e^3) + 2\*log(e\*x^2 + d)/e^3) + b\*integrate(x^5\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^5\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(ex^2+d)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^5\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out



$$3.105 \quad \int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=157

$$\frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{bcx (c^2 d + 2e) \tan^{-1} \left( \frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{\sqrt{c^2 d + e}} \right)}{8de^{3/2} \sqrt{c^2 x^2} (c^2 d + e)^{3/2}} + \frac{bcx \sqrt{c^2 x^2 - 1}}{8e \sqrt{c^2 x^2} (c^2 d + e) (d + ex^2)}$$

[Out] 1/4\*x^4\*(a+b\*arcsec(c\*x))/d/(e\*x^2+d)^2-1/8\*b\*c\*(c^2\*d+2\*e)\*x\*arctan(e^(1/2)\*(c^2\*x^2-1)^(1/2)/(c^2\*d+e)^(1/2))/d/e^(3/2)/(c^2\*d+e)^(3/2)/(c^2\*x^2)^(1/2)+1/8\*b\*c\*x\*(c^2\*x^2-1)^(1/2)/e/(c^2\*d+e)/(e\*x^2+d)/(c^2\*x^2)^(1/2)

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {264, 5238, 12, 446, 78, 63, 205}

$$\frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{bcx (c^2 d + 2e) \tan^{-1} \left( \frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{\sqrt{c^2 d + e}} \right)}{8de^{3/2} \sqrt{c^2 x^2} (c^2 d + e)^{3/2}} + \frac{bcx \sqrt{c^2 x^2 - 1}}{8e \sqrt{c^2 x^2} (c^2 d + e) (d + ex^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] (b\*c\*x\*sqrt[-1 + c^2\*x^2])/(8\*e\*(c^2\*d + e)\*sqrt[c^2\*x^2]\*(d + e\*x^2)) + (x^4\*(a + b\*ArcSec[c\*x]))/(4\*d\*(d + e\*x^2)^2) - (b\*c\*(c^2\*d + 2\*e)\*x\*ArcTan[(sqrt[e]\*sqrt[-1 + c^2\*x^2])/sqrt[c^2\*d + e]])/(8\*d\*e^(3/2)\*(c^2\*d + e)^(3/2)\*sqrt[c^2\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx &= \frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bcx) \int \frac{x^3}{4d \sqrt{-1+c^2x^2} (d+ex^2)^2} dx}{\sqrt{c^2x^2}} \\
&= \frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bcx) \int \frac{x^3}{\sqrt{-1+c^2x^2} (d+ex^2)^2} dx}{4d \sqrt{c^2x^2}} \\
&= \frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bcx) \text{Subst} \left( \int \frac{x}{\sqrt{-1+c^2x} (d+ex)^2} dx, x, x^2 \right)}{8d \sqrt{c^2x^2}} \\
&= \frac{bcx \sqrt{-1 + c^2x^2}}{8e (c^2d + e) \sqrt{c^2x^2} (d + ex^2)} + \frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bc (c^2d + 2e) x) \text{Subst} \left( \int \frac{x}{\sqrt{-1+c^2x} (d+ex)^2} dx, x, x^2 \right)}{16de (c^2d + e)} \\
&= \frac{bcx \sqrt{-1 + c^2x^2}}{8e (c^2d + e) \sqrt{c^2x^2} (d + ex^2)} + \frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(b (c^2d + 2e) x) \text{Subst} \left( \int \frac{x}{\sqrt{-1+c^2x} (d+ex)^2} dx, x, x^2 \right)}{8cde (c^2d + e)} \\
&= \frac{bcx \sqrt{-1 + c^2x^2}}{8e (c^2d + e) \sqrt{c^2x^2} (d + ex^2)} + \frac{x^4 (a + b \sec^{-1}(cx))}{4d (d + ex^2)^2} - \frac{bc (c^2d + 2e) x \tan^{-1} \left( \frac{\sqrt{e} \sqrt{-1+c^2x}}{\sqrt{c^2}} \right)}{8de^{3/2} (c^2d + e)^{3/2} \sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 1.42, size = 389, normalized size = 2.48

$$\frac{\frac{8a}{d+ex^2} - \frac{4ad}{(d+ex^2)^2} + \frac{b\sqrt{e}(c^2d+2e) \log\left(\frac{16de^{3/2}\sqrt{c^2(-d)-e}\left(\sqrt{e}+cx\left(-\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}+ic\sqrt{d}\right)\right)}{b(c^2d+2e)(\sqrt{e}x+i\sqrt{d})}\right)}{d(c^2(-d)-e)^{3/2}} + \frac{b\sqrt{e}(c^2d+2e) \log\left(\frac{16ide^{3/2}\sqrt{c^2(-d)-e}\left(-\sqrt{e}+cx\left(-\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}+ic\sqrt{d}\right)\right)}{b(c^2d+2e)}\right)}{d(c^2(-d)-e)^{3/2}}}{16e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] -1/16\*((-4\*a\*d)/(d + e\*x^2)^2 + (8\*a)/(d + e\*x^2) - (2\*b\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/((c^2\*d + e)\*(d + e\*x^2)) + (4\*b\*(d + 2\*e\*x^2)\*ArcSec[c\*x])/(d + e\*x^2)^2 + (4\*b\*ArcSin[1/(c\*x)]/d + (b\*Sqrt[e]\*(c^2\*d + 2\*e)\*Log[(-16\*d\*Sqrt[-(c^2\*d) - e]\*e^(3/2)\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e])\*Sqrt[1 - 1/(c^2\*x^2)])\*x])/(b\*(c^2\*d + 2\*e)\*(I\*Sqrt[d] + Sqrt[e]\*x)))]/(d\*(-(

$$\frac{c^2 d - e)^{3/2} + (b \sqrt{e} (c^2 d + 2e) \operatorname{Log}[\frac{(16I) d \sqrt{-(c^2 d - e)} - e)^{3/2} (-\sqrt{e} + c(Ic \sqrt{d} + \sqrt{-(c^2 d - e)} \sqrt{1 - 1/(c^2 * x^2)}) * x)}{(b(c^2 d + 2e)(\sqrt{d} + I \sqrt{e} * x))}] / (d * (-(c^2 d - e)^{3/2}))]}{e^2}$$

**fricas [B]** time = 1.80, size = 1015, normalized size = 6.46

$$\frac{4ac^4d^4 + 8ac^2d^3e + 4ad^2e^2 + 8(ac^4d^3e + 2ac^2d^2e^2 + ade^3)x^2 + (bc^2d^3 + (bc^2de^2 + 2be^3)x^4 + 2bd^2e + 2(bc^2d^2e + 2(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{-(c^2*d*e - e^2)}*\log((c^2*e*x^2 - c^2*d + 2*\sqrt{-(c^2*d*e - e^2)}*\sqrt{c^2*x^2 - 1}) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\operatorname{arcsec}(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\operatorname{arctan}(-c*x + \sqrt{c^2*x^2 - 1}) - 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}}{(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{c^2*d*e + e^2}*\operatorname{arctan}(\sqrt{c^2*d*e + e^2}*\sqrt{c^2*x^2 - 1})/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\operatorname{arcsec}(c*x) - 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\operatorname{arctan}(-c*x + \sqrt{c^2*x^2 - 1}) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}}{(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16\*(4\*a\*c^4\*d^4 + 8\*a\*c^2\*d^3\*e + 4\*a\*d^2\*e^2 + 8\*(a\*c^4\*d^3\*e + 2\*a\*c^2\*d^2\*e^2 + a\*d\*e^3)\*x^2 + (b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + 2\*b\*e^3)\*x^4 + 2\*b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + 2\*b\*d\*e^2)\*x^2)\*sqrt(-(c^2\*d\*e - e^2))\*log((c^2\*e\*x^2 - c^2\*d + 2\*sqrt(-(c^2\*d\*e - e^2))\*sqrt(c^2\*x^2 - 1) - 2\*e)/(e\*x^2 + d)) + 4\*(b\*c^4\*d^4 + 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + 2\*(b\*c^4\*d^3\*e + 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*arcsec(c\*x) - 8\*(b\*c^4\*d^4 + 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^4\*d^2\*e^2 + 2\*b\*c^2\*d\*e^3 + b\*e^4)\*x^4 + 2\*(b\*c^4\*d^3\*e + 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) - 2\*(b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*sqrt(c^2\*x^2 - 1))/(c^4\*d^5\*e^2 + 2\*c^2\*d^4\*e^3 + d^3\*e^4 + (c^4\*d^3\*e^4 + 2\*c^2\*d^2\*e^5 + d\*e^6)\*x^4 + 2\*(c^4\*d^4\*e^3 + 2\*c^2\*d^3\*e^4 + d^2\*e^5)\*x^2), -1/8\*(2\*a\*c^4\*d^4 + 4\*a\*c^2\*d^3\*e + 2\*a\*d^2\*e^2 + 4\*(a\*c^4\*d^3\*e + 2\*a\*c^2\*d^2\*e^2 + a\*d\*e^3)\*x^2 + (b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + 2\*b\*e^3)\*x^4 + 2\*b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + 2\*b\*d\*e^2)\*x^2)\*sqrt(c^2\*d\*e + e^2)\*arctan(sqrt(c^2\*d\*e + e^2)\*sqrt(c^2\*x^2 - 1)/(c^2\*d + e)) + 2\*(b\*c^4\*d^4 + 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + 2\*(b\*c^4\*d^3\*e + 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*arcsec(c\*x) - 4\*(b\*c^4\*d^4 + 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^4\*d^2\*e^2 + 2\*b\*c^2\*d\*e^3 + b\*e^4)\*x^4 + 2\*(b\*c^4\*d^3\*e + 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) - (b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*sqrt(c^2\*x^2 - 1))/(c^4\*d^5\*e^2 + 2\*c^2\*d^4\*e^3 + d^3\*e^4 + (c^4\*d^3\*e^4 + 2\*c^2\*d^2\*e^5 + d\*e^6)\*x^4 + 2\*(c^4\*d^4\*e^3 + 2\*c^2\*d^3\*e^4 + d^2\*e^5)\*x^2)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [B]** time = 0.08, size = 1870, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (x^3(a+b\operatorname{arcsec}(cx)))/(e^x d^2+x)^3, x$

[Out] 
$$\begin{aligned} & -1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*b*\operatorname{arcsec}(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*b*\operatorname{arcsec}(c*x)/e^2*d/(c^2*e*x^2+c^2*d)^2+1/4*c^3*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\operatorname{arctan}(1/(c^2*x^2-1)^{(1/2)})+1/4*c^3*b*(c^2*x^2-1)^{(1/2)}/e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\operatorname{arctan}(1/(c^2*x^2-1)^{(1/2)})-1/16*c^3*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))-1/16*c^3*b*(c^2*x^2-1)^{(1/2)}/e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))-1/16*c^3*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e*x+(-c^2*e*d)^{(1/2)}))-1/16*c^3*b*(c^2*x^2-1)^{(1/2)}/e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e*x+(-c^2*e*d)^{(1/2)})))+1/4*c*b*(c^2*x^2-1)^{(1/2)}*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\operatorname{arctan}(1/(c^2*x^2-1)^{(1/2)})+1/4*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\operatorname{arctan}(1/(c^2*x^2-1)^{(1/2)})-1/8*c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})-1/8*c*b*(c^2*x^2-1)^{(1/2)}*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))-1/8*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))-1/8*c*b*($$

$$c^2*x^2-1)^{(1/2)}*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(-(c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e*x+(-c^2*e*d)^{(1/2)}))-1/8*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/(-(c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e*x+(-c^2*e*d)^{(1/2)}))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(2ex^2 + d)a}{4(e^4x^4 + 2de^3x^2 + d^2e^2)} \left( (2ex^2 + d) \arctan(\sqrt{cx+1}\sqrt{cx-1}) - (e^4x^4 + 2de^3x^2 + d^2e^2) \int \frac{1}{c^2e^4x^6 + (2c^2de^3 - e^4)x^4 - d^2e^2} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] -1/4\*(2\*e\*x^2 + d)\*a/(e^4\*x^4 + 2\*d\*e^3\*x^2 + d^2\*e^2) - 1/4\*((2\*e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - 4\*(e^4\*x^4 + 2\*d\*e^3\*x^2 + d^2\*e^2)\*integrate(1/4\*(2\*c^2\*e\*x^3 + c^2\*d\*x)\*e^(1/2\*log(c\*x + 1) + 1/2\*log(c\*x - 1))/(c^2\*e^4\*x^6 + (2\*c^2\*d\*e^3 - e^4)\*x^4 - d^2\*e^2 + (c^2\*d^2\*e^2 - 2\*d\*e^3)\*x^2 + (c^2\*e^4\*x^6 + (2\*c^2\*d\*e^3 - e^4)\*x^4 - d^2\*e^2 + (c^2\*d^2\*e^2 - 2\*d\*e^3)\*x^2)\*e^(log(c\*x + 1) + log(c\*x - 1))), x))\*b/(e^4\*x^4 + 2\*d\*e^3\*x^2 + d^2\*e^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^3\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.106 \quad \int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$$

Optimal. Leaf size=193

$$-\frac{a+b \sec^{-1}(cx)}{4e(d+ex^2)^2} + \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}} - \frac{bcx(3c^2d+2e) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}(c^2d+e)^{3/2}} - \frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}(c^2d+e)(d+ex^2)}$$

[Out]  $1/4*(-a-b*\text{arcsec}(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*\text{arctan}((c^2*x^2-1)^{(1/2)})/d^2/e/(c^2*x^2)^{(1/2)}-1/8*b*c*(3*c^2*d+2*e)*x*\text{arctan}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)})/(c^2*d+e)^{(1/2)}/d^2/(c^2*d+e)^{(3/2)}/e^{(1/2)}/(c^2*x^2)^{(1/2)}-1/8*b*c*x*(c^2*x^2-1)^{(1/2)}/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5236, 446, 103, 156, 63, 205}

$$-\frac{a+b \sec^{-1}(cx)}{4e(d+ex^2)^2} + \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}} - \frac{bcx(3c^2d+2e) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}(c^2d+e)^{3/2}} - \frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}(c^2d+e)(d+ex^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(a + b*\text{ArcSec}[c*x]))/(d + e*x^2)^3, x]$

[Out]  $-(b*c*x*\text{Sqrt}[-1 + c^2*x^2])/(8*d*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*(d + e*x^2)) - (a + b*\text{ArcSec}[c*x])/(4*e*(d + e*x^2)^2) + (b*c*x*\text{ArcTan}[\text{Sqrt}[-1 + c^2*x^2]])/(4*d^2*e*\text{Sqrt}[c^2*x^2]) - (b*c*(3*c^2*d + 2*e)*x*\text{ArcTan}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/\text{Sqrt}[c^2*d + e]])/(8*d^2*\text{Sqrt}[e]*(c^2*d + e)^{(3/2)}*\text{Sqrt}[c^2*x^2])$

### Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 103

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] := \text{Simp}[(b*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}*(e + f*x$

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

### Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 5236

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSec[c*x]))/(2*e*(p + 1)), x
] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x(a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx &= -\frac{a + b \sec^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^2} dx}{4e\sqrt{c^2x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex)^2} dx, x, x^2\right)}{8e\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \sec^{-1}(cx)}{4e(d + ex^2)^2} - \frac{(bcx) \text{Subst}\left(\int \frac{-c^2d-e+\frac{1}{2}c^2ex}{x\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{8de(c^2d+e)\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \sec^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{8d^2e\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \sec^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bx) \text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{2}} dx, x, \sqrt{-1+c^2x^2}\right)}{4cd^2e\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \sec^{-1}(cx)}{4e(d + ex^2)^2} + \frac{bcx \tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} - \frac{bc(3d+e)}{4d^2e\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.98, size = 386, normalized size = 2.00

$$\frac{1}{16} \left( -\frac{4a}{e(d+ex^2)^2} - \frac{b(3c^2d+2e) \log\left(-\frac{16d^2\sqrt{e}\sqrt{c^2(-d)-e}\left(\sqrt{e+cx}\left(-\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}+ic\sqrt{d}\right)\right)}{b(3c^2d+2e)(\sqrt{e+ix}\sqrt{d})}\right)}{d^2\sqrt{e}(c^2(-d)-e)^{3/2}} - \frac{b(3c^2d+2e) \log\left(\frac{16d^2\sqrt{e}\sqrt{c^2(-d)-e}\left(\sqrt{e+cx}\left(-\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}+ic\sqrt{d}\right)\right)}{b(3c^2d+2e)(\sqrt{e+ix}\sqrt{d})}\right)}{d^2\sqrt{e}(c^2(-d)-e)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] ((-4\*a)/(e\*(d + e\*x^2)^2) - (2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(d\*(c^2\*d + e)\*(d + e\*x^2)) - (4\*b\*ArcSec[c\*x])/(e\*(d + e\*x^2)^2) - (4\*b\*ArcSin[1/(c\*x)])/(d^2\*e) - (b\*(3\*c^2\*d + 2\*e)\*Log[(-16\*d^2\*Sqrt[-(c^2\*d) - e]\*Sqrt[e]\*(Sqrt[

$$e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)]*x))/(b*(3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x)))/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]) - (b*(3*c^2*d + 2*e)*Log[((16*I)*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)]*x)))/(b*(3*c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x)))]/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]))/16$$

**fricas [B]** time = 1.89, size = 888, normalized size = 4.60

$$\left[ \frac{4ac^4d^4 + 8ac^2d^3e + 4ad^2e^2 + (3bc^2d^3 + (3bc^2de^2 + 2be^3)x^4 + 2bd^2e + 2(3bc^2d^2e + 2bde^2)x^2)\sqrt{-c^2de - e^2}}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out]  $[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{-c^2*d*e - e^2}*\log((c^2*e*x^2 - c^2*d + 2*\sqrt{-c^2*d*e - e^2})*\sqrt{c^2*x^2 - 1} - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*\arcsin(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}]/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{c^2*d*e + e^2}*\arctan(\sqrt{c^2*d*e + e^2}*\sqrt{c^2*x^2 - 1}/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*\arcsin(c*x) - 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}]/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]$

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign

by intervals (correct if the argument is real): Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [B]** time = 0.07, size = 1840, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x*(a+b*\text{arcsec}(c*x))/(e*x^2+d)^3, x)$

[Out] 
$$\begin{aligned} & -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*\text{arcsec}(c*x) \\ & +1/4*c^3*b*(c^2*x^2-1)^{(1/2)}*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\text{arctan}(1/(c^2*x^2-1)^{(1/2)}) \\ & +1/4*c^3*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\text{arctan}(1/(c^2*x^2-1)^{(1/2)})- \\ & 3/16*c^3*b*(c^2*x^2-1)^{(1/2)}*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2* \\ & ((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))-3/16*c^3*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x \\ & /(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x \\ & -e)/(c*e*x+(-c^2*e*d)^{(1/2)}))-3/16*c^3*b*(c^2*x^2-1)^{(1/2)}*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/ \\ & (c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e*x+(-c^2*e*d)^{(1/2)}))-3/16*c^3*b*(c^2*x^2-1)^{(1/2)}/ \\ & ((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e*x+(-c^2*e*d)^{(1/2)}))+1/4*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d^2/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\text{arctan}(1/(c^2*x^2-1)^{(1/2)})*e^2+1/4*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\text{arctan}(1/(c^2*x^2-1)^{(1/2)})*e+1/8*c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*e-1/8*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*e-1/8*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d^2/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))*e^2-1/8*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(c*e*x+(-c^2*e*d)^{(1/2)}))*e-1/8*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d^2/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)})*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-c*e$$

$*x+(-c^2*e*d)^{(1/2)}) * e^{-2-1/8*c*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(-c^2*d+e)/e)^{(1/2)}/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^{(1/2)})/(c*e*x+(-c^2*e*d)^{(1/2)}) * \ln(-2*((-c^2*d+e)/e)^{(1/2)*(c^2*x^2-1)^{(1/2)*e+(-c^2*e*d)^{(1/2)*c*x-e}/(-c*e*x+(-c^2*e*d)^{(1/2))}) * e$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left( (c^2 e^3 x^4 + 2 c^2 d e^2 x^2 + c^2 d^2 e) \int \frac{x e^{\left(\frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1)\right)}}{c^2 e^3 x^6 + (2 c^2 d e^2 - e^3) x^4 + (c^2 e^3 x^6 + (2 c^2 d e^2 - e^3) x^4 - d^2 e + (c^2 d^2 e - 2 d e^2) x^2)(cx+1)(cx-1) - d^2 e + (c^2 d^2 e - 2 d e^2) x^2} dx \right)}{4 (e^3 x^4 + 2 d e^2 x^2 + d^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out]  $\frac{1}{4} * (4 * (c^2 * e^3 * x^4 + 2 * c^2 * d * e^2 * x^2 + c^2 * d^2 * e) * \text{integrate}(1/4 * x * e^{(1/2 * \log(c * x + 1) + 1/2 * \log(c * x - 1))} / (c^2 * e^3 * x^6 + (2 * c^2 * d * e^2 - e^3) * x^4 - d^2 * e + (c^2 * d^2 * e - 2 * d * e^2) * x^2 + (c^2 * e^3 * x^6 + (2 * c^2 * d * e^2 - e^3) * x^4 - d^2 * e + (c^2 * d^2 * e - 2 * d * e^2) * x^2) * e^{(\log(c * x + 1) + \log(c * x - 1))}, x) - a * \text{rctan}(\sqrt{c * x + 1} * \sqrt{c * x - 1})) * b / (e^3 * x^4 + 2 * d * e^2 * x^2 + d^2 * e) - 1/4 * a / (e^3 * x^4 + 2 * d * e^2 * x^2 + d^2 * e)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(e x^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.107 \quad \int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^3} dx$$

**Optimal.** Leaf size=685

$$\frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b \sec^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

[Out]  $\frac{1}{4}e^{2(a+b \operatorname{arcsec}(cx))/d^3}(e+d/x^2)^{-2} - e^{(a+b \operatorname{arcsec}(cx))/d^3}(e+d/x^2)^{-1} + \frac{1}{2}I^*(a+b \operatorname{arcsec}(cx))^2/b/d^3 - \frac{1}{2}(a+b \operatorname{arcsec}(cx)) \ln(1 - c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (c^2*d+e)^{(1/2)}))/d^3 - \frac{1}{2}(a+b \operatorname{arcsec}(cx)) \ln(1 + c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (c^2*d+e)^{(1/2)}))/d^3 - \frac{1}{2}(a+b \operatorname{arcsec}(cx)) \ln(1 - c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (c^2*d+e)^{(1/2)}))/d^3 - \frac{1}{2}(a+b \operatorname{arcsec}(cx)) \ln(1 + c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (c^2*d+e)^{(1/2)}))/d^3 + \frac{1}{2}I*b*\operatorname{polylog}(2, -c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (c^2*d+e)^{(1/2)}))/d^3 + \frac{1}{2}I*b*\operatorname{polylog}(2, c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (c^2*d+e)^{(1/2)}))/d^3 + \frac{1}{2}I*b*\operatorname{polylog}(2, -c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (c^2*d+e)^{(1/2)}))/d^3 + \frac{1}{2}I*b*\operatorname{polylog}(2, c*(1/c/x + I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (c^2*d+e)^{(1/2)}))/d^3 + \frac{1}{8}b*(c^2*d+2*e)*\arctan((c^2*d+e)^{(1/2)}/c/x/e^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})*e^{(1/2)}/d^3/(c^2*d+e)^{(3/2)} - b*\arctan((c^2*d+e)^{(1/2)}/c/x/e^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})*e^{(1/2)}/d^3/(c^2*d+e)^{(1/2)} + \frac{1}{8}b*c*e*(1-1/c^2/x^2)^{(1/2)}/d^2/(c^2*d+e)/(e+d/x^2)/x$

**Rubi [A]** time = 1.29, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$ , Rules used = {5240, 4734, 4730, 382, 377, 205, 4742, 4520, 2190, 2279, 2391}

$$\frac{ib \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} + \frac{ib \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{i \sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^3), x]

[Out]  $(b*c*e*\sqrt{1 - 1/(c^2*x^2)})/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^{2*(a + b*ArcSec[c*x])})/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcSec[c*x]))/(d^3*(e +$

$$\begin{aligned} & d/x^2)) + ((I/2)*(a + b*\text{ArcSec}[c*x])^2)/(b*d^3) - (b*\text{Sqrt}[e]*\text{ArcTan}[\text{Sqrt}[c^2*d + e]/(c*\text{Sqrt}[e]*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)])/(d^3*\text{Sqrt}[c^2*d + e]) + (b*\text{Sqrt}[e]*(c^2*d + 2*e)*\text{ArcTan}[\text{Sqrt}[c^2*d + e]/(c*\text{Sqrt}[e]*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d + e)^{(3/2)}) - ((a + b*\text{ArcSec}[c*x])*Log[1 - (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]/(2*d^3) - ((a + b*\text{ArcSec}[c*x])*Log[1 + (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]/(2*d^3) - ((a + b*\text{ArcSec}[c*x])*Log[1 - (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(2*d^3) - ((a + b*\text{ArcSec}[c*x])*Log[1 + (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(2*d^3) + ((I/2)*b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])))]/d^3 + ((I/2)*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])))]/d^3 + ((I/2)*b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])))]/d^3 + ((I/2)*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{(I*\text{ArcSec}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])))]/d^3 \end{aligned}$$
Rule 205

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \text{ :> } \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ /; } \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$$
Rule 377

$$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}/((c_ + (d_)*(x_)^{(n_)}), x\_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$$
Rule 382

$$\begin{aligned} & \text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \\ & \text{:> } -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, n, q\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*(p+q+2) + 1, 0] \ \&\& \ (\text{LtQ}[p, -1] \ \|\ \text{!LtQ}[q, -1]) \ \&\& \ \text{NeQ}[p, -1] \end{aligned}$$
Rule 2190

$$\begin{aligned} & \text{Int}[(F_)^{((g_)*((e_ + (f_)*(x_)))^{(n_)*((c_ + (d_)*(x_))^{(m_)})))/((a_ + (b_)*((F_)^{((g_)*((e_ + (f_)*(x_)))^{(n_)}))^{(n_)}), x\_Symbol] \text{ :> } \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a] ]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a] ], x], x] \text{ /; } \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \text{IGtQ}[m, 0] \end{aligned}$$
Rule 2279

$$\text{Int}[\text{Log}[(a_ + (b_)*((F_)^{((e_)*((c_ + (d_)*(x_)))^{(n_)}))], x\_Symbol]$$

```

:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rule 4520

```

Int[(((e_.) + (f_.)*(x_)^(m_.))*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

```

### Rule 4730

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x]))/(2*e*(p + 1)), x]
+ Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]

```

### Rule 4734

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

```

### Rule 4742

```

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.)^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
:= -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

```

### Rule 5240

```

Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)^(n_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]

```

Rubi steps



$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{x^5 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{e^2 x \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^3} - \frac{2ex \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} + \frac{x \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{x^{a+b \cos^{-1} \left( \frac{x}{c} \right)}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \frac{x^{a+b \cos^{-1} \left( \frac{x}{c} \right)}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \text{Subst} \left( \int \frac{x^{a+b \cos^{-1} \left( \frac{x}{c} \right)}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{e^2 (a + b \sec^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \sec^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int \left( -\frac{\sqrt{-d} (a+b \cos^{-1} \left( \frac{x}{c} \right))}{2d(\sqrt{e}-\sqrt{-d}x)} + \frac{\sqrt{-d} (a+b \cos^{-1} \left( \frac{x}{c} \right))}{2d(\sqrt{e}+\sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \sec^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \sec^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int \frac{a+b \cos^{-1} \left( \frac{x}{c} \right)}{\sqrt{e}-\sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2(-d)} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \sec^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \sec^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{-d} (a+b \cos^{-1} \left( \frac{x}{c} \right))}{c\sqrt{e}x} \right)}{d^3 \sqrt{c^2 d + e}} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \sec^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \sec^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \sec^{-1}(cx))}{2bd^3} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \sec^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \sec^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \sec^{-1}(cx))}{2bd^3} \\
&= \frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \sec^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \sec^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \sec^{-1}(cx))}{2bd^3}
\end{aligned}$$

**Mathematica** [F] time = 52.49, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x(d + ex^2)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^3), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^3), x]

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{e^3 x^7 + 3 d e^2 x^5 + 3 d^2 e x^3 + d^3 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^3,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign  
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy  
 m(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Va  
 lue

**maple** [C] time = 6.52, size = 5373, normalized size = 7.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} a \left( \frac{2ex^2 + 3d}{d^2e^2x^4 + 2d^3ex^2 + d^4} - \frac{2 \log(ex^2 + d)}{d^3} + \frac{4 \log(x)}{d^3} \right) + b \int \frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4\*a\*((2\*e\*x^2 + 3\*d)/(d^2\*e^2\*x^4 + 2\*d^3\*e\*x^2 + d^4) - 2\*log(e\*x^2 + d)/d^3 + 4\*log(x)/d^3) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{x(e x^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x\*(d + e\*x^2)^3),x)

[Out] int((a + b\*acos(1/(c\*x)))/(x\*(d + e\*x^2)^3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.108 \quad \int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=1124

$$\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{-d}}{16e^3}$$

[Out]  $3/16*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}-3/16*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}+3/16*(a+b*\text{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}-3/16*(a+b*\text{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}-3/16*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}-3/16*I*b*\text{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}+3/16*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}+3/16*I*b*\text{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/e^{(5/2)}/(-d)^{(1/2)}+1/16*b*\text{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}))/e/(c^2*d+e)^{(3/2)}/d^{(1/2)}+1/16*b*\text{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}))/e/(c^2*d+e)^{(3/2)}/d^{(1/2)}+1/16*(a+b*\text{arcsec}(c*x))*(-d)^{(1/2)}/e^{(3/2)}/(-d/x+(-d)^{(1/2)}*e^{(1/2)})^2+3/16*(a+b*\text{arcsec}(c*x))/e^2/(-d/x+(-d)^{(1/2)}*e^{(1/2)})-1/16*(a+b*\text{arcsec}(c*x))*(-d)^{(1/2)}/e^{(3/2)}/(d/x+(-d)^{(1/2)}*e^{(1/2)})^2-3/16*(a+b*\text{arcsec}(c*x))/e^2/(d/x+(-d)^{(1/2)}*e^{(1/2)})+3/16*b*\text{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}))/e^2/d^{(1/2)}/(c^2*d+e)^{(1/2)}+3/16*b*\text{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)}))/e^2/d^{(1/2)}/(c^2*d+e)^{(1/2)}+1/16*b*c*(-d)^{(1/2)}*(1-1/c^2/x^2)^{(1/2)}/e^{(3/2)}/(c^2*d+e)/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/16*b*c*(-d)^{(1/2)}*(1-1/c^2/x^2)^{(1/2)}/e^{(3/2)}/(c^2*d+e)/(d/x+(-d)^{(1/2)}*e^{(1/2)})$

**Rubi [A]** time = 1.58, antiderivative size = 1124, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$ , Rules used = {5240, 4668, 4744, 731, 725, 206, 4742, 4520, 2190, 2279,

2391}

$$\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{-d}}{16e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] (b\*c\*Sqrt[-d]\*Sqrt[1 - 1/(c^2\*x^2)])/(16\*e^(3/2)\*(c^2\*d + e)\*(Sqrt[-d]\*Sqrt[e] - d/x)) + (b\*c\*Sqrt[-d]\*Sqrt[1 - 1/(c^2\*x^2)])/(16\*e^(3/2)\*(c^2\*d + e)\*(Sqrt[-d]\*Sqrt[e] + d/x)) + (Sqrt[-d]\*(a + b\*ArcSec[c\*x]))/(16\*e^(3/2)\*(Sqrt[-d]\*Sqrt[e] - d/x)^2) + (3\*(a + b\*ArcSec[c\*x]))/(16\*e^2\*(Sqrt[-d]\*Sqrt[e] - d/x)) - (Sqrt[-d]\*(a + b\*ArcSec[c\*x]))/(16\*e^(3/2)\*(Sqrt[-d]\*Sqrt[e] + d/x)^2) - (3\*(a + b\*ArcSec[c\*x]))/(16\*e^2\*(Sqrt[-d]\*Sqrt[e] + d/x)) + (b\*ArcTanh[(c^2\*d - (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*Sqrt[d]\*e\*(c^2\*d + e)^(3/2)) + (3\*b\*ArcTanh[(c^2\*d - (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*Sqrt[d]\*e^2\*Sqrt[c^2\*d + e]) + (b\*ArcTanh[(c^2\*d + (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*Sqrt[d]\*e^2\*Sqrt[c^2\*d + e]) + (3\*(a + b\*ArcSec[c\*x])\*Log[1 - (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(16\*Sqrt[-d]\*e^(5/2)) - (3\*(a + b\*ArcSec[c\*x])\*Log[1 + (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(16\*Sqrt[-d]\*e^(5/2)) + (3\*(a + b\*ArcSec[c\*x])\*Log[1 - (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(16\*Sqrt[-d]\*e^(5/2)) - (3\*(a + b\*ArcSec[c\*x])\*Log[1 + (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(16\*Sqrt[-d]\*e^(5/2)) + (((3\*I)/16)\*b\*PolyLog[2, -((c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e]))])/(Sqrt[-d]\*e^(5/2)) - (((3\*I)/16)\*b\*PolyLog[2, (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(Sqrt[-d]\*e^(5/2)) + (((3\*I)/16)\*b\*PolyLog[2, -((c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e]))])/(Sqrt[-d]\*e^(5/2)) - (((3\*I)/16)\*b\*PolyLog[2, (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(Sqrt[-d]\*e^(5/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 731

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4520

```
Int[(((e_) + (f_)*(x_))^(m_)*Sin[(c_) + (d_)*(x_)])/(Cos[(c_) + (d_)
*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 4668

```
Int[((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_)^2)^(p_), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
```

tQ[p, 0] || IGtQ[n, 0])

#### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
  :> -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] +
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))
  )/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

#### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
  :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{d^3 (a + b \cos^{-1}\left(\frac{x}{c}\right))}{8(-d)^{3/2} e^{3/2} (\sqrt{-d} \sqrt{e} - dx)^3} - \frac{3d (a + b \cos^{-1}\left(\frac{x}{c}\right))}{16e^2 (\sqrt{-d} \sqrt{e} - dx)^2} - \frac{d^3 (a + b \cos^{-1}\left(\frac{x}{c}\right))}{8(-d)^{3/2} e^{3/2} (\sqrt{-d} \sqrt{e} + dx)^3} \right. \right. \\
&\quad \left. \left. (3d) \text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d} \sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right) \quad (3d) \text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d} \sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right) \quad (3d) \text{Subst} \left( \int \frac{a + b \cos^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d} \sqrt{e} - dx)^3} dx, x, \frac{1}{x} \right) \right. \right. \\
&= \frac{\quad}{16e^2} + \frac{\quad}{16e^2} + \frac{\quad}{16e^2} \\
&= \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})^2} + \frac{3(a + b \sec^{-1}(cx))}{16e^2 (\sqrt{-d} \sqrt{e} - \frac{d}{x})} - \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} + \frac{d}{x})^2} - \frac{3(a + b \sec^{-1}(cx))}{16e^2 (\sqrt{-d} \sqrt{e} + \frac{d}{x})} \\
&= \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} + \frac{d}{x})} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})} \\
&= \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} + \frac{d}{x})} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})} \\
&= \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} + \frac{d}{x})} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})} \\
&= \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} + \frac{d}{x})} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})} \\
&= \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} + \frac{d}{x})} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})} \\
&= \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{-d} \sqrt{1 - \frac{1}{c^2 x^2}}}{16e^{3/2} (c^2 d + e) (\sqrt{-d} \sqrt{e} + \frac{d}{x})} + \frac{\sqrt{-d} (a + b \sec^{-1}(cx))}{16e^{3/2} (\sqrt{-d} \sqrt{e} - \frac{d}{x})}
\end{aligned}$$



**Mathematica [A]** time = 6.20, size = 1819, normalized size = 1.62

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] (a\*d\*x)/(4\*e^2\*(d + e\*x^2)^2) - (5\*a\*x)/(8\*e^2\*(d + e\*x^2)) + (3\*a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*Sqrt[d]\*e^(5/2)) + b\*((5\*(-(ArcSec[c\*x]/(I\*Sqrt[d]\*Sqrt[e] + e\*x)) + (I\*(ArcSin[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x))/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e]))/Sqrt[d]))/(16\*e^2) + (5\*(-(ArcSec[c\*x]/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x)) - (I\*(ArcSin[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x))/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e]))/Sqrt[d]))/(16\*e^2) + ((I/16)\*Sqrt[d]\*(-(ArcSec[c\*x]/(Sqrt[e]\*((-I)\*Sqrt[d] + Sqrt[e]\*x)^2)) + (ArcSin[1/(c\*x)]/Sqrt[e] - I\*((c\*Sqrt[d]\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(c^2\*d + e)\*((-I)\*Sqrt[d] + Sqrt[e]\*x)) + ((2\*c^2\*d + e)\*Log[(-4\*d\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] - Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x))/((2\*c^2\*d + e)\*((-I)\*Sqrt[d] + Sqrt[e]\*x)))]/(c^2\*d + e)^(3/2))/d)/e^2 - ((I/16)\*Sqrt[d]\*((I\*c\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(Sqrt[d]\*(c^2\*d + e)\*(I\*Sqrt[d] + Sqrt[e]\*x)) - ArcSec[c\*x]/(Sqrt[e]\*(I\*Sqrt[d] + Sqrt[e]\*x)^2) + ArcSin[1/(c\*x)]/(d\*Sqrt[e]) - (I\*(2\*c^2\*d + e)\*Log[(4\*d\*Sqrt[e]\*Sqrt[c^2\*d + e]\*((-I)\*Sqrt[e] + c\*(c\*Sqrt[d] + Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x))/((2\*c^2\*d + e)\*(I\*Sqrt[d] + Sqrt[e]\*x)))]/(d\*(c^2\*d + e)^(3/2)))/e^2 + (3\*(8\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (4\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (4\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (2\*I)\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])]) - 2\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - 2\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])])/(32\*Sqrt[d]\*e^(5/2)) - (3\*(8\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*ArcTan[((-I)\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (4\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] - (2\*I)\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])] + (4\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])])

$t[c^2*d + e])*E^{(I*ArcSec[c*x])}/(c*sqrt[d])) + (2*I)*ArcSec[c*x]*Log[1 + E^{((2*I)*ArcSec[c*x])}] - 2*PolyLog[2, ((-I)*(-sqrt[e] + sqrt[c^2*d + e])*E^{(I*ArcSec[c*x])}/(c*sqrt[d])) - 2*PolyLog[2, (I*(sqrt[e] + sqrt[c^2*d + e])*E^{(I*ArcSec[c*x])}/(c*sqrt[d])) + PolyLog[2, -E^{((2*I)*ArcSec[c*x])})])]/(32*sqrt[d]*e^{(5/2)})$

**fricas** [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arcsec}(cx) + ax^4}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*x^4\*arcsec(c\*x) + a\*x^4)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [C] time = 4.41, size = 3223, normalized size = 2.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x)

[Out]  $-3/8*I*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^{(1/2)}-I/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{(1/2)}+3/8*I*b*(-(c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^{(1/2)}$

$$\begin{aligned}
& 1/2) - 5/4 * I / c^2 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * ( \\
& 1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - 2 * e) * d)^{1/2}) / \\
& e / (c^2 * d + e) / d^2 - 5/4 * I / c^2 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arc} \\
& \tan(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d) \\
& ^{1/2}) / e / (c^2 * d + e) / d^2 + I / c^4 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} \\
& ) * \operatorname{arctanh}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - \\
& 2 * e) * d)^{1/2}) / (c^2 * d + e)^2 / d^3 * (e * (c^2 * d + e))^{1/2} - 5/8 * c^6 * b * x^3 / (c^2 * e * x^2 \\
& + c^2 * d)^2 / (c^2 * d + e) / e * \operatorname{arcsec}(c * x) * d - 3/8 * c^6 * b * x / (c^2 * e * x^2 + c^2 * d)^2 / (c^2 * d + \\
& e) / e^2 * \operatorname{arcsec}(c * x) * d^2 + 1/8 * c^5 * b * x^2 / (c^2 * e * x^2 + c^2 * d)^2 / (c^2 * d + e) / e * ((c^2 * \\
& x^2 - 1) / c^2 / x^2)^{1/2} * d - 3/8 * c^4 * b * x / (c^2 * e * x^2 + c^2 * d)^2 / (c^2 * d + e) / e * \operatorname{arcsec}( \\
& c * x) * d + I / c^4 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * (1 / \\
& c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - 2 * e) * d)^{1/2}) / (c \\
& ^2 * d + e)^2 * e / d^3 + I / c^4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctan}( \\
& c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} \\
& ) / (c^2 * d + e)^2 * e / d^3 + I / c^4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctan}( \\
& c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * \\
& d)^{1/2}) / e / (c^2 * d + e) / d^3 * (e * (c^2 * d + e))^{1/2} + 3/4 * I / c^2 * b * ((c^2 * d + 2 * (e * (c^2 \\
& * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctan}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((c^2 * d \\
& + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2}) / e^2 / (c^2 * d + e) / d^2 * (e * (c^2 * d + e))^{1/2} \\
& + 5/4 * I / c^2 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * (1 / c / \\
& x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - 2 * e) * d)^{1/2}) / (c^2 \\
& * d + e)^2 / e / d^2 * (e * (c^2 * d + e))^{1/2} - 5/4 * I / c^2 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} \\
& + 2 * e) * d)^{1/2} * \operatorname{arctan}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((c^2 * d + 2 * (e * (c^2 * d \\
& + e))^{1/2} + 2 * e) * d)^{1/2}) / (c^2 * d + e)^2 / e / d^2 * (e * (c^2 * d + e))^{1/2} - 3/4 * I / c^2 * b \\
& * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * (1 / c / x + I * (1 - 1 / c^2 \\
& / x^2)^{1/2}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - 2 * e) * d)^{1/2}) / e^2 / (c^2 * d + e) / d^ \\
& 2 * (e * (c^2 * d + e))^{1/2} - I / c^4 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \\
& \operatorname{arctanh}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - 2 * \\
& e) * d)^{1/2}) / e / (c^2 * d + e) / d^3 * (e * (c^2 * d + e))^{1/2} + 3/8 * a / e^2 / (d * e)^{1/2} * \operatorname{arct} \\
& \tan(e * x / (d * e)^{1/2}) - 5/8 * c^4 * a / (c^2 * e * x^2 + c^2 * d)^2 / e * x^3 - I / c^4 * b * (- (c^2 * d - 2 * \\
& (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) \\
& ) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} - 2 * e) * d)^{1/2}) / (c^2 * d + e) / d^3 + 3/16 * I * c^3 * b / e \\
& ^2 / (c^2 * d + e) * d * \operatorname{sum}(_R1 / (_R1^2 * c^2 * d + c^2 * d + 2 * e) * (I * \operatorname{arcsec}(c * x) * \ln((\_R1 - 1 / c / x \\
& - I * (1 - 1 / c^2 / x^2)^{1/2}) / \_R1) + \operatorname{dilog}((\_R1 - 1 / c / x - I * (1 - 1 / c^2 / x^2)^{1/2}) / \_R1)), \\
& \_R1 = \operatorname{RootOf}(c^2 * d * \_Z^4 + (2 * c^2 * d + 4 * e) * \_Z^2 + c^2 * d)) - 3/16 * I * c^3 * b / e^2 / (c^2 * d + e) \\
& * d * \operatorname{sum}(1 / \_R1 / (_R1^2 * c^2 * d + c^2 * d + 2 * e) * (I * \operatorname{arcsec}(c * x) * \ln((\_R1 - 1 / c / x - I * (1 - 1 / c^ \\
& 2 / x^2)^{1/2}) / \_R1) + \operatorname{dilog}((\_R1 - 1 / c / x - I * (1 - 1 / c^2 / x^2)^{1/2}) / \_R1)), \_R1 = \operatorname{RootOf} \\
& (c^2 * d * \_Z^4 + (2 * c^2 * d + 4 * e) * \_Z^2 + c^2 * d)) + 7/4 * I / c^2 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e)) \\
& ^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((-c^2 * d + 2 * ( \\
& e * (c^2 * d + e))^{1/2} - 2 * e) * d)^{1/2}) / (c^2 * d + e)^2 / d^2 - I / c^4 * b * ((c^2 * d + 2 * (e * (c^2 \\
& * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctan}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ((c^2 * d \\
& + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2}) / (c^2 * d + e) / d^3 + 7/4 * I / c^2 * b * ((c^2 * d + 2 * ( \\
& e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctan}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / ( \\
& (c^2 * d + 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2}) / (c^2 * d + e)^2 / d^2 + 3/4 * I * b * (- (c^2 * \\
& d - 2 * (e * (c^2 * d + e))^{1/2} + 2 * e) * d)^{1/2} * \operatorname{arctanh}(c * d * (1 / c / x + I * (1 - 1 / c^2 / x^2)^{1/2}) / (
\end{aligned}$$

$$\frac{1}{2}) / ((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2}) / e / (c^2*d+e)^2 / d - 3/8 * I * b * ((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} * \arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2})) / ((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}) / e^2 / (c^2*d+e) / d - 3/8 * I * b * (-c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2})) / ((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2}) / e^2 / (c^2*d+e) / d + 3/4 * I * b * ((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} * \arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2})) / ((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}) / e / (c^2*d+e)^2 / d - 5/8 * c^4 * b * x^3 / (c^2*e*x^2+c^2*d)^2 / (c^2*d+e) * \operatorname{arcsec}(c*x) + 1/8 * c^5 * b * x^4 / (c^2*e*x^2+c^2*d)^2 / (c^2*d+e) * ((c^2*x^2-1)/c^2/x^2)^{1/2} + 3/16 * I * c * b * e / (c^2*d+e) * \sum(\_R1 / (\_R1^2 * c^2*d + c^2*d+2*e) * (I * \operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{1/2})) / \_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{1/2}) / \_R1)), \_R1 = \operatorname{RootOf}(c^2*d*_Z^4 + (2*c^2*d+4*e)*_Z^2 + c^2*d)) - 3/16 * I * c * b * e / (c^2*d+e) * \sum(1 / \_R1 / (\_R1^2 * c^2*d + c^2*d+2*e) * (I * \operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{1/2})) / \_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{1/2}) / \_R1)), \_R1 = \operatorname{RootOf}(c^2*d*_Z^4 + (2*c^2*d+4*e)*_Z^2 + c^2*d)) - 3/8 * c^4 * a / (c^2*e*x^2+c^2*d)^2 / e^2 * d * x$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8} a \left( \frac{5ex^3 + 3dx}{e^4x^4 + 2de^3x^2 + d^2e^2} - \frac{3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} \right) + b \int \frac{x^4 \arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] -1/8\*a\*((5\*e\*x^3 + 3\*d\*x)/(e^4\*x^4 + 2\*d\*e^3\*x^2 + d^2\*e^2) - 3\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e^2)) + b\*integrate(x^4\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mapad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^4\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

$$3.109 \quad \int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^3} dx$$

Optimal. Leaf size=1124

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{a+b \sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b \sec^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

[Out]  $-1/16*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(3/2)}/(c^2*d+e)^{(3/2)}-1/16*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(3/2)}/(c^2*d+e)^{(3/2)}-1/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*(a+b*\operatorname{arcsec}(c*x))/(-d)^{(1/2)}/e^{(1/2)}/(-d/x+(-d)^{(1/2)}*e^{(1/2)})^2+1/16*(a+b*\operatorname{arcsec}(c*x))/d/e/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/16*(-a-b*\operatorname{arcsec}(c*x))/(-d)^{(1/2)}/e^{(1/2)}/(d/x+(-d)^{(1/2)}*e^{(1/2)})^2+1/16*(-a-b*\operatorname{arcsec}(c*x))/d/e/(d/x+(-d)^{(1/2)}*e^{(1/2)})+1/16*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(3/2)}/e/(c^2*d+e)^{(1/2)}+1/16*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(3/2)}/e/(c^2*d+e)^{(1/2)}+1/16*b*c*(1-1/c^2/x^2)^{(1/2)}/(c^2*d+e)/(-d)^{(1/2)}/e^{(1/2)}/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/16*b*c*(1-1/c^2/x^2)^{(1/2)}/(c^2*d+e)/(-d)^{(1/2)}/e^{(1/2)}/(d/x+(-d)^{(1/2)}*e^{(1/2)})$

**Rubi [A]** time = 3.02, antiderivative size = 1124, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5240, 4734, 4668, 4744, 731, 725, 206, 4742, 4520, 2190,

2279, 2391}

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3, x]

[Out] (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]/(16\*Sqrt[-d]\*Sqrt[e]\*(c^2\*d + e)\*(Sqrt[-d]\*Sqrt[e] - d/x)) + (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]/(16\*Sqrt[-d]\*Sqrt[e]\*(c^2\*d + e)\*(Sqrt[-d]\*Sqrt[e] + d/x)) + (a + b\*ArcSec[c\*x])/(16\*Sqrt[-d]\*Sqrt[e]\*(Sqrt[-d]\*Sqrt[e] - d/x)^2) + (a + b\*ArcSec[c\*x])/(16\*d\*e\*(Sqrt[-d]\*Sqrt[e] - d/x)) - (a + b\*ArcSec[c\*x])/(16\*Sqrt[-d]\*Sqrt[e]\*(Sqrt[-d]\*Sqrt[e] + d/x)^2) - (a + b\*ArcSec[c\*x])/(16\*d\*e\*(Sqrt[-d]\*Sqrt[e] + d/x)) - (b\*ArcTanh[(c^2\*d - (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*d^(3/2)\*(c^2\*d + e)^(3/2)) + (b\*ArcTanh[(c^2\*d - (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*d^(3/2)\*e\*Sqrt[c^2\*d + e]) - (b\*ArcTanh[(c^2\*d + (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*d^(3/2)\*(c^2\*d + e)^(3/2)) + (b\*ArcTanh[(c^2\*d + (Sqrt[-d]\*Sqrt[e])/x)/(c\*Sqrt[d]\*Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])])/(16\*d^(3/2)\*e\*Sqrt[c^2\*d + e]) - ((a + b\*ArcSec[c\*x])\*Log[1 - (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) + ((a + b\*ArcSec[c\*x])\*Log[1 + (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) - ((a + b\*ArcSec[c\*x])\*Log[1 - (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) + ((a + b\*ArcSec[c\*x])\*Log[1 + (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) - ((I/16)\*b\*PolyLog[2, -(c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) + ((I/16)\*b\*PolyLog[2, (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) - ((I/16)\*b\*PolyLog[2, -(c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2)) + ((I/16)\*b\*PolyLog[2, (c\*Sqrt[-d]\*E^(I\*ArcSec[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(16\*(-d)^(3/2)\*e^(3/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 731

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4520

```
Int[(((e_) + (f_)*(x_))^(m_)*Sin[(c_) + (d_)*(x_)])/(Cos[(c_) + (d_)
*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 4668

```
Int[((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_)^2)^(p_), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
```



tQ[p, 0] || IGtQ[n, 0])

#### Rule 4734

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Subst[Int[((a + b*x)^n*Sin[x])/(c*d + e*Cos[x]), x], x, ArcCos[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(e*(m + 1)), x] + Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

#### Rule 5240

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{x^2 (a + b \cos^{-1}(\frac{x}{c}))}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{e (a + b \cos^{-1}(\frac{x}{c}))}{d (e + dx^2)^3} + \frac{a + b \cos^{-1}(\frac{x}{c})}{d (e + dx^2)^2} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1}(\frac{x}{c})}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \cos^{-1}(\frac{x}{c})}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{\text{Subst} \left( \int \left( -\frac{d(a + b \cos^{-1}(\frac{x}{c}))}{4e(\sqrt{-d} \sqrt{e} - dx)^2} - \frac{d(a + b \cos^{-1}(\frac{x}{c}))}{4e(\sqrt{-d} \sqrt{e} + dx)^2} - \frac{d(a + b \cos^{-1}(\frac{x}{c}))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \cos^{-1}(\frac{x}{c})}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{3 \text{Subst} \left( \int \frac{a + b \cos^{-1}(\frac{x}{c})}{(\sqrt{-d} \sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{16e} - \frac{3 \text{Subst} \left( \int \frac{a + b \cos^{-1}(\frac{x}{c})}{(\sqrt{-d} \sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{16e} + \frac{\text{Subst} \left( \int \frac{a + b \cos^{-1}(\frac{x}{c})}{(-de - d^2x^2)} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{a + b \sec^{-1}(cx)}{16\sqrt{-d} \sqrt{e} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)^2} + \frac{a + b \sec^{-1}(cx)}{16de \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \sec^{-1}(cx)}{16\sqrt{-d} \sqrt{e} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)^2} - \frac{a + b \sec^{-1}(cx)}{16de \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{a}{16\sqrt{-d} \sqrt{e}} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{a}{16\sqrt{-d} \sqrt{e}} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{a}{16\sqrt{-d} \sqrt{e}} \\
&= \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{bc \sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d} \sqrt{e} (c^2d + e) \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{a}{16\sqrt{-d} \sqrt{e}}
\end{aligned}$$

**Mathematica [A]** time = 6.14, size = 1827, normalized size = 1.63

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$\begin{aligned} & -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x)))/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(d*e) \\ & - (-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x)))/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*(-(ArcSec[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x)))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)))/((c^2*d + e)^(3/2))))/d)/Sqrt[d]*e + ((I/16)*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x)))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)))/d*(c^2*d + e)^(3/2))/Sqrt[d]*e + (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])])/(32*d^(3/2)*e^(3/2)) - (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) - (4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) + (4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]) \end{aligned}$$

$I \cdot \text{ArcSec}[c*x]) / (c \cdot \text{Sqrt}[d]) + (2*I) \cdot \text{ArcSec}[c*x] \cdot \text{Log}[1 + E^{((2*I) \cdot \text{ArcSec}[c*x])}] - 2 \cdot \text{PolyLog}[2, ((-I) \cdot (-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]) \cdot E^{(I \cdot \text{ArcSec}[c*x])}) / (c \cdot \text{Sqrt}[d])] - 2 \cdot \text{PolyLog}[2, (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]) \cdot E^{(I \cdot \text{ArcSec}[c*x])}) / (c \cdot \text{Sqrt}[d])] + \text{PolyLog}[2, -E^{((2*I) \cdot \text{ArcSec}[c*x])})] / (32*d^{(3/2)} \cdot e^{(3/2)})]$

**fricas** [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{bx^2 \text{arcsec}(cx) + ax^2}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")`

[Out] `integral((b*x^2*arcsec(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [C] time = 6.71, size = 2357, normalized size = 2.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x)`

[Out]  $-1/8*c^6*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*arcsec(c*x)*d-1/8*c^5*b*x^4/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*((c^2*x^2-1)/c^2/x^2)^{(1/2)}+1/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*arcsec(c*x)+1/4*I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/(c^2*d+e)^2*e/d^3+1/4*I/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{(1/2)}-1/8*I*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*arctan(c*d*$

$$\begin{aligned}
& 1/c/x+I*(1-1/c^2/x^2)^{(1/2)} / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} / ( \\
& c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^{(1/2)}+1/8*I*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)} \\
& +2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2 \\
& *d+e))^{(1/2)}-2*e)*d)^{(1/2)} / (c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^{(1/2)}-1/4*I/c^2 \\
& *b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2 \\
& /x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} / (c^2*d+e)^2/d^3*( \\
& e*(c^2*d+e))^{(1/2)}+1/4*I/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\
& *\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2 \\
& *e)*d)^{(1/2)} / (c^2*d+e)^2*e/d^3+1/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e) \\
& *\operatorname{arcsec}(c*x)-1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x \\
& ^2)^{(1/2)}-1/8*c^4*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arcsec}(c*x)+1/4*I*b*(-( \\
& c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2 \\
& )^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} / (c^2*d+e)^2/d^2+1/4* \\
& I*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^ \\
& 2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} / (c^2*d+e)^2/d^2- \\
& 1/16*I*c^3*b/e/(c^2*d+e)*\operatorname{sum}(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln \\
& ((_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{( \\
& 1/2)})/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/16*I*c*b/d/( \\
& c^2*d+e)*\operatorname{sum}(\_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1- \\
& 1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\operatorname{Ro \\
& otOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/16*I*c*b/d/(c^2*d+e)*\operatorname{sum}(1/_R1 \\
& /(_R1^2*c^2*d+c^2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2) \\
& )/_R1)+\operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+ \\
& (2*c^2*d+4*e)*_Z^2+c^2*d))+1/16*I*c^3*b/e/(c^2*d+e)*\operatorname{sum}(\_R1/(_R1^2*c^2*d+c^ \\
& 2*d+2*e)*(I*\operatorname{arcsec}(c*x)*\ln((\_R1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((\_R \\
& 1-1/c/x-I*(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z \\
& ^2+c^2*d))-1/8*I*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(c*d*( \\
& 1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/e \\
& / (c^2*d+e)/d^2-1/4*I/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arc \\
& tanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)* \\
& d)^{(1/2)}) / (c^2*d+e)/d^3-1/4*I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{( \\
& 1/2)}*\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^2*d+e))^{(1/2) \\
& +2*e)*d)^{(1/2)}) / (c^2*d+e)/d^3-1/8*I*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d \\
& )^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{( \\
& 1/2)}-2*e)*d)^{(1/2)}) / e/(c^2*d+e)/d^2+1/4*I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1 \\
& /2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}) / ((c^2*d+2*(e*(c^ \\
& 2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}) / e/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)}-1/4*I/c^2* \\
& b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^ \\
& 2/x^2)^{(1/2)}) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}) / e/(c^2*d+e)/d^3 \\
& *(e*(c^2*d+e))^{(1/2)}+1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/d*x^3-1/8*c^4*a/(c^2*e*x \\
& ^2+c^2*d)^2/e*x+1/8*a/d/e/(d*e)^{(1/2)}*\operatorname{arctan}(e*x/(d*e)^{(1/2)})
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} a \left( \frac{ex^3 - dx}{de^3x^4 + 2d^2e^2x^2 + d^3e} + \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}de} \right) + b \int \frac{x^2 \arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/8\*a\*((e\*x^3 - d\*x)/(d\*e^3\*x^4 + 2\*d^2\*e^2\*x^2 + d^3\*e) + arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d\*e)) + b\*integrate(x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^2\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.110 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=1114

$$\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

[Out]  $1/16*b*e*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(5/2)}/(c^2*d+e)^{(3/2)}+1/16*b*e*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(5/2)}/(c^2*d+e)^{(3/2)}+3/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}-3/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}+3/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}-3/16*(a+b*\operatorname{arcsec}(c*x))*\ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}-3/16*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}+3/16*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}-3/16*I*b*\operatorname{polylog}(2,c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}+3/16*I*b*\operatorname{polylog}(2,-c*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(c^2*d+e)^{(1/2)}))/(-d)^{(5/2)}/e^{(1/2)}+1/16*(a+b*\operatorname{arcsec}(c*x))*e^{(1/2)}/(-d)^{(3/2)}/(-d/x+(-d)^{(1/2)}*e^{(1/2)})^2-5/16*(a+b*\operatorname{arcsec}(c*x))/d^2/(-d/x+(-d)^{(1/2)}*e^{(1/2)})-1/16*(a+b*\operatorname{arcsec}(c*x))*e^{(1/2)}/(-d)^{(3/2)}/(d/x+(-d)^{(1/2)}*e^{(1/2)})^2+5/16*(a+b*\operatorname{arcsec}(c*x))/d^2/(d/x+(-d)^{(1/2)}*e^{(1/2)})-5/16*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(5/2)}/(c^2*d+e)^{(1/2)}-5/16*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d+e)^{(1/2)}/(1-1/c^2/x^2)^{(1/2)})/d^{(5/2)}/(c^2*d+e)^{(1/2)}+1/16*b*c*e^{(1/2)}*(1-1/c^2/x^2)^{(1/2)}/(-d)^{(3/2)}/(c^2*d+e)/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/16*b*c*e^{(1/2)}*(1-1/c^2/x^2)^{(1/2)}/(-d)^{(3/2)}/(c^2*d+e)/(d/x+(-d)^{(1/2)}*e^{(1/2)})$

**Rubi [A]** time = 3.79, antiderivative size = 1114, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5230, 4734, 4668, 4744, 731, 725, 206, 4742, 4520, 2190,

2279, 2391}

$$\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^3, x]

```
[Out] (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcSec[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcSec[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSec[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSec[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) - (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) - (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) + (3*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/((-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/((-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 725



```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 731

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4520

```
Int[(((e_) + (f_)*(x_))^(m_)*Sin[(c_) + (d_)*(x_)])/(Cos[(c_) + (d_)
*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 4668

```
Int[((a_) + ArcCos[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_)^2)^(p_), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
```

tQ[p, 0] || IGtQ[n, 0])

#### Rule 4734

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*ArcCos[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2\*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 4742

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Subst[Int[((a + b\*x)^n\*Sin[x])/(c\*d + e\*Cos[x]), x], x, ArcCos[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 4744

Int[((a\_.) + ArcCos[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^n)/(e\*(m + 1)), x] + Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcCos[c\*x])^(n - 1))/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 5230

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcCos[x/c])^n)/x^(2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{x^4 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{e^2 \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^3} - \frac{2e \left( a + b \cos^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{\text{Subst} \left( \int \left( \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \left( -\frac{d(a + b \cos^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{3 \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{16d} + \frac{3 \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{16d} + \frac{3 \text{Subst} \left( \int \frac{a + b \cos^{-1} \left( \frac{x}{c} \right)}{-de} dx, x, \frac{1}{x} \right)}{8d} \\
&= \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{16(-d)^{3/2} (\sqrt{-d}\sqrt{e} - \frac{d}{x})^2} - \frac{5(a + b \sec^{-1}(cx))}{16d^2 (\sqrt{-d}\sqrt{e} - \frac{d}{x})} - \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{16(-d)^{3/2} (\sqrt{-d}\sqrt{e} + \frac{d}{x})^2} + \frac{5(a + b \sec^{-1}(cx))}{16d^2 (\sqrt{-d}\sqrt{e} + \frac{d}{x})} \\
&= \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{16(-d)^{3/2} (c^2d + e)} \\
&= \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{16(-d)^{3/2} (c^2d + e)} \\
&= \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{16(-d)^{3/2} (c^2d + e)} \\
&= \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{bc\sqrt{e} \sqrt{1 - \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d + e) (\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{\sqrt{e} (a + b \sec^{-1}(cx))}{16(-d)^{3/2} (c^2d + e)}
\end{aligned}$$

**Mathematica** [A] time = 6.06, size = 1812, normalized size = 1.63

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^3,x]

[Out] (a\*x)/(4\*d\*(d + e\*x^2)^2) + (3\*a\*x)/(8\*d^2\*(d + e\*x^2)) + (3\*a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*Sqrt[e]) + b\*((-3\*(-(ArcSec[c\*x]/(I\*Sqrt[d]\*Sqrt[e] + e\*x)) + (I\*(ArcSin[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x)))/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e]))/Sqrt[d]))/(16\*d^2) - (3\*(-(ArcSec[c\*x]/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x)) - (I\*(ArcSin[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x)))/(Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) - e]))/Sqrt[d]))/(16\*d^2) + ((I/16)\*(-(ArcSec[c\*x]/(Sqrt[e]\*((-I)\*Sqrt[d] + Sqrt[e]\*x)^2)) + (ArcSin[1/(c\*x)]/Sqrt[e] - I\*((c\*Sqrt[d]\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(c^2\*d + e))\*((-I)\*Sqrt[d] + Sqrt[e]\*x)) + ((2\*c^2\*d + e)\*Log[(-4\*d\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] - Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x)))/((2\*c^2\*d + e)\*((-I)\*Sqrt[d] + Sqrt[e]\*x)))/(c^2\*d + e)^(3/2))/d)/d^(3/2) - ((I/16)\*((I\*c\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(Sqrt[d]\*(c^2\*d + e)\*(I\*Sqrt[d] + Sqrt[e]\*x)) - ArcSec[c\*x]/(Sqrt[e]\*(I\*Sqrt[d] + Sqrt[e]\*x)^2) + ArcSin[1/(c\*x)]/(d\*Sqrt[e]) - (I\*(2\*c^2\*d + e)\*Log[(4\*d\*Sqrt[e]\*Sqrt[c^2\*d + e]\*((-I)\*Sqrt[e] + c\*(c\*Sqrt[d] + Sqrt[c^2\*d + e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x)))/((2\*c^2\*d + e)\*(I\*Sqrt[d] + Sqrt[e]\*x)))/(d\*(c^2\*d + e)^(3/2)))/d^(3/2) + (3\*(8\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - (4\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(Sqrt[e] - Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + (4\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + (2\*I)\*ArcSec[c\*x]\*Log[1 + E^((2\*I)\*ArcSec[c\*x])] - 2\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - 2\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + PolyLog[2, -E^((2\*I)\*ArcSec[c\*x])])/(32\*d^(5/2)\*Sqrt[e]) - (3\*(8\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*ArcTan[(((I\*c\*Sqrt[d] + Sqrt[e])\*Tan[ArcSec[c\*x]/2])/Sqrt[c^2\*d + e]] - (2\*I)\*ArcSec[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - (4\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(-Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) - (2\*I)\*ArcSec[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])\*E^(I\*ArcSec[c\*x]))/(c\*Sqrt[d])]) + (4\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 - (I\*(Sqrt[e] + Sqrt[c^2\*d + e])

$$\int \frac{E^{\arcsin(cx)}}{c\sqrt{d}} + (2I)\arcsin(cx)\log[1 + E^{(2I)\arcsin(cx)}] - 2\text{PolyLog}[2, (-1)(-\sqrt{e} + \sqrt{c^2d + e})E^{\arcsin(cx)}] - 2\text{PolyLog}[2, (I)(\sqrt{e} + \sqrt{c^2d + e})E^{\arcsin(cx)}] + \text{PolyLog}[2, -E^{(2I)\arcsin(cx)}]}{32d^{5/2}\sqrt{e}}$$

**fricas** [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
 INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [C] time = 7.83, size = 3214, normalized size = 2.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x)

[Out] 
$$-9/4I/c^2b*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}e^2\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))/((-c^2d+2*(e*(c^2d+e))^{1/2}-2e)*d)^{1/2}/d^4/(c^2d+e)^2+5/4I/c^2b*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))/((-c^2d+2*(e*(c^2d+e))^{1/2}-2e)*d)^{1/2}*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}/d^4/(c^2d+e)*(e*(c^2d+e))^{1/2}-9/4I/c^2b*((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}e^2\operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{1/2}))/((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}/d^4/(c^2d+e)^2-I/c^4b*(-(c^2d-2$$

$$\begin{aligned}
& * (e^{(c^2*d+e)})^{(1/2)+2*e} * d^{(1/2)} * e^3 * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} / d^5 / (c^2*d+e)^2 + 7/4 * I/c^2 * b * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * e/d^4 / (c^2*d+e) \\
& - I/c^4 * b * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * e^3 * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / d^5 / (c^2*d+e)^2 + 7/4 * I/c^2 * b * (-c^2*d-2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} * e/d^4 / (c^2*d+e) - 5/4 * I/c^2 * b * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / d^4 / (c^2*d+e) * (e^{(c^2*d+e)})^{(1/2)} + 5/8 * c^6 * b * x / (c^2 * e * x^2 + c^2 * d)^2 / (c^2 * d + e) * \operatorname{arcsec}(c*x) + 5/8 * I * b * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / d^3 / (c^2*d+e) + 5/8 * I * b * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} * (-c^2*d-2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / d^3 / (c^2*d+e) + 3/16 * I * c^3 * b / d / (c^2*d+e) * \sum(_R1/(_R1^2*c^2*d+c^2*d+2*e) * (I * \operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1), _R1 = \operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d) - 3/16 * I * c^3 * b / d / (c^2*d+e) * \sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e) * (I * \operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1), _R1 = \operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d) + 3/16 * I * c * b / d^2 / (c^2*d+e) * e * \sum(_R1/(_R1^2*c^2*d+c^2*d+2*e) * (I * \operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1), _R1 = \operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d) - 3/16 * I * c * b / d^2 / (c^2*d+e) * e * \sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e) * (I * \operatorname{arcsec}(c*x) * \ln((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1) + \operatorname{dilog}((\_R1-1/c/x-I*(1-1/c^2/x^2))^{(1/2)})/_R1), _R1 = \operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d) - 5/8 * I * b * (-c^2*d-2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} / (c^2*d+e)^2 / d^3 * (e^{(c^2*d+e)})^{(1/2)} + 5/8 * I * b * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / d^3 / (c^2*d+e)^2 * (e^{(c^2*d+e)})^{(1/2)} - 5/4 * I * b * (-c^2*d-2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} / (c^2*d+e)^2 * e / d^3 - 5/4 * I * b * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / (c^2*d+e)^2 * e / d^3 + I/c^4 * b * (-c^2*d-2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} * e / d^5 / (c^2*d+e) * (e^{(c^2*d+e)})^{(1/2)} + I/c^4 * b * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * e^2 * \operatorname{arctan}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} / d^5 / (c^2*d+e)^2 * (e^{(c^2*d+e)})^{(1/2)} + 1/4 * c^4 * a * x / d / (c^2 * e * x^2 + c^2 * d)^2 + 3/8 * c^2 * a / d^2 * x / (c^2 * e * x^2 + c^2 * d) + 3/8 * a / d^2 / (d * e)^{(1/2)} * \operatorname{arctan}(e*x/(d*e)^{(1/2)}) - I/c^4 * b * (-c^2*d-2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * e^2 * \operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2))^{(1/2)}) / ((-c^2*d+2*(e^{(c^2*d+e)})^{(1/2)}-2*e)*d)^{(1/2)} / d^5 / (c^2*d+e)^2 * (e^{(c^2*d+e)})^{(1/2)} - I/c^4 * b * ((c^2*d+2*(e^{(c^2*d+e)})^{(1/2)+2*e})*d)^{(1/2)} * \operatorname{arctan}
\end{aligned}$$

$(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e/d^5/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}+7/4*I/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^4/(c^2*d+e)^2*(e*(c^2*d+e))^{(1/2)}-7/4*I/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e)^2*(e*(c^2*d+e))^{(1/2)}+3/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*\operatorname{arcsec}(c*x)+1/8*c^5*b*x^4/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*e^2+1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*e+3/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*\operatorname{arcsec}(c*x)*e^2+5/8*c^4*b*x/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*\operatorname{arcsec}(c*x)*e+I/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})*e^2/d^5/(c^2*d+e)+I/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})*e^2/d^5/(c^2*d+e)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} a \left( \frac{3ex^3 + 5dx}{d^2e^2x^4 + 2d^3ex^2 + d^4} + \frac{3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d^2} \right) + b \int \frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/8\*a\*((3\*e\*x^3 + 5\*d\*x)/(d^2\*e^2\*x^4 + 2\*d^3\*e\*x^2 + d^4) + 3\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^2)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^3,x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```



### 3.111 $\int x^5 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=403

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^3} + \frac{8bcd^{7/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{c}\right)}{105e^3}$$

[Out]  $\frac{1}{3}d^2(e^2x^2+d)^{3/2}(a+b\operatorname{arcsec}(cx))/e^3 - \frac{2}{5}d(e^2x^2+d)^{5/2}(a+b\operatorname{arcsec}(cx))/e^3 + \frac{1}{7}(e^2x^2+d)^{7/2}(a+b\operatorname{arcsec}(cx))/e^3 + \frac{8bcd^{7/2}x \arctan\left(\frac{\sqrt{d+ex^2}}{c}\right)}{105e^3}$

**Rubi [A]** time = 1.27, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5238, 12, 1615, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^3} + \frac{bx\sqrt{c^2x^2 - 1}}{105e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5 \sqrt{d + ex^2} (a + b \operatorname{ArcSec}[cx]), x]$

[Out]  $(b(23c^4d^2 + 12c^2de - 75e^2)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}) / (1680c^5e^2\sqrt{c^2x^2}) + (b(29c^2d - 25e)x\sqrt{-1 + c^2x^2}(d + ex^2)^{3/2}) / (840c^3e^2\sqrt{c^2x^2}) - (b x \sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}) / (42c^2e^2\sqrt{c^2x^2}) + (d^2(d + ex^2)^{3/2}(a + b \operatorname{ArcSec}[cx])) / (3e^3) - (2d(d + ex^2)^{5/2}(a + b \operatorname{ArcSec}[cx])) / (5e^3) + ((d + ex^2)^{7/2}(a + b \operatorname{ArcSec}[cx])) / (7e^3) + (8b^2cd^{7/2}x \operatorname{ArcTan}[\sqrt{d + ex^2} / (\sqrt{d} \sqrt{-1 + c^2x^2})]) / (105e^3\sqrt{c^2x^2}) - (b(105c^6d^3 - 35c^4d^2e + 63c^2de^2 + 75e^3)x \operatorname{ArcTanh}[(\sqrt{e} \sqrt{-1 + c^2x^2}) / (c \sqrt{d + ex^2})]) / (1680c^6e^{5/2}\sqrt{c^2x^2})$

**Rule 12**

$\text{Int}[(a_*) (u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*) (v_)] /; \text{FreeQ}[b, x]$

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x_
)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*(e + f*x)^p/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1615

Int[(Px\_)\*((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k\*(a + b\*x)^(m + q - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*b^(q - 1)\*(m + n + p + q + 1)), x] + Dist[1/(d\*f\*b^q\*(m + n + p + q + 1)), Int[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p\*ExpandToSum[d\*f\*b^q\*(m + n + p + q + 1)\*Px - d\*f\*k\*(m + n + p + q + 1)\*(a + b\*x)^q + k\*(a + b\*x)^(q - 2)\*(a^2\*d\*f\*(m + n + p + q + 1) - b\*(b\*c\*e\*(m + q - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*(m + q) + n + p) - b\*(d\*e\*(m + q + n) + c\*f\*(m + q + p)))\*x), x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int x^5 \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx &= \frac{d^2 (d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2} (a+b \sec^{-1}(cx))}{7e^3} \\
&= \frac{d^2 (d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2} (a+b \sec^{-1}(cx))}{7e^3} \\
&= \frac{d^2 (d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2} (a+b \sec^{-1}(cx))}{7e^3} \\
&= -\frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^3} \\
&= \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{(d+ex^2)^{7/2} (a+b \sec^{-1}(cx))}{7e^3} \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}}{840c^3e^2} \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}}{840c^3e^2} \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}}{840c^3e^2} \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}}{840c^3e^2} \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}}{840c^3e^2}
\end{aligned}$$

**Mathematica [C]** time = 0.67, size = 366, normalized size = 0.91

$$\frac{\sqrt{d+ex^2} \left( 16ac^5 (8d^3 - 4d^2ex^2 + 3de^2x^4 + 15e^3x^6) + 16bc^5 \sec^{-1}(cx) (8d^3 - 4d^2ex^2 + 3de^2x^4 + 15e^3x^6) - bex\sqrt{d+ex^2} \right)}{1680c^5e^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]),x]

[Out] 
$$-1/3360*(b*(-128*c^4*d^4*\text{Sqrt}[1 + d/(e*x^2)]*(-1 + c^2*x^2)*\text{AppellF1}[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))]) + e*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^4*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]*\text{AppellF1}[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])) / (c^5*e^3*x*(-1 + c^2*x^2)*\text{Sqrt}[d + e*x^2]) + (\text{Sqrt}[d + e*x^2]*(16*a*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6) - b*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*\text{ArcSec}[c*x])) / (1680*c^5*e^3)$$

**fricas [A]** time = 7.03, size = 1701, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] 
$$[1/6720*(128*b*c^7*\text{sqrt}(-d)*d^3*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*\text{sqrt}(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*\text{sqrt}(e*x^2 + d)*\text{sqrt}(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*\text{sqrt}(e)*\log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*\text{sqrt}(c^2*x^2 - 1)*\text{sqrt}(e*x^2 + d)*\text{sqrt}(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*\text{arcsec}(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*\text{sqrt}(c^2*x^2 - 1))*\text{sqrt}(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^7*d^(7/2)*\text{arctan}(-1/2*\text{sqrt}(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*\text{sqrt}(e*x^2 + d)*\text{sqrt}(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*\text{sqrt}(e)*\log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*\text{sqrt}(c^2*x^2 - 1)*\text{sqrt}(e*x^2 + d)*\text{sqrt}(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*\text{arcsec}(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 +$$

```

25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*(
64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d
*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(
-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b
*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240
*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 +
16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)
*arcsec(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c
*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d))/(c^7*e^3), 1/3360*(128*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*(
c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*
e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^
3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a*
c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*
(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*ar
csec(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^
3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d))/(c^7*e^3)]

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^5, x)
```

**maple** [F] time = 4.86, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/105\*(15\*(e\*x^2 + d)^(3/2)\*x^4/e - 12\*(e\*x^2 + d)^(3/2)\*d\*x^2/e^2 + 8\*(e\*x^2 + d)^(3/2)\*d^2/e^3)\*a - 1/105\*(105\*e^3\*integrate((105\*(c^2\*e^3\*x^7 - e^3\*x^5 + (c^2\*e^3\*x^7 - e^3\*x^5)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)\*log(x) + (105\*c^2\*e^3\*x^7\*log(c) - 105\*e^3\*x^5\*log(c) + (15\*(7\*c^2\*log(c) + c^2)\*e^3\*x^7 - 4\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x + 3\*(c^2\*d\*e^2 - 35\*e^3\*log(c))\*x^5)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d))/(c^2\*e^3\*x^2 - e^3 + (c^2\*e^3\*x^2 - e^3)\*e^(log(c\*x + 1) + log(c\*x - 1))), x) - (15\*e^3\*x^6 + 3\*d\*e^2\*x^4 - 4\*d^2\*e\*x^2 + 8\*d^3)\*sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/e^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \sqrt{ex^2 + d} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^5\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*asec(c\*x))\*(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

### 3.112 $\int x^3 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=294

$$-\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}$$

[Out]  $-1/3*d*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/e^2+1/5*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/e^2-2/15*b*c*d^{(5/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})/e^2/(c^2*x^2)^{(1/2)}+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^4/e^{(3/2)}/(c^2*x^2)^{(1/2)}-1/20*b*x*(e*x^2+d)^{(3/2)}*(c^2*x^2-1)^{(1/2)}/c/e/(c^2*x^2)^{(1/2)}-1/120*b*(c^2*d+9*e)*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^3/e/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.40, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5238, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}} + \frac{bx(15c^4d^2-10c^2de-9e^2)\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{120ce\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-(b*(c^2*d + 9*e)*x*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(120*c^3*e*\text{Sqrt}[c^2*x^2]) - (b*x*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c*e*\text{Sqrt}[c^2*x^2]) - (d*(d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(3*e^2) + ((d + e*x^2)^{(5/2)}*(a + b*\text{ArcSec}[c*x]))/(5*e^2) - (2*b*c*d^{(5/2)}*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])])/(15*e^2*\text{Sqrt}[c^2*x^2]) + (b*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(c*\text{Sqrt}[d + e*x^2])])/(120*c^4*e^{(3/2)}*\text{Sqrt}[c^2*x^2])$

#### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 43

$\text{Int}[(a_*) + (b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_))^{(n_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{Le}[\dots]))$



$Q[7*m + 4*n + 4, 0] \parallel LtQ[9*m + 5*(n + 1), 0] \parallel GtQ[m + n + 2, 0]$

### Rule 63

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] := With[$   
 $\{p = Denominator[m]\}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +$   
 $(d*x^p)/b]^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[\{a, b, c, d\}, x] \&\& NeQ$   
 $[b*c - a*d, 0] \&\& LtQ[-1, m, 0] \&\& LeQ[-1, n, 0] \&\& LeQ[Denominator[n], Den$   
 $ominator[m]] \&\& IntLinearQ[a, b, c, d, m, n, x]$

### Rule 93

$Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_$   
 $_)), x\_Symbol] := With[\{q = Denominator[m]\}, Dist[q, Subst[Int[x^(q*(m + 1)$   
 $- 1)/(b*e - a*f - (d*e - c*f)*x^q], x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)$   
 $], x] /; FreeQ[\{a, b, c, d, e, f\}, x] \&\& EqQ[m + n + 1, 0] \&\& RationalQ[n]$   
 $\&\& LtQ[-1, m, 0] \&\& SimplerQ[a + b*x, c + d*x]$

### Rule 154

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)$   
 $)^(p_)*((g_.) + (h_.)*(x_)), x\_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n$   
 $+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p$   
 $+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +$   
 $p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))] + (b*d*f*g*(m + n + p$   
 $+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /$   
 $; FreeQ[\{a, b, c, d, e, f, g, h, n, p\}, x] \&\& GtQ[m, 0] \&\& NeQ[m + n + p +$   
 $2, 0] \&\& IntegersQ[2*m, 2*n, 2*p]$

### Rule 157

$Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_$   
 $_)))/((a_.) + (b_.)*(x_)), x\_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^$   
 $p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]$   
 $, x] /; FreeQ[\{a, b, c, d, e, f, g, h, n, p\}, x]$

### Rule 204

$Int[((a_.) + (b_.)*(x_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[$   
 $-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b] \&\& (LtQ[$   
 $a, 0] \parallel LtQ[b, 0])$

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*(e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

### Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{d+ex^2} (a+b \sec^{-1}(cx)) dx &= -\frac{d(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^2} - \frac{(bcx)}{e} \\
&= -\frac{d(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^2} - \frac{(bcx)}{e} \\
&= -\frac{d(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^2} - \frac{(bcx)}{e} \\
&= -\frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5e^2} \\
&= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(a+bx \sec^{-1}(cx))}{e} \\
&= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(a+bx \sec^{-1}(cx))}{e} \\
&= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(a+bx \sec^{-1}(cx))}{e} \\
&= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(a+bx \sec^{-1}(cx))}{e} \\
&= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(a+bx \sec^{-1}(cx))}{e}
\end{aligned}$$

**Mathematica [A]** time = 0.77, size = 327, normalized size = 1.11

$$\frac{\sqrt{d+ex^2} \left( 8ac^3 (-2d^2 + dex^2 + 3e^2x^4) + 8bc^3 \sec^{-1}(cx) (-2d^2 + dex^2 + 3e^2x^4) - bex\sqrt{1-\frac{1}{c^2x^2}} (c^2(7d+6ex^2)) \right)}{120c^3e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]),x]

[Out]  $(\sqrt{d + e x^2} * (8 a^3 c^3 (-2 d^2 + d e x^2 + 3 e^2 x^4) - b e \sqrt{1 - 1/(c^2 x^2)} * x * (9 e + c^2 (7 d + 6 e x^2)) + 8 b c^3 (-2 d^2 + d e x^2 + 3 e^2 x^4) * \text{ArcSec}[c x])) / (120 c^3 e^2) + (b \sqrt{1 - 1/(c^2 x^2)} * x * (\sqrt{c^2} * \text{Sqrt}[e] * \text{Sqrt}[c^2 d + e] * (15 c^4 d^2 - 10 c^2 d e - 9 e^2) * \text{Sqrt}[(c^2 (d + e x^2)) / (c^2 d + e)] * \text{ArcSinh}[(c \text{Sqrt}[e] * \text{Sqrt}[-1 + c^2 x^2]) / (\text{Sqrt}[c^2] * \text{Sqrt}[c^2 d + e])]) + 16 c^7 d^{5/2} * \text{Sqrt}[d + e x^2] * \text{ArcTan}[(\text{Sqrt}[d] * \text{Sqrt}[-1 + c^2 x^2]) / \text{Sqrt}[d + e x^2]])) / (120 c^6 e^2 * \text{Sqrt}[-1 + c^2 x^2] * \text{Sqrt}[d + e x^2])$

**fricas [A]** time = 4.13, size = 1383, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out]  $[1/480 * (16 b^5 c^5 \sqrt{-d} d^2 \log(((c^4 d^2 - 6 c^2 d e + e^2) x^4 - 8 (c^2 d^2 - d e) x^2 + 4 \sqrt{c^2 x^2 - 1}) * ((c^2 d - e) x^2 - 2 d) * \sqrt{e x^2 + d}) * \sqrt{-d} + 8 d^2) / x^4) - (15 b^5 c^4 d^2 - 10 b^5 c^2 d e - 9 b^5 e^2) * \sqrt{e} * \log(8 c^4 e^2 x^4 + c^4 d^2 - 6 c^2 d e + 8 (c^4 d e - c^2 e^2) x^2 - 4 (2 c^3 e x^2 + c^3 d - c e) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d} * \sqrt{e} + e^2) + 4 * (24 a^5 c^5 e^2 x^4 + 8 a^5 c^5 d e x^2 - 16 a^5 c^5 d^2 + 8 (3 b^5 c^5 e^2 x^4 + b^5 c^5 d e x^2 - 2 b^5 c^5 d^2) * \text{arcsec}(c x) - (6 b^5 c^3 e^2 x^2 + 7 b^5 c^3 d e + 9 b^5 c e^2) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d}) / (c^5 e^2), -1/480 * (32 b^5 c^5 d^{5/2} * \arctan(-1/2 * \sqrt{c^2 x^2 - 1}) * ((c^2 d - e) x^2 - 2 d) * \sqrt{e x^2 + d}) * \sqrt{d} / (c^2 d e x^4 + (c^2 d^2 - d e) x^2 - d^2) + (15 b^5 c^4 d^2 - 10 b^5 c^2 d e - 9 b^5 e^2) * \sqrt{e} * \log(8 c^4 e^2 x^4 + c^4 d^2 - 6 c^2 d e + 8 (c^4 d e - c^2 e^2) x^2 - 4 (2 c^3 e x^2 + c^3 d - c e) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d} * \sqrt{e} + e^2) - 4 * (24 a^5 c^5 e^2 x^4 + 8 a^5 c^5 d e x^2 - 16 a^5 c^5 d^2 + 8 (3 b^5 c^5 e^2 x^4 + b^5 c^5 d e x^2 - 2 b^5 c^5 d^2) * \text{arcsec}(c x) - (6 b^5 c^3 e^2 x^2 + 7 b^5 c^3 d e + 9 b^5 c e^2) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d}) / (c^5 e^2), 1/240 * (8 b^5 c^5 \sqrt{-d} d^2 \log(((c^4 d^2 - 6 c^2 d e + e^2) x^4 - 8 (c^2 d^2 - d e) x^2 + 4 \sqrt{c^2 x^2 - 1}) * ((c^2 d - e) x^2 - 2 d) * \sqrt{e x^2 + d}) * \sqrt{-d} + 8 d^2) / x^4) - (15 b^5 c^4 d^2 - 10 b^5 c^2 d e - 9 b^5 e^2) * \sqrt{-e} * \arctan(1/2 * (2 c^2 e x^2 + c^2 d - e) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d} * \sqrt{-e} / (c^3 e^2 x^4 - c d e + (c^3 d e - c e^2) x^2) + 2 * (24 a^5 c^5 e^2 x^4 + 8 a^5 c^5 d e x^2 - 16 a^5 c^5 d^2 + 8 (3 b^5 c^5 e^2 x^4 + b^5 c^5 d e x^2 - 2 b^5 c^5 d^2) * \text{arcsec}(c x) - (6 b^5 c^3 e^2 x^2 + 7 b^5 c^3 d e + 9 b^5 c e^2) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d}) / (c^5 e^2), -1/240 * (16 b^5 c^5 d^{5/2} * \arctan(-1/2 * \sqrt{c^2 x^2 - 1}) * ((c^2 d - e) x^2 - 2 d) * \sqrt{e x^2 + d}) * \sqrt{d} / (c^2 d e x^4 + (c^2 d^2 - d e) x^2 - d^2) + (15 b^5 c^4 d^2 - 10 b^5 c^2 d e - 9 b^5 e^2) * \sqrt{-e} * \arctan(1/2 * (2 c^2 e x^2 + c^2 d - e) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d} * \sqrt{-e} / (c^3 e^2 x^4 - c d e + (c^3 d e - c e^2) x^2)) - 2 * (24 a^5 c^5 e^2 x^4 + 8 a^5 c^5 d e x^2 - 16 a^5 c^5 d^2 + 8 (3 b^5 c^5 e^2 x^4 + b^5 c^5 d e x^2 - 2 b^5 c^5 d^2) * \text{arcsec}(c x) - (6 b^5 c^3 e^2 x^2 + 7 b^5 c^3 d e + 9 b^5 c e^2) * \sqrt{c^2 x^2 - 1}) * \sqrt{e x^2 + d}) / (c^5 e^2)$

$e^2x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*\operatorname{arcsec}(c*x) - (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*\sqrt{c^2*x^2 - 1})*\sqrt{e*x^2 + d})/(c^5*e^2)]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^3, x)`

**maple** [F] time = 4.49, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)`

[Out] `int(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `1/15*(3*(e*x^2 + d)^(3/2)*x^2/e - 2*(e*x^2 + d)^(3/2)*d/e^2)*a - 1/15*(15*e^2*integrate((15*(c^2*e^2*x^5 - e^2*x^3 + (c^2*e^2*x^5 - e^2*x^3)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (15*c^2*e^2*x^5*log(c) - 15*e^2*x^3*log(c) + (3*(5*c^2*log(c) + c^2)*e^2*x^5 - 2*c^2*d^2*x + (c^2*d*e - 15*e^2*log(c))*x^3)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/(c^2*e^2*x^2 - e^2 + (c^2*e^2*x^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1))), x) - (3*e^2*x^4 + d*e*x^2 - 2*d^2)*sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/e^2`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)`

[Out] `int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{asec}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*asec(c*x))*(e*x**2+d)**(1/2), x)`

[Out] `Integral(x**3*(a + b*asec(c*x))*sqrt(d + e*x**2), x)`

### 3.113 $\int x\sqrt{d+ex^2} (a+b\sec^{-1}(cx)) dx$

**Optimal.** Leaf size=195

$$\frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} - \frac{bx(3c^2d+e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

[Out]  $1/3*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/e+1/3*b*c*d^{(3/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)))/e/(c^2*x^2)^{(1/2)}-1/6*b*(3*c^2*d+e)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2))}/c^2/e^{(1/2)}/(c^2*x^2)^{(1/2)}-1/6*b*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5236, 446, 102, 157, 63, 217, 206, 93, 204}

$$\frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} - \frac{bx(3c^2d+e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{Sqrt}[d+e*x^2]*(a+b*\text{ArcSec}[c*x]),x]$

[Out]  $-(b*x*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])/(6*c*\text{Sqrt}[c^2*x^2]) + ((d+e*x^2)^{(3/2)}*(a+b*\text{ArcSec}[c*x]))/(3*e) + (b*c*d^{(3/2)}*x*\text{ArcTan}[\text{Sqrt}[d+e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1+c^2*x^2])])/(3*e*\text{Sqrt}[c^2*x^2]) - (b*(3*c^2*d+e)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1+c^2*x^2])/(c*\text{Sqrt}[d+e*x^2])])/(6*c^2*\text{Sqrt}[e]*\text{Sqrt}[c^2*x^2])$

#### Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 93

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}, x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n]$

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 102

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 5236



```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))*(x_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSec[c*x]))/(2*e*(p + 1)), x
] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int x\sqrt{d+ex^2} (a+b\sec^{-1}(cx)) dx &= \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} - \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{x\sqrt{-1+c^2x^2}} dx}{3e\sqrt{c^2x^2}} \\
&= \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} - \frac{(bcx) \text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} - \frac{(bcx) \text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} - \frac{(bcd^2x) \text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} - \frac{(bcd^2x) \text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\sec^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.49, size = 266, normalized size = 1.36

$$\frac{\sqrt{d+ex^2} \left( 2ac(d+ex^2) - bex\sqrt{1-\frac{1}{c^2x^2}} + 2bc\sec^{-1}(cx)(d+ex^2) \right) - bxc\sqrt{1-\frac{1}{c^2x^2}} \left( \sqrt{c^2}\sqrt{e}\sqrt{c^2d+e}(3c^2d+e) \right)}{6ce}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]),x]

[Out] (Sqrt[d + e\*x^2]\*(-(b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x) + 2\*a\*c\*(d + e\*x^2) + 2\*b\*c\*(d + e\*x^2)\*ArcSec[c\*x]))/(6\*c\*e) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(3\*c^2\*d + e)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])]) + 2\*c^5\*d^(3/2)\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(6\*c^4\*e\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [A] time = 2.29, size = 1100, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/24\*(2\*b\*c^3\*sqrt(-d)\*d\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (3\*b\*c^2\*d + b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d - sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e), 1/24\*(4\*b\*c^3\*d^(3/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (3\*b\*c^2\*d + b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d - sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e), 1/12\*(b\*c^3\*sqrt(-d)\*d\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (3\*b\*c^2\*d + b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d - sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e), 1/12\*(2\*b\*c^3\*d^(3/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (3\*b\*c^2\*d + b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d - sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a) x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*x, x)

**maple** [F] time = 3.34, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int(x\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex^2 + d)^{\frac{3}{2}} a}{3e} + \frac{\frac{1}{2} \left( 2(ex^2 + d)^{\frac{3}{2}} \arctan(\sqrt{cx+1} \sqrt{cx-1}) - \frac{3c^4 d \left( \frac{2\sqrt{ex^2+de}}{c^4} + \frac{(c^2 de + e^2) \log\left(\frac{\sqrt{ex^2+d}c^2 - \sqrt{c^2 d + ec}}{\sqrt{ex^2+d}c^2 + \sqrt{c^2 d + ec}}\right)}{\sqrt{c^2 d + ec}^5} \right)}{e^2} \right)}{3c^4} + \frac{2(ex^2 + d)^{\frac{3}{2}}}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/3\*(e\*x^2 + d)^(3/2)\*a/e + 1/3\*((e\*x^2 + d)^(3/2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - 3\*e\*integrate((3\*(c^2\*e\*x^3 - e\*x + (c^2\*e\*x^3 - e\*x))\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)\*log(x) + (3\*c^2\*e\*x^3\*log(c) - 3\*e\*x\*log(c) + ((3\*c^2\*log(c) + c^2)\*e\*x^3 + (c^2\*d - 3\*e\*log(c))\*x))\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d))/(c^2\*e\*x^2 + (c^2\*e\*x^2 - e)\*e^(log(c\*x + 1) + log(c\*x - 1)) - e), x))\*b/e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{asec}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*asec(c*x))*(e*x**2+d)**(1/2), x)`

[Out] `Integral(x*(a + b*asec(c*x))*sqrt(d + e*x**2), x)`

$$3.114 \quad \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x,x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x,x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x} dx$$

Mathematica [A] time = 4.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x,x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x, x]

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2+d} (b \text{arcsec}(cx) + a)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x, x)

**maple** [A] time = 2.21, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x,x)

[Out] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d} \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right) - \sqrt{ex^2 + d}\right)a + b \int \frac{\sqrt{ex^2 + d} \arctan\left(\frac{\sqrt{cx + 1} \sqrt{cx - 1}}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="maxima")

[Out] -(sqrt(d)\*arcsinh(d/(sqrt(d\*e)\*abs(x)))) - sqrt(e\*x^2 + d))\*a + b\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ex^2 + d} \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x,x)`

[Out] `int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x, x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) \sqrt{d + ex^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x,x)`

[Out] `Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x, x)`

$$3.115 \quad \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^3, x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^3, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3} dx = \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Mathematica [A] time = 4.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^3, x]

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2+d} (b \text{arcsec}(cx) + a)}{x^3}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^3, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^3, x)

**maple** [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x)

[Out] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( \frac{e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{\sqrt{d}} - \frac{\sqrt{ex^2 + d}e}{d} + \frac{(ex^2 + d)^{\frac{3}{2}}}{dx^2} \right) a + b \int \frac{\sqrt{ex^2 + d} \arctan\left(\frac{\sqrt{cx + 1} \sqrt{cx - 1}}{x^3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] -1/2\*(e\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/sqrt(d) - sqrt(e\*x^2 + d)\*e/d + (e\*x^2 + d)^(3/2)/(d\*x^2))\*a + b\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^3, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) \sqrt{d + ex^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**3, x)
```

$$3.116 \quad \int x^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Optimal. Leaf size=26

$$\text{Int}\left(x^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)), x\right)$$

[Out] Unintegrable(x^2\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2), x)

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int][x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int x^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx = \int x^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

**Mathematica** [A] time = 9.23, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

**fricas** [A] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2 \text{arcsec}(cx) + ax^2\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral((b\*x^2\*arcsec(c\*x) + a\*x^2)\*sqrt(e\*x^2 + d), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*x^2, x)

**maple** [A] time = 3.90, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int(x^2\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} \left( \frac{2(ex^2 + d)^{\frac{3}{2}}x}{e} - \frac{\sqrt{ex^2 + d} dx}{e} - \frac{d^2 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}} \right) a + b \int \sqrt{ex^2 + d} x^2 \arctan\left(\sqrt{cx + 1} \sqrt{cx - 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/8\*(2\*(e\*x^2 + d)^(3/2)\*x/e - sqrt(e\*x^2 + d)\*d\*x/e - d^2\*arsinh(e\*x/sqrt(d\*e))/e^(3/2))\*a + b\*integrate(sqrt(e\*x^2 + d)\*x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^2\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asec}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asec(c*x))*(e*x**2+d)**(1/2), x)`

[Out] `Integral(x**2*(a + b*asec(c*x))*sqrt(d + e*x**2), x)`

$$3.117 \quad \int \sqrt{d + ex^2} \left( a + b \sec^{-1}(cx) \right) dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\sqrt{d + ex^2} \left( a + b \sec^{-1}(cx) \right), x\right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \sqrt{d + ex^2} \left( a + b \sec^{-1}(cx) \right) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int][Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int \sqrt{d + ex^2} \left( a + b \sec^{-1}(cx) \right) dx = \int \sqrt{d + ex^2} \left( a + b \sec^{-1}(cx) \right) dx$$

**Mathematica [A]** time = 4.37, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex^2} \left( a + b \sec^{-1}(cx) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

**fricas [A]** time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex^2 + d}(b \text{arcsec}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a), x)

**maple** [A] time = 2.31, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \sqrt{ex^2 + d} x + \frac{d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}} \right) a + b \int \sqrt{ex^2 + d} \arctan\left(\sqrt{cx + 1} \sqrt{cx - 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/2\*(sqrt(e\*x^2 + d)\*x + d\*arcsinh(e\*x/sqrt(d\*e))/sqrt(e))\*a + b\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int((d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asec}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2), x)
```



$$3.118 \quad \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^2, x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^2, x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^2, x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^2, x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2+d} (b \text{arcsec}(cx) + a)}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^2, x)

**maple** [A] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x)

[Out] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\left( \sqrt{e} \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right) - \frac{\sqrt{ex^2 + d}}{x} \right) a + b \int \frac{\sqrt{ex^2 + d} \arctan\left(\frac{\sqrt{cx + 1} \sqrt{cx - 1}}{x}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] (sqrt(e)\*arcsinh(e\*x/sqrt(d\*e)) - sqrt(e\*x^2 + d)/x)\*a + b\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2, x)`

[Out] `int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2, x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) \sqrt{d + ex^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**2, x)`

[Out] `Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**2, x)`

$$3.119 \quad \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=328

$$-\frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} + \frac{2bc\sqrt{c^2x^2-1} (c^2d+2e) \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} + \frac{bx\sqrt{1-c^2x^2} (c^2d+2e)}{9a}$$

[Out]  $-1/3*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/d/x^3+2/9*b*c*(c^2*d+2*e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}+1/9*b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}-2/9*b*c^2*(c^2*d+2*e)*x*\text{EllipticE}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*\text{EllipticF}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.42, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {264, 5238, 12, 474, 583, 524, 427, 426, 424, 421, 419}

$$-\frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} + \frac{2bc\sqrt{c^2x^2-1} (c^2d+2e) \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} + \frac{bx\sqrt{1-c^2x^2} (c^2d+2e)}{9a}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^4, x]

[Out]  $(2*b*c*(c^2*d+2*e)*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])/(9*d*\text{Sqrt}[c^2*x^2]) + (b*c*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])/(9*x^2*\text{Sqrt}[c^2*x^2]) - ((d+e*x^2)^{(3/2)}*(a+b*\text{ArcSec}[c*x]))/(3*d*x^3) - (2*b*c^2*(c^2*d+2*e)*x*\text{Sqrt}[1-c^2*x^2]*\text{Sqrt}[d+e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(9*d*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[1+(e*x^2)/d]) + (b*(c^2*d+e)*(2*c^2*d+3*e)*x*\text{Sqrt}[1-c^2*x^2]*\text{Sqrt}[1+(e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(9*d*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 264**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n,

p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

#### Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

#### Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

#### Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(
q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
+ a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*
(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^(n*(
m + 1))), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^4} dx &= -\frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} - \frac{(bcx) \int -\frac{(d+ex^2)^{3/2}}{3dx^4 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} + \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{x^4 \sqrt{-1+c^2x^2}} dx}{3d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} - \frac{(bcx) \int \frac{-2d(c^2d+2e)}{x^2\sqrt{-1+c^2x^2}} dx}{9d\sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d+2e) \sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{2bc(c^2d+2e) \sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{2bc(c^2d+2e) \sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{2bc(c^2d+2e) \sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{2bc(c^2d+2e) \sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{2bc(c^2d+2e) \sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{3dx^3}
\end{aligned}$$

**Mathematica [C]** time = 0.65, size = 247, normalized size = 0.75

$$\frac{\sqrt{d+ex^2} \left( -3a(d+ex^2) + bcx\sqrt{1-\frac{1}{c^2x^2}}(2c^2dx^2+d+4ex^2) - 3b \sec^{-1}(cx)(d+ex^2) \right)}{9dx^3} - \frac{ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}}}{9d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^4, x]

[Out] (Sqrt[d + e\*x^2]\*(-3\*a\*(d + e\*x^2) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + 2\*c^2\*d\*x^2 + 4\*e\*x^2) - 3\*b\*(d + e\*x^2)\*ArcSec[c\*x]))/(9\*d\*x^3) - ((1/9)\*b\*c\*Sq

```
rt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*EllipticE[
I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2)*El
lipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)))]/(Sqrt[-c^2]*d*Sqrt[1 - c^2
*x^2]*Sqrt[d + e*x^2])
```

**fricas** [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^4, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^4, x)
```

**maple** [F] time = 3.30, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex^2 + d)^{\frac{3}{2}} a}{3 dx^3} - \frac{\left( (ex^2 + d)^{\frac{3}{2}} \arctan(\sqrt{cx + 1} \sqrt{cx - 1}) - 3(ex^2 \log(c) + d \log(c) + (ex^2 + d) \log(x)) \sqrt{ex^2 + d} \right) b}{3 dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```



```
[Out] -1/3*(e*x^2 + d)^(3/2)*a/(d*x^3) - 1/3*(3*d*x^3*integrate((3*(c^2*d*x^2 + (
c^2*d*x^2 - d)*e^(log(c*x + 1) + log(c*x - 1)) - d)*sqrt(e*x^2 + d)*log(x)
+ (3*c^2*d*x^2*log(c) - (c^2*e*x^4 - (3*c^2*log(c) - c^2)*d*x^2 + 3*d*log(c
))*e^(log(c*x + 1) + log(c*x - 1)) - 3*d*log(c))*sqrt(e*x^2 + d))/(c^2*d*x^
6 - d*x^4 + (c^2*d*x^6 - d*x^4)*e^(log(c*x + 1) + log(c*x - 1))), x) + (e*x
^2 + d)^(3/2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(d*x^3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) \sqrt{d + ex^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**4, x)
```

```
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**4, x)
```

$$3.120 \quad \int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=453

$$\frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{bc\sqrt{c^2x^2-1} (12c^2d-e) \sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}}{25dx^4}$$

[Out]  $-1/5*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/d/x^5+2/15*e*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/d^2/x^3-2/15*b*c*e^2*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}+1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}+1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}+1/25*b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/x^4/(c^2*x^2)^{(1/2)}+1/45*b*c*e*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(c^2*x^2)^{(1/2)}+1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(c^2*x^2)^{(1/2)}+2/15*b*c^2*e^2*x*\text{EllipticE}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}-1/45*b*c^2*e*(2*c^2*d+e)*x*\text{EllipticE}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}-1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*\text{EllipticE}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*\text{EllipticF}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}+2/45*b*c^2*e*(c^2*d+e)*x*\text{EllipticF}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}-2/15*b*e^2*(c^2*d+e)*x*\text{EllipticF}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.60, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {271, 264, 5238, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{bc\sqrt{c^2x^2-1} (24c^4d^2+19c^2de-31e^2) \sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^6, x]

[Out]  $(b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(225*d^2*\text{Sqrt}[c^2*x^2]) + (b*c*(12*c^2*d - e)*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d +$

$$\frac{e*x^2)}{(225*d*x^2*\text{Sqrt}[c^2*x^2]) + (b*c*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(3/2)})/(25*d*x^4*\text{Sqrt}[c^2*x^2]) - ((d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(5*d*x^5) + (2*e*(d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(15*d^2*x^3) - (b*c^2*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(225*d^2*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]) + (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(225*d^2*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])$$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

### Rule 421

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/(Sqrt[a + b\*x^2]\*Sqrt[1 + (d\*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1
))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x
```

```

_)^2)^(p_.), x_Symbol] :=> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= -\frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= \frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3} \\
&= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)^3}{15d^2x^3}
\end{aligned}$$

**Mathematica** [C] time = 0.74, size = 325, normalized size = 0.72

$$\frac{\sqrt{d+ex^2} \left( -15a(3d^2+dex^2-2e^2x^4) + bcx\sqrt{1-\frac{1}{c^2x^2}} \left( dex^2(19c^2x^2+8) + 3d^2(8c^4x^4+4c^2x^2+3) - 31e^2x^4 \right) - 1 \right)}{225d^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/x^6,x]

[Out] (Sqrt[d + e\*x^2]\*(-15\*a\*(3\*d^2 + d\*e\*x^2 - 2\*e^2\*x^4) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(-31\*e^2\*x^4 + d\*e\*x^2\*(8 + 19\*c^2\*x^2) + 3\*d^2\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(3\*d^2 + d\*e\*x^2 - 2\*e^2\*x^4)\*ArcSec[c\*x]))/(225\*d^2\*x^5) - ((I/225)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(24\*c^4\*d^2 + 19\*c^2\*d\*e - 31\*e^2)\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] + (-24\*c^6\*d^3 - 31\*c^4\*d^2\*e + 23\*c^2\*d\*e^2 + 30\*e^3)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))]))/(Sqrt[-c^2]\*d^2\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{x^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^6, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/x^6, x)

**maple** [F] time = 4.68, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsec}(cx)) \sqrt{ex^2 + d}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x)

[Out] int((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15} a \left( \frac{2(ex^2 + d)^{\frac{3}{2}} e}{d^2 x^3} - \frac{3(ex^2 + d)^{\frac{3}{2}}}{dx^5} \right) - \frac{\left( (2e^2 x^4 - dex^2 - 3d^2) \sqrt{ex^2 + d} \arctan(\sqrt{cx + 1} \sqrt{cx - 1}) - 15(2e^2 x^4 + 1) \right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="maxima")

[Out] 1/15\*a\*(2\*(e\*x^2 + d)^(3/2)\*e/(d^2\*x^3) - 3\*(e\*x^2 + d)^(3/2)/(d\*x^5)) - 1/15\*(15\*d^2\*x^5\*integrate((15\*(c^2\*d^2\*x^2 - d^2 + (c^2\*d^2\*x^2 - d^2)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)\*log(x) + (15\*c^2\*d^2\*x^2\*log(c) - 15\*d^2\*log(c) + (2\*c^2\*e^2\*x^6 - c^2\*d\*e\*x^4 + 3\*(5\*c^2\*log(c) - c^2)\*d^2\*x^2 - 15\*d^2\*log(c))\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d))/(c^2\*d^2\*x^8 - d^2\*x^6 + (c^2\*d^2\*x^8 - d^2\*x^6)\*e^(log(c\*x + 1) + log(c\*x - 1))), x) - (2\*e^2\*x^4 - d\*e\*x^2 - 3\*d^2)\*sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/(d^2\*x^5)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))))/x^6,x)

[Out] int(((d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))))/x^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) \sqrt{d + ex^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*6,x)

[Out] Integral((a + b\*asec(c\*x))\*sqrt(d + e\*x\*\*2)/x\*\*6, x)

### 3.121 $\int x^3 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=374

$$-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2} - \frac{2bcd^{7/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{35e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{42ce\sqrt{c^2x^2}}$$

[Out]  $-1/5*d*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/e^2+1/7*(e*x^2+d)^{(7/2)}*(a+b*\text{arcsec}(c*x))/e^2-2/35*b*c*d^{(7/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})/e^2/(c^2*x^2)^{(1/2)}+1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*e^3)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^6/e^{(3/2)}/(c^2*x^2)^{(1/2)}-1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^{(3/2)}*(c^2*x^2-1)^{(1/2)}/c^3/e/(c^2*x^2)^{(1/2)}-1/42*b*x*(e*x^2+d)^{(5/2)}*(c^2*x^2-1)^{(1/2)}/c/e/(c^2*x^2)^{(1/2)}+1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^5/e/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.51, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5238, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2} + \frac{bx\sqrt{c^2x^2-1}(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $(b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/ (560*c^5*e*\text{Sqrt}[c^2*x^2]) - (b*(13*c^2*d + 25*e)*x*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(3/2)})/(840*c^3*e*\text{Sqrt}[c^2*x^2]) - (b*x*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(5/2)})/(42*c*e*\text{Sqrt}[c^2*x^2]) - (d*(d + e*x^2)^{(5/2)}*(a + b*\text{ArcSec}[c*x]))/(5*e^2) + ((d + e*x^2)^{(7/2)}*(a + b*\text{ArcSec}[c*x]))/(7*e^2) - (2*b*c*d^{(7/2)})*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])]/(35*e^2*\text{Sqrt}[c^2*x^2]) + (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(c*\text{Sqrt}[d + e*x^2])])/(560*c^6*e^{(3/2)}*\text{Sqrt}[c^2*x^2])$

#### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$



Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 573

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_))^(r\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q\*(e + f\*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int x^3 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx &= -\frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= -\frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= -\frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= -\frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= -\frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= \frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= \frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= \frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= \frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{ \\
&= \frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \sec^{-1}(cx))}{7e^2} - \frac{(b}{
\end{aligned}$$

**Mathematica** [C] time = 0.63, size = 339, normalized size = 0.91

$$\frac{\sqrt{d+ex^2} \left( 48ac^5 (2d-5ex^2) (d+ex^2)^2 + 48bc^5 \sec^{-1}(cx) (2d-5ex^2) (d+ex^2)^2 + bex \sqrt{1 - \frac{1}{c^2x^2}} \left( c^4 (57d^2 + 106d^2e + 40e^2x^2) + 48b^2c^5 (2d-5ex^2) (d+ex^2)^2 \right) \right)}{1680c^5e^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]),x]

[Out] -1/1120\*(b\*(32\*c^4\*d^4\*Sqrt[1 + d/(e\*x^2)]\*(-1 + c^2\*x^2)\*AppellF1[1, 1/2, 1/2, 2, 1/(c^2\*x^2), -(d/(e\*x^2))]) + e\*(-35\*c^6\*d^3 + 35\*c^4\*d^2\*e + 63\*c^2\*d\*e^2 + 25\*e^3)\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*AppellF1[1, 1/2, 1/2, 2, c^2\*x^2, -((e\*x^2)/d)])/(c^5\*e^2\*x\*(-1 + c^2\*x^2)\*Sqrt[d + e\*x^2]) - (Sqrt[d + e\*x^2]\*(48\*a\*c^5\*(2\*d - 5\*e\*x^2)\*(d + e\*x^2)^2 + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(75\*e^2 + 2\*c^2\*e\*(82\*d + 25\*e\*x^2) + c^4\*(57\*d^2 + 106\*d\*e\*x^2 + 40\*e^2\*x^4)) + 48\*b\*c^5\*(2\*d - 5\*e\*x^2)\*(d + e\*x^2)^2\*ArcSec[c\*x]))/(1680\*c^5\*e^2)

**fricas** [A] time = 12.02, size = 1701, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] [1/6720\*(96\*b\*c^7\*sqrt(-d)\*d^3\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) - 3\*(35\*b\*c^6\*d^3 - 35\*b\*c^4\*d^2\*e - 63\*b\*c^2\*d\*e^2 - 25\*b\*e^3)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(240\*a\*c^7\*e^3\*x^6 + 384\*a\*c^7\*d\*e^2\*x^4 + 48\*a\*c^7\*d^2\*e\*x^2 - 96\*a\*c^7\*d^3 + 48\*(5\*b\*c^7\*e^3\*x^6 + 8\*b\*c^7\*d\*e^2\*x^4 + b\*c^7\*d^2\*e\*x^2 - 2\*b\*c^7\*d^3)\*arcsec(c\*x) - (40\*b\*c^5\*e^3\*x^4 + 57\*b\*c^5\*d^2\*e + 164\*b\*c^3\*d\*e^2 + 75\*b\*c\*e^3 + 2\*(53\*b\*c^5\*d\*e^2 + 25\*b\*c^3\*e^3)\*x^2)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d))/(c^7\*e^2), -1/6720\*(192\*b\*c^7\*d^(7/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d))/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2) + 3\*(35\*b\*c^6\*d^3 - 35\*b\*c^4\*d^2\*e - 63\*b\*c^2\*d\*e^2 - 25\*b\*e^3)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) - 4\*(240\*a\*c^7\*e^3\*x^6 + 384\*a\*c^7\*d\*e^2\*x^4 + 48\*a\*c^7\*d^2\*e\*x^2 - 96\*a\*c^7\*d^3 + 48\*(5\*b\*c^7\*e^3\*x^6 + 8\*b\*c^7\*d\*e^2\*x^4 + b\*c^7\*d^2\*e\*x^2 - 2\*b\*c^7\*d^3)\*arcsec(c\*x) - (40\*b\*c^5\*e^3\*x^4 + 57\*b\*c^5\*d^2\*e + 164\*b\*c^3\*d\*e^2 + 75\*b\*c\*e^3 + 2\*(53\*b\*c^5\*d\*e^2 + 25\*b\*c^3\*e^3)\*x^2)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d))/(c^7\*e^2), 1/3360\*(48

```

*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*
e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e
*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*
a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 4
8*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arc
sec(c*x) - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^
3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d))/(c^7*e^2), -1/3360*(96*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^
2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*
x^2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)
)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
+ d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(240*a*c^
7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5
*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(
c*x) - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 +
2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/
(c^7*e^2)]

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)\*x^3, x)

**maple** [F] time = 4.33, size = 0, normalized size = 0.00

$$\int x^3 (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

[Out] int(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/35\*(5\*(e\*x^2 + d)^(5/2)\*x^2/e - 2\*(e\*x^2 + d)^(5/2)\*d/e^2)\*a - 1/35\*(35\*e^2\*integrate((35\*(c^2\*e^3\*x^7 - d\*e^2\*x^3 + (c^2\*d\*e^2 - e^3)\*x^5 + (c^2\*e^3\*x^7 - d\*e^2\*x^3 + (c^2\*d\*e^2 - e^3)\*x^5))\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)\*log(x) + (35\*c^2\*e^3\*x^7\*log(c) - 35\*d\*e^2\*x^3\*log(c) + 35\*(c^2\*d\*e^2\*log(c) - e^3\*log(c))\*x^5 + (5\*(7\*c^2\*log(c) + c^2)\*e^3\*x^7 - 2\*c^2\*d^3\*x + ((35\*c^2\*log(c) + 8\*c^2)\*d\*e^2 - 35\*e^3\*log(c))\*x^5 + (c^2\*d^2\*e - 35\*d\*e^2\*log(c))\*x^3)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)/(c^2\*e^2\*x^2 - e^2 + (c^2\*e^2\*x^2 - e^2)\*e^(log(c\*x + 1) + log(c\*x - 1))), x) - (5\*e^3\*x^6 + 8\*d\*e^2\*x^4 + d^2\*e\*x^2 - 2\*d^3)\*sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/e^2

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (ex^2 + d)^{3/2} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int(x^3\*(d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*asec(c\*x)),x)

[Out] Timed out

### 3.122 $\int x (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=262

$$\frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} + \frac{bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} - \frac{bx(15c^4d^2 + 10c^2de + 3e^2)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

[Out]  $1/5*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/e+1/5*b*c*d^{(5/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})/e/(c^2*x^2)^{(1/2)}-1/40*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^4/e^{(1/2)}/(c^2*x^2)^{(1/2)}-1/20*b*x*(e*x^2+d)^{(3/2)}*(c^2*x^2-1)^{(1/2)}/c/(c^2*x^2)^{(1/2)}-1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^3/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.27, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5236, 446, 102, 154, 157, 63, 217, 206, 93, 204}

$$\frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} - \frac{bx(15c^4d^2 + 10c^2de + 3e^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}} + \frac{bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}} - \frac{bx(15c^4d^2 + 10c^2de + 3e^2)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-(b*(7*c^2*d + 3*e)*x*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(40*c^3*\text{Sqrt}[c^2*x^2]) - (b*x*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c*\text{Sqrt}[c^2*x^2]) + ((d + e*x^2)^{(5/2)}*(a + b*\text{ArcSec}[c*x]))/(5*e) + (b*c*d^{(5/2)}*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])])/(5*e*\text{Sqrt}[c^2*x^2]) - (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(c*\text{Sqrt}[d + e*x^2])])/(40*c^4*\text{Sqrt}[e]*\text{Sqrt}[c^2*x^2])$

#### Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 102

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)), x_Symbol] := Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 154

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 204

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 206

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```



$Q[a, 0] \parallel \text{LtQ}[b, 0]$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x \ \&\& \ !\text{GtQ}[a, 0]$

### Rule 446

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}*((c_) + (d_)*(x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p * (c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 5236

$\text{Int}[(a_) + \text{ArcSec}[(c_)*(x_)]*(b_)]*(x_)*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSec}[c*x])]/(2*e*(p + 1)), x] - \text{Dist}[(b*c*x)/(2*e*(p + 1)*\text{Sqrt}[c^2*x^2]), \text{Int}[(d + e*x^2)^{(p + 1)}/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \ \text{NeQ}[p, -1]$

### Rubi steps

$$\begin{aligned}
\int x (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx &= \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} - \frac{(bcx) \int \frac{(d+ex^2)^{5/2}}{x\sqrt{-1+c^2x^2}} dx}{5e\sqrt{c^2x^2}} \\
&= \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} - \frac{(bcx) \text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{10e\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} - \frac{(bcx) \text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{10e\sqrt{c^2x^2}} \\
&= -\frac{b(7c^2d + 3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} \\
&= -\frac{b(7c^2d + 3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} \\
&= -\frac{b(7c^2d + 3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} \\
&= -\frac{b(7c^2d + 3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e} \\
&= -\frac{b(7c^2d + 3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2} (d + ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e}
\end{aligned}$$

**Mathematica [A]** time = 0.77, size = 305, normalized size = 1.16

$$\frac{\sqrt{d+ex^2} \left( 8ac^3 (d+ex^2)^2 + 8bc^3 \sec^{-1}(cx) (d+ex^2)^2 - bex\sqrt{1-\frac{1}{c^2x^2}} (c^2(9d+2ex^2)+3e) \right) - bx\sqrt{1-\frac{1}{c^2x^2}} \left( 8c^7d + 3e \right)}{40c^3e}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 - b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(3*e
+ c^2*(9*d + 2*e*x^2)) + 8*b*c^3*(d + e*x^2)^2*ArcSec[c*x]))/(40*c^3*e) - (
b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*(15*c^4*d^2 +
10*c^2*d*e + 3*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*
Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])]) + 8*c^7*d^(5/2)*Sqrt[d + e
*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))/(40*c^6*e*Sqrt
[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**fricas** [A] time = 3.17, size = 1377, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*
d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d
)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*
log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*
c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) +
4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2
*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e +
3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), 1/160*(16*b*c^5*d^(
5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*
sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c
^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e
*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*
d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^
3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*
(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqr
t(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d
)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e^2*
x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 +
b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(
c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), 1/80*(8*b*c^5*d^(5/2)*arctan(-1/2*s
qrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x
^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*
sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
+ d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e
^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^
2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sq
rt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a) x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)\*x, x)

**maple** [F] time = 3.27, size = 0, normalized size = 0.00

$$\int x (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

[Out] int(x\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{5}(ex^2 + d)^{5/2} a/e + \frac{1}{5}((e^2x^4 + 2d*ex^2 + d^2)\sqrt{ex^2 + d}) \arctan(\sqrt{cx + 1}\sqrt{cx - 1}) - 5e \int ((5(c^2e^2x^5 + (c^2d*e - e^2)x^3 - d*ex + (c^2e^2x^5 + (c^2d*e - e^2)x^3 - d*ex))e^{(\log(cx + 1) + \log(cx - 1))})\sqrt{ex^2 + d}\log(x) + (5c^2e^2x^5\log(c) + 5(c^2d*e\log(c) - e^2\log(c))x^3 - 5d*ex\log(c) + ((5c^2\log(c) + c^2)e^2x^5 + ((5c^2\log(c) + 2c^2)d*e - 5e^2\log(c))x^3 + (c^2d^2 - 5d*e\log(c))x))e^{(\log(cx + 1) + \log(cx - 1))})\sqrt{ex^2 + d})/(c^2e*x^2 + (c^2e*x^2 - e)e^{(\log(cx + 1) + \log(cx - 1)) - e}), x) * b/e$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (ex^2 + d)^{3/2} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))),x)

```
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

```
[Out] Timed out
```

$$3.123 \quad \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x, x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x, x]

[Out] Defer[Int] [((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x} dx = \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x} dx$$

Mathematica [A] time = 4.97, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x, x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x, x]

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)) \sqrt{ex^2 + d}}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsec}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)/x, x)

**maple** [A] time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arcsec}(cx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left( 3d^{\frac{3}{2}} \operatorname{arsinh} \left( \frac{d}{\sqrt{de}|x|} \right) - (ex^2 + d)^{\frac{3}{2}} - 3\sqrt{ex^2 + d}d \right) a + \left( e \int \sqrt{ex^2 + d} x \arctan \left( \sqrt{cx + 1} \sqrt{cx - 1} \right) dx + d \int \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x,x, algorithm="maxima")

[Out] -1/3\*(3\*d^(3/2)\*arcsinh(d/(sqrt(d\*e)\*abs(x))) - (e\*x^2 + d)^(3/2) - 3\*sqrt(e\*x^2 + d)\*d)\*a + (e\*integrate(sqrt(e\*x^2 + d)\*x\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)), x) + d\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x, x))\*b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x, x)`

[Out] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x, x)`

[Out] `Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x, x)`



$$3.124 \quad \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^3, x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out] Defer[Int](((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^3, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3} dx = \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Mathematica [A] time = 5.54, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^3, x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^3, x]

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)) \sqrt{ex^2 + d}}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^3,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/x^3, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsec}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)/x^3, x)

**maple** [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arcsec}(cx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^3,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( 3 \sqrt{d} e \operatorname{arsinh} \left( \frac{d}{\sqrt{d} e |x|} \right) - 3 \sqrt{ex^2 + d} e - \frac{(ex^2 + d)^{\frac{3}{2}} e}{d} + \frac{(ex^2 + d)^{\frac{5}{2}}}{dx^2} \right) a + \left( e \int \frac{\sqrt{ex^2 + d} \arctan \left( \frac{\sqrt{cx + 1} \sqrt{cx - 1}}{x} \right)}{x} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^3,x, algorithm="maxima")

[Out] -1/2\*(3\*sqrt(d)\*e\*arcsinh(d/(sqrt(d\*e)\*abs(x)))) - 3\*sqrt(e\*x^2 + d)\*e - (e\*x^2 + d)^(3/2)\*e/d + (e\*x^2 + d)^(5/2)/(d\*x^2))\*a + (e\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x, x) + d\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^3, x))\*b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3, x)`

[Out] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**3, x)`

[Out] `Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**3, x)`

$$3.125 \quad \int x^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Optimal. Leaf size=26

$$\text{Int}\left(x^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)), x\right)$$

[Out] Unintegrable( $x^2*(e*x^2+d)^{(3/2)*(a+b*\text{arcsec}(c*x))}$ ), x]

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int [ $x^2*(d + e*x^2)^{(3/2)*(a + b*\text{ArcSec}[c*x])}$ ], x]

[Out] Defer[Int] [ $x^2*(d + e*x^2)^{(3/2)*(a + b*\text{ArcSec}[c*x])}$ ], x]

Rubi steps

$$\int x^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx = \int x^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Mathematica [A] time = 9.59, size = 0, normalized size = 0.00

$$\int x^2 (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate [ $x^2*(d + e*x^2)^{(3/2)*(a + b*\text{ArcSec}[c*x])}$ ], x]

[Out] Integrate [ $x^2*(d + e*x^2)^{(3/2)*(a + b*\text{ArcSec}[c*x])}$ ], x]

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^4 + adx^2 + (bex^4 + bdx^2) \text{arcsec}(cx)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e\*x^4 + a\*d\*x^2 + (b\*e\*x^4 + b\*d\*x^2)\*arcsec(c\*x))\*sqrt(e\*x^2 + d), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)\*x^2, x)

**maple** [A] time = 3.94, size = 0, normalized size = 0.00

$$\int x^2 (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

[Out] int(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48} \left( \frac{8(ex^2 + d)^{\frac{5}{2}}x}{e} - \frac{2(ex^2 + d)^{\frac{3}{2}}dx}{e} - \frac{3\sqrt{ex^2 + d}d^2x}{e} - \frac{3d^3 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}} \right) a + b \int (ex^4 + dx^2)\sqrt{ex^2 + d} \operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/48\*(8\*(e\*x^2 + d)^(5/2)\*x/e - 2\*(e\*x^2 + d)^(3/2)\*d\*x/e - 3\*sqrt(e\*x^2 + d)\*d^2\*x/e - 3\*d^3\*arcsinh(e\*x/sqrt(d\*e))/e^(3/2))\*a + b\*integrate((e\*x^4 + d\*x^2)\*sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 (ex^2 + d)^{3/2} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

```
[Out] Timed out
```

$$3.126 \quad \int (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Optimal. Leaf size=23

$$\text{Int}\left((d + ex^2)^{3/2} (a + b \sec^{-1}(cx)), x\right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)), x)

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int] [(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx = \int (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

**Mathematica** [A] time = 5.53, size = 0, normalized size = 0.00

$$\int (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

**fricas** [A] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a), x)

**maple** [A] time = 2.26, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} \left( 2 (ex^2 + d)^{\frac{3}{2}} x + 3 \sqrt{ex^2 + d} dx + \frac{3d^2 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}} \right) a + b \int (ex^2 + d)^{\frac{3}{2}} \arctan\left(\sqrt{cx + 1} \sqrt{cx - 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] 1/8\*(2\*(e\*x^2 + d)^(3/2)\*x + 3\*sqrt(e\*x^2 + d)\*d\*x + 3\*d^2\*arcsinh(e\*x/sqrt(d\*e))/sqrt(e))\*a + b\*integrate((e\*x^2 + d)^(3/2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (ex^2 + d)^{\frac{3}{2}} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int((d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))), x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asec}(cx)) (d + ex^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*asec(c\*x)),x)

[Out] Integral((a + b\*asec(c\*x))\*(d + e\*x\*\*2)\*\*(3/2), x)

$$3.127 \quad \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^2,x)

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^2,x]

[Out] Defer[Int] [((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2} dx = \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2} dx$$

**Mathematica [A]** time = 8.89, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^2,x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^2, x]

**fricas [A]** time = 1.04, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)) \sqrt{ex^2 + d}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^2,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsec}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)/x^2, x)

**maple** [A] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arcsec}(cx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^2,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( 3 \sqrt{ex^2 + d} ex + 3d\sqrt{e} \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right) - \frac{2(ex^2 + d)^{\frac{3}{2}}}{x} \right) a + \left( e \int \sqrt{ex^2 + d} \arctan\left(\sqrt{cx + 1} \sqrt{cx - 1}\right) dx + d \int \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^2,x, algorithm="maxima")

[Out] 1/2\*(3\*sqrt(e\*x^2 + d)\*e\*x + 3\*d\*sqrt(e)\*arcsinh(e\*x/sqrt(d\*e)) - 2\*(e\*x^2 + d)^(3/2)/x)\*a + (e\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)), x) + d\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^2, x))\*b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{\frac{3}{2}} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2, x)`

[Out] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**2, x)`

[Out] `Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**2, x)`

$$3.128 \quad \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^4, x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^4, x]

[Out] Defer[Int][((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^4, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4} dx = \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4} dx$$

Mathematica [A] time = 11.48, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^4, x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^4, x]

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)) \sqrt{ex^2 + d}}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^4,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/x^4, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsec}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^4,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)/x^4, x)

**maple** [A] time = 3.18, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arcsec}(cx))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^4,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^4,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{3 \sqrt{ex^2 + d} e^2 x}{d} + 3 e^{\frac{3}{2}} \operatorname{arsinh} \left( \frac{ex}{\sqrt{de}} \right) - \frac{2 (ex^2 + d)^{\frac{3}{2}} e}{dx} - \frac{(ex^2 + d)^{\frac{5}{2}}}{dx^3} \right) a + \left( e \int \frac{\sqrt{ex^2 + d} \arctan(\sqrt{cx + 1} \sqrt{cx - 1})}{x^2} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^4,x, algorithm="maxima")

[Out] 1/3\*(3\*sqrt(e\*x^2 + d)\*e^2\*x/d + 3\*e^(3/2)\*arcsinh(e\*x/sqrt(d\*e)) - 2\*(e\*x^2 + d)^(3/2)\*e/(d\*x) - (e\*x^2 + d)^(5/2)/(d\*x^3))\*a + (e\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^2, x) + d\*integrate(sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^4, x))\*b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4, x)`

[Out] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asec}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**4, x)`

[Out] `Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**4, x)`

$$3.129 \quad \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=416

$$-\frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{4bc\sqrt{c^2x^2-1} (c^2d+2e) \sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1} (d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1} (8c^4d^2+23c^2de+23e^2) \sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$

[Out]  $-1/5*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^{(3/2)}*(c^2*x^2-1)^{(1/2)}/x^4/(c^2*x^2)^{(1/2)}+1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}+4/75*b*c*(c^2*d+2*e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/x^2/(c^2*x^2)^{(1/2)}-1/75*b*c^2*(8*c^4*d^2+23*c^2*d*e+23*e^2)*x*\text{EllipticE}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+1/75*b*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*\text{EllipticF}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.54, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {264, 5238, 12, 474, 580, 583, 524, 427, 426, 424, 421, 419}

$$-\frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{bc\sqrt{c^2x^2-1} (8c^4d^2+23c^2de+23e^2) \sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{bc\sqrt{1-c^2x^2} (c^2d+e) (8c^4d^2+23c^2de+23e^2) \sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^6,x]

[Out]  $(b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])/(75*d*\text{Sqrt}[c^2*x^2])+(4*b*c*(c^2*d+2*e)*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])/(75*x^2*\text{Sqrt}[c^2*x^2])+(b*c*\text{Sqrt}[-1+c^2*x^2]*(d+e*x^2)^{(3/2)})/(25*x^4*\text{Sqrt}[c^2*x^2])-(d+e*x^2)^{(5/2)}*(a+b*\text{ArcSec}[c*x])/(5*d*x^5)-(b*c^2*(8*c^4*d^2+23*c^2*d*e+23*e^2)*x*\text{Sqrt}[1-c^2*x^2]*\text{Sqrt}[d+e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(75*d*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[1+(e*x^2)/d])+(b*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*\text{Sqrt}[1-c^2*x^2]*\text{Sqrt}[1+(e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(75*d*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1+c^2*x^2]*\text{Sqrt}[d+e*x^2])$

Rule 12



Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

### Rule 419

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)^2]\*Sqrt[(c\_) + (d\_)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

### Rule 421

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)^2]\*Sqrt[(c\_) + (d\_)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1+(d\*x^2)/c]/Sqrt[c+d\*x^2], Int[1/(Sqrt[a+b\*x^2]\*Sqrt[1+(d\*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

### Rule 424

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

### Rule 426

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[a+b\*x^2]/Sqrt[1+(b\*x^2)/a], Int[Sqrt[1+(b\*x^2)/a]/Sqrt[c+d\*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

### Rule 427

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[1+(d\*x^2)/c]/Sqrt[c+d\*x^2], Int[Sqrt[a+b\*x^2]/Sqrt[1+(d\*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]

### Rule 474

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a+b\*x^n)^(p+1)\*(c+d\*x^n)^

```
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
+ a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

### Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

### Rule 580

```
Int[((g_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

### Rule 583

```
Int[((g_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

### Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} - \frac{(bcx) \int -\frac{(d+ex^2)^{5/2}}{5dx^6 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} + \frac{(bcx) \int \frac{(d+ex^2)^{5/2}}{x^6 \sqrt{-1+c^2x^2}} dx}{5d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x^6 \sqrt{-1+c^2x^2}} dx}{5d\sqrt{c^2x^2}} \\
&= \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{5dx^5}
\end{aligned}$$

**Mathematica [C]** time = 0.68, size = 303, normalized size = 0.73

$$\frac{\sqrt{d+ex^2} \left( -15a(d+ex^2)^2 + bcx\sqrt{1-\frac{1}{c^2x^2}} (dex^2(23c^2x^2+11) + d^2(8c^4x^4+4c^2x^2+3) + 23e^2x^4) - 15b \sec^{-1}(cx) \right)}{75dx^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^6,x]

[Out] (Sqrt[d + e\*x^2]\*(-15\*a\*(d + e\*x^2)^2 + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(23\*e^2\*x^4 + d\*e\*x^2\*(11 + 23\*c^2\*x^2) + d^2\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(d + e\*x^2)^2\*ArcSec[c\*x]))/(75\*d\*x^5) - ((I/75)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(8\*c^4\*d^2 + 23\*c^2\*d\*e + 23\*e^2)\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] - (8\*c^6\*d^3 + 27\*c^4\*d^2\*e + 34\*c^2\*d\*e^2 + 15\*e^3)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))]))/(Sqrt[-c^2]\*d\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arcsec}(cx))\sqrt{ex^2 + d}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/x^6, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsec}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)/x^6, x)

**maple** [F] time = 4.56, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arcsec}(cx))}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^6,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^6,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex^2 + d)^{\frac{5}{2}} a}{5 dx^5} - \frac{\left((e^2 x^4 + 2 dex^2 + d^2) \sqrt{ex^2 + d} \arctan\left(\sqrt{cx + 1} \sqrt{cx - 1}\right) - 5(e^2 x^4 \log(c) + 2 dex^2 \log(c) + d^2 \log(c))\right)}{5 dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^6,x, algorithm="maxima")

[Out] 
$$-1/5*(e*x^2 + d)^{(5/2)}*a/(d*x^5) - 1/5*(5*d*x^5*\text{integrate}((5*(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2 + (c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)*e^{(\log(c*x + 1) + \log(c*x - 1))})*\text{sqrt}(e*x^2 + d)*\log(x) + (5*c^2*d*e*x^4*\log(c) + 5*(c^2*d^2*\log(c) - d*e*\log(c))*x^2 - 5*d^2*\log(c) - (c^2*e^2*x^6 - (5*c^2*\log(c) - 2*c^2)*d*e*x^4 - ((5*c^2*\log(c) - c^2)*d^2 - 5*d*e*\log(c))*x^2 + 5*d^2*\log(c))*e^{(\log(c*x + 1) + \log(c*x - 1))})*\text{sqrt}(e*x^2 + d))/(c^2*d*x^8 - d*x^6 + (c^2*d*x^8 - d*x^6)*e^{(\log(c*x + 1) + \log(c*x - 1))}), x) + (e^2*x^4 + 2*d*e*x^2 + d^2)*\text{sqrt}(e*x^2 + d)*\text{arctan}(\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)))*b/(d*x^5)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))))/x^6,x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))))/x^6, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*asec(c\*x))/x\*\*6,x)

[Out] Timed out

$$3.130 \quad \int \frac{(d+ex^2)^{3/2} (a+b \sec^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=554

$$\frac{2e(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{35d^2x^5} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{7dx^7} + \frac{bc\sqrt{c^2x^2-1} (d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1} (30c^2d - 1225dx^4 + \dots)}{1225dx^4 + \dots}$$

[Out]  $-1/7*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/d/x^7+2/35*e*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/d^2/x^5+1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^{(3/2)}*(c^2*x^2-1)^{(1/2)}/d/x^4/(c^2*x^2)^{(1/2)}+1/49*b*c*(e*x^2+d)^{(5/2)}*(c^2*x^2-1)^{(1/2)}/d/x^6/(c^2*x^2)^{(1/2)}+1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}+1/3675*b*c*(120*c^4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(c^2*x^2)^{(1/2)}-1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*x*\text{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+2/3675*b*(c^2*d+e)*(120*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*\text{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.78, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {271, 264, 5238, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{35d^2x^5} - \frac{(d+ex^2)^{5/2} (a+b \sec^{-1}(cx))}{7dx^7} + \frac{bc\sqrt{c^2x^2-1} (528c^4d^2e + 240c^6d^3 + 193c^2de^2 - \dots)}{3675d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^8,x]

[Out]  $(b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(3675*d^2*\text{Sqrt}[c^2*x^2]) + (b*c*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(3675*d*x^2*\text{Sqrt}[c^2*x^2]) + (b*c*(30*c^2*d + 11*e)*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(3/2)})/(1225*d*x^4*\text{Sqrt}[c^2*x^2]) + (b*c*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(5/2)})/(49*d*x^6*\text{Sqrt}[c^2*x^2]) - ((d + e*x^2)^{(5/2)}*(a + b*\text{ArcSec}[c*x]))/(7*d*x^7) + (2*e*(d + e*x^2)^{(5/2)}*(a + b*\text{ArcSec}[c*x]))/(35*d^2*x^5) - (b*c^2*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -e/(c^2*d)])/(3675*d^2*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2])$

$$2*x^2*\text{Sqrt}[1 + (e*x^2)/d] + (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 105*e^3)*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -(e/(c^2*d))]/(3675*d^2*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])$$
Rule 12

$$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$$
Rule 264

$$\text{Int}[(c_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[(m+1)/n + p + 1, 0] \&\& \text{NeQ}[m, -1]$$
Rule 271

$$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1)+1))/(a*(m+1)), \text{Int}[x^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{ILtQ}[\text{Simplify}[(m+1)/n + p + 1], 0] \&\& \text{NeQ}[m, -1]$$
Rule 419

$$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*(x_)^2]*\text{Sqrt}[(c_) + (d_)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& \text{!}(\text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])$$
Rule 421

$$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*(x_)^2]*\text{Sqrt}[(c_) + (d_)*(x_)^2]), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[1 + (d*x^2)/c]), x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{!GtQ}[c, 0]$$
Rule 424

$$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/\text{Sqrt}[(c_) + (d_)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$$
Rule 426

$$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/\text{Sqrt}[(c_) + (d_)*(x_)^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (b*x^2)/a], \text{Int}[\text{Sqrt}[1 + (b*x^2)/a]/\text{Sqrt}[c + d*x^2]$$

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

### Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

### Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

### Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1
))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

### Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

### Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
```





**Mathematica** [C] time = 0.86, size = 383, normalized size = 0.69

$$\frac{\sqrt{d+ex^2} \left( -105a(5d-2ex^2)(d+ex^2)^2 + bcx\sqrt{1-\frac{1}{c^2x^2}}(de^2x^4(193c^2x^2+71) + 3d^2ex^2(176c^4x^4+83c^2x^2+61)) \right)}{3675d^2x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]))/x^8,x]

[Out] (Sqrt[d + e\*x^2]\*(-105\*a\*(5\*d - 2\*e\*x^2)\*(d + e\*x^2)^2 + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(-247\*e^3\*x^6 + d\*e^2\*x^4\*(71 + 193\*c^2\*x^2) + 3\*d^2\*e\*x^2\*(61 + 83\*c^2\*x^2 + 176\*c^4\*x^4) + 15\*d^3\*(5 + 6\*c^2\*x^2 + 8\*c^4\*x^4 + 16\*c^6\*x^6) - 105\*b\*(5\*d - 2\*e\*x^2)\*(d + e\*x^2)^2\*ArcSec[c\*x]))/(3675\*d^2\*x^7) - ((I/3675)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(240\*c^6\*d^3 + 528\*c^4\*d^2\*e + 193\*c^2\*d\*e^2 - 247\*e^3)\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] - 2\*(120\*c^8\*d^4 + 324\*c^6\*d^3\*e + 221\*c^4\*d^2\*e^2 - 88\*c^2\*d\*e^3 - 105\*e^4)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))]))/(Sqrt[-c^2]\*d^2\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arcsec}(cx))\sqrt{ex^2 + d}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)/x^8, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsec}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x))/x^8,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)/x^8, x)

**maple** [F] time = 5.49, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arcsec}(cx))}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)`

[Out] `int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35} a \left( \frac{2 (ex^2 + d)^{\frac{5}{2}} e}{d^2 x^5} - \frac{5 (ex^2 + d)^{\frac{5}{2}}}{dx^7} \right) - \frac{\left( (2e^3 x^6 - de^2 x^4 - 8d^2 ex^2 - 5d^3) \sqrt{ex^2 + d} \arctan(\sqrt{cx + 1} \sqrt{cx - 1}) - \dots \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")`

[Out] `1/35*a*(2*(e*x^2 + d)^(5/2)*e/(d^2*x^5) - 5*(e*x^2 + d)^(5/2)/(d*x^7)) - 1/35*(35*d^2*x^7*integrate((35*(c^2*d^2*e*x^4 - d^3 + (c^2*d^3 - d^2*e)*x^2 + (c^2*d^2*e*x^4 - d^3 + (c^2*d^3 - d^2*e)*x^2))*e^(log(c*x + 1) + log(c*x - 1))*sqrt(e*x^2 + d)*log(x) + (35*c^2*d^2*e*x^4*log(c) - 35*d^3*log(c) + 35*(c^2*d^3*log(c) - d^2*e*log(c))*x^2 + (2*c^2*e^3*x^8 - c^2*d*e^2*x^6 + (35*c^2*log(c) - 8*c^2)*d^2*e*x^4 - 35*d^3*log(c) + 5*((7*c^2*log(c) - c^2)*d^3 - 7*d^2*e*log(c))*x^2))*e^(log(c*x + 1) + log(c*x - 1))*sqrt(e*x^2 + d))/(c^2*d^2*x^10 - d^2*x^8 + (c^2*d^2*x^10 - d^2*x^8)*e^(log(c*x + 1) + log(c*x - 1))), x) - (2*e^3*x^6 - d*e^2*x^4 - 8*d^2*e*x^2 - 5*d^3)*sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(d^2*x^7)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8,x)`

[Out] `int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8, x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**8,x)`

[Out] Timed out

$$3.131 \quad \int \frac{x^5(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=321

$$\frac{d^2\sqrt{d+ex^2}(a+b \sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b \sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b \sec^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{c}\right)}{15e^3\sqrt{c^2}}$$

[Out]  $-2/3*d*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/e^3+1/5*(e*x^2+d)^{(5/2)}*(a+b*\text{arcsec}(c*x))/e^3+8/15*b*c*d^{(5/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2))}/e^3/(c^2*x^2)^{(1/2)}-1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2))}/c^4/e^{(5/2)}/(c^2*x^2)^{(1/2)}-1/20*b*x*(e*x^2+d)^{(3/2)}*(c^2*x^2-1)^{(1/2)}/c/e^2/(c^2*x^2)^{(1/2)}+d^2*(a+b*\text{arcscec}(c*x))*(e*x^2+d)^{(1/2)}/e^3+1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^3/e^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 1.01, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5238, 12, 1615, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d^2\sqrt{d+ex^2}(a+b \sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b \sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b \sec^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{c}\right)}{15e^3\sqrt{c^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5*(a + b*\text{ArcSec}[c*x]))/\text{Sqrt}[d + e*x^2], x]$

[Out]  $(b*(19*c^2*d - 9*e)*x*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/((120*c^3*e^2*\text{Sqrt}[c^2*x^2]) - (b*x*\text{Sqrt}[-1 + c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c*e^2*\text{Sqrt}[c^2*x^2]) + (d^2*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSec}[c*x]))/e^3 - (2*d*(d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(3*e^3) + ((d + e*x^2)^{(5/2)}*(a + b*\text{ArcSec}[c*x]))/(5*e^3) + (8*b*c*d^{(5/2)}*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])])/(15*e^3*\text{Sqrt}[c^2*x^2]) - (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(c*\text{Sqrt}[d + e*x^2])])/(120*c^4*e^{(5/2)}*\text{Sqrt}[c^2*x^2])$

**Rule 12**

$\text{Int}[(a_*)(u_), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

**Rule 43**

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1615

Int[(Px\_)\*((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_))\*((e\_) + (f\_)\*(x\_)^(p\_)), x\_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k\*(a + b\*x)^(m + q - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*b^(q - 1)\*(m + n + p + q + 1)), x] + Dist[1/(d\*f\*b^q\*(m + n + p + q + 1)), Int[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p\*ExpandToSum[d\*f\*b^q\*(m + n + p + q + 1)\*Px - d\*f\*k\*(m + n + p + q + 1)\*(a + b\*x)^q + k\*(a + b\*x)^(q - 2)\*(a^2\*d\*f\*(m + n + p + q + 1) - b\*(b\*c\*e\*(m + q - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*(m + q) + n + p) - b\*(d\*e\*(m + q + n) + c\*f\*(m + q + p)))\*x], x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2\*m, 2\*n, 2\*p]

Rule 5238

Int[((a\_) + ArcSec[(c\_)\*(x\_)])\*(b\_))\*((f\_)\*(x\_)^(m\_))\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{d^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^3} \\
&= \frac{d^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^3} \\
&= \frac{d^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \sec^{-1}(cx))}{5e^3} \\
&= -\frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} \\
&= \frac{b(19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} - \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2}}{5e^3} \\
&= \frac{b(19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} - \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2}}{5e^3} \\
&= \frac{b(19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} - \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2}}{5e^3} \\
&= \frac{b(19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} - \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2}}{5e^3} \\
&= \frac{b(19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} - \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2}}{5e^3}
\end{aligned}$$

**Mathematica [A]** time = 0.75, size = 328, normalized size = 1.02

$$\frac{\sqrt{d + ex^2} \left( 8ac^3 (8d^2 - 4dex^2 + 3e^2x^4) + 8bc^3 \sec^{-1}(cx) (8d^2 - 4dex^2 + 3e^2x^4) + bex \sqrt{1 - \frac{1}{c^2x^2}} (c^2 (13d - 6ex^2)) \right)}{120c^3 e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2],x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(8\*d^2 - 4\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(-9\*e + c^2\*(13\*d - 6\*e\*x^2)) + 8\*b\*c^3\*(8\*d^2 - 4\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcSec[c\*x]))/(120\*c^3\*e^3) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(45\*c^4\*d^2 - 10\*c^2\*d\*e + 9\*e^2)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])]) + 64\*c^7\*d^(5/2)\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]]))/(120\*c^6\*e^3\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [A] time = 5.85, size = 1385, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/480\*(64\*b\*c^5\*sqrt(-d)\*d^2\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (45\*b\*c^4\*d^2 - 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + 8\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*arcsec(c\*x) - (6\*b\*c^3\*e^2\*x^2 - 13\*b\*c^3\*d\*e + 9\*b\*c\*e^2)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d))/(c^5\*e^3), 1/480\*(12\*8\*b\*c^5\*d^(5/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (45\*b\*c^4\*d^2 - 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + 8\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*arcs ec(c\*x) - (6\*b\*c^3\*e^2\*x^2 - 13\*b\*c^3\*d\*e + 9\*b\*c\*e^2)\*sqrt(c^2\*x^2 - 1)\*s qrt(e\*x^2 + d))/(c^5\*e^3), 1/240\*(32\*b\*c^5\*sqrt(-d)\*d^2\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (45\*b\*c^4\*d^2 - 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + 8\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*arcsec(c\*x) - (6\*b\*c^3\*e^2\*x^2 - 13\*b\*c^3\*d\*e + 9\*b\*c\*e^2)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d))/(c^5\*e^3), 1/240\*(64\*b\*c^5\*d^(5/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*s qrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (45\*b\*c^4\*d^2 - 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2))



$d*e - c*e^2)*x^2)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*\text{arcsec}(c*x) - (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c^3*e^2)*\sqrt{c^2*x^2 - 1})*\sqrt{e*x^2 + d})/(c^5*e^3)]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^5}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^5/sqrt(e\*x^2 + d), x)

**maple** [F] time = 5.36, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arcsec}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{15}*(3*\sqrt{e*x^2 + d})*x^4/e - 4*\sqrt{e*x^2 + d}*d*x^2/e^2 + 8*\sqrt{e*x^2 + d}*d^2/e^3)*a - \frac{1}{15}*(15*e^3*\int((15*c^2*e^3*x^7*\log(c) - 15*e^3*x^5*\log(c) + (3*(5*c^2*\log(c) + c^2)*e^3*x^7 + 4*c^2*d^2*e*x^3 + 8*c^2*d^3*x - (c^2*d*e^2 + 15*e^3*\log(c))*x^5)*e^{(\log(cx + 1) + \log(cx - 1))} + 15*(c^2*e^3*x^7 - e^3*x^5 + (c^2*e^3*x^7 - e^3*x^5)*e^{(\log(cx + 1) + \log(cx - 1))})*\log(x))/((c^2*e^3*x^2 - e^3 + (c^2*e^3*x^2 - e^3)*e^{(\log(cx + 1) + \log(cx - 1))})*\sqrt{e*x^2 + d}), x) - (3*e^2*x^4 - 4*d*e*x^2 + 8*d^2)*\sqrt{e*x^2 + d})*\arctan(\sqrt{cx + 1}*\sqrt{cx - 1}))*b/e^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

[Out] `int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{asec}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(1/2), x)`

[Out] `Integral(x**5*(a + b*asec(c*x))/sqrt(d + e*x**2), x)`

$$3.132 \quad \int \frac{x^3(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=225

$$\frac{d\sqrt{d+ex^2}(a+b \sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b \sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{bx(3c^2d-e) \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

[Out]  $1/3*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/e^2-2/3*b*c*d^{(3/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})/e^2/(c^2*x^2)^{(1/2)}+1/6*b*(3*c^2*d-e)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^2/e^{(3/2)}/(c^2*x^2)^{(1/2)}-d*(a+b*\text{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/e^2-1/6*b*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c/e/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5238, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d\sqrt{d+ex^2}(a+b \sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b \sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{bx(3c^2d-e) \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x]

[Out]  $-(b*x*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(6*c*e*\text{Sqrt}[c^2*x^2]) - (d*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSec}[c*x]))/e^2 + ((d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(3*e^2) - (2*b*c*d^{(3/2)}*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])])/(3*e^2*\text{Sqrt}[c^2*x^2]) + (b*(3*c^2*d - e)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(c*\text{Sqrt}[d + e*x^2])])/(6*c^2*e^{(3/2)}*\text{Sqrt}[c^2*x^2])$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))] + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))*x, x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x
], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

### Rule 266

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 573

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}*((c_) + (d_)*(x_)^{(n_)})^{(q_)}*((e_) + (f_)*(x_)^{(n_)})^{(r_)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p, q, r\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 5238

$\text{Int}[(a_) + \text{ArcSec}[(c_)*(x_)]*(b_)]*((f_)*(x_)^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcSec}[c*x], u, x] - \text{Dist}[(b*c*x)/\text{Sqrt}[c^2*x^2], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m, p\}, x] \&\& ((\text{IGtQ}[p, 0] \&\& \text{!(ILtQ}[(m - 1)/2, 0] \&\& \text{GtQ}[m + 2*p + 3, 0])) \parallel (\text{IGtQ}[(m + 1)/2, 0] \&\& \text{!(ILtQ}[p, 0] \&\& \text{GtQ}[m + 2*p + 3, 0])) \parallel (\text{ILtQ}[(m + 2*p + 1)/2, 0] \&\& \text{!ILtQ}[(m - 1)/2, 0]))$

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx &= -\frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} - \frac{(bcx) \int \frac{(-2d+ex^2)\sqrt{d}}{3e^2x\sqrt{-1+c^2x^2}}}{\sqrt{c^2x^2}} \\
&= -\frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} - \frac{(bcx) \int \frac{(-2d+ex^2)\sqrt{d}}{x\sqrt{-1+c^2x^2}}}{3e^2\sqrt{c^2x^2}} \\
&= -\frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} - \frac{(bcx) \text{Subst}\left(\int \frac{(-2d+ex^2)\sqrt{d}}{x}\right)}{6e^2\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^2}
\end{aligned}$$

**Mathematica [A]** time = 0.52, size = 272, normalized size = 1.21

$$\frac{bx\sqrt{1 - \frac{1}{c^2x^2}} \left( \sqrt{c^2} \sqrt{e} (3c^2d - e) \sqrt{c^2d + e} \sqrt{\frac{c^2(d+ex^2)}{c^2d+e}} \sinh^{-1} \left( \frac{c\sqrt{e} \sqrt{c^2x^2-1}}{\sqrt{c^2} \sqrt{c^2d+e}} \right) + 4c^5d^{3/2} \sqrt{d + ex^2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{c^2x^2-1}}{\sqrt{d+ex^2}} \right) \right)}{6c^4e^2\sqrt{c^2x^2 - 1} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] -1/6\*(Sqrt[d + e\*x^2]\*(4\*a\*c\*d + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x - 2\*a\*c\*e\*x^2 + 2\*b\*c\*(2\*d - e\*x^2)\*ArcSec[c\*x]))/(c\*e^2) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(S

```

qrt[c^2]*(3*c^2*d - e)*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])]
+ 4*c^5*d^(3/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(6*c^4*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])

```

**fricas** [A] time = 1.40, size = 1111, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^3}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^3/sqrt(e\*x^2 + d), x)

**maple** [F] time = 4.82, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsec}(cx))}{\sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{\sqrt{e x^2 + d} x^2}{e} - \frac{2 \sqrt{e x^2 + d} d}{e^2} \right) a - \frac{\frac{1}{2} \left( \frac{6 c^2 d^2 \left( \frac{\log \left( \frac{\sqrt{e x^2 + d} c^2 - \sqrt{c^2 d + e c}}{\sqrt{e x^2 + d} c^2 + \sqrt{c^2 d + e c}} \right) - \log \left( \frac{\sqrt{e x^2 + d} - \sqrt{d}}{\sqrt{e x^2 + d} + \sqrt{d}} \right)}{\sqrt{c^2 d + e c}} \right)}{e} - \frac{3 c^4 d \left( \frac{2 \sqrt{e x^2 + d}}{c^4} + \frac{e \log \left( \frac{\sqrt{e x^2 + d} c^2 - \sqrt{c^2 d + e c}}{\sqrt{e x^2 + d} c^2 + \sqrt{c^2 d + e c}} \right)}{\sqrt{c^2 d + e c}^5} \right)}{e^2} \right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/3\*(sqrt(e\*x^2 + d)\*x^2/e - 2\*sqrt(e\*x^2 + d)\*d/e^2)\*a - 1/3\*(3\*sqrt(e\*x^2 + d)\*e^2\*integrate((3\*c^2\*e^2\*x^5\*log(c) - 3\*e^2\*x^3\*log(c) + ((3\*c^2\*log(c) + c^2)\*e^2\*x^5 - 2\*c^2\*d^2\*x - (c^2\*d\*e + 3\*e^2\*log(c))\*x^3)\*e^(log(c\*x + 1) + log(c\*x - 1)) + 3\*(c^2\*e^2\*x^5 - e^2\*x^3 + (c^2\*e^2\*x^5 - e^2\*x^3)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*log(x))/((c^2\*e^2\*x^2 - e^2 + (c^2\*e^2\*x^2 - e^2)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)), x) - (e^2\*x^4 - d\*e\*x^2 - 2\*d^2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/(sqrt(e\*x^2 + d)\*e^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right)}{\sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

[Out] `int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{asec}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(1/2), x)`

[Out] `Integral(x**3*(a + b*asec(c*x))/sqrt(d + e*x**2), x)`

$$3.133 \quad \int \frac{x(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=132

$$\frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{e} + \frac{bc\sqrt{d} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}} - \frac{bx \tanh^{-1}\left(\frac{\sqrt{e} \sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e} \sqrt{c^2x^2}}$$

[Out] b\*c\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(c^2\*x^2-1)^(1/2))\*d^(1/2)/e/(c^2\*x^2)^(1/2)-b\*x\*arctanh(e^(1/2)\*(c^2\*x^2-1)^(1/2)/c/(e\*x^2+d)^(1/2))/e^(1/2)/(c^2\*x^2)^(1/2)+(a+b\*arcsec(c\*x))\*(e\*x^2+d)^(1/2)/e

**Rubi [A]** time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {5236, 446, 105, 63, 217, 206, 93, 204}

$$\frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{e} + \frac{bc\sqrt{d} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}} - \frac{bx \tanh^{-1}\left(\frac{\sqrt{e} \sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e} \sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/e + (b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(e\*Sqrt[c^2\*x^2]) - (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[c^2\*x^2])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5236

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSec[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}} dx}{e\sqrt{c^2x^2}} \\
&= \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} - \frac{(bcx) \text{Subst} \left( \int \frac{\sqrt{d+ex}}{x\sqrt{-1+c^2x}} dx, x, x^2 \right)}{2e\sqrt{c^2x^2}} \\
&= \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} - \frac{(bcx) \text{Subst} \left( \int \frac{1}{\sqrt{-1+c^2x} \sqrt{d+ex}} dx, x, x^2 \right)}{2\sqrt{c^2x^2}} - \frac{(bcdx) \text{Subst} \left( \int \frac{1}{\sqrt{-1+c^2x}} dx, x, x^2 \right)}{2\sqrt{c^2x^2}} \\
&= \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} - \frac{(bx) \text{Subst} \left( \int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}} dx, x, \sqrt{-1+c^2x^2} \right)}{c\sqrt{c^2x^2}} - \frac{(bcdx) \text{Subst} \left( \int \frac{1}{\sqrt{-1+c^2x}} dx, x, x^2 \right)}{2\sqrt{c^2x^2}} \\
&= \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} + \frac{bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1+c^2x^2}} \right)}{e\sqrt{c^2x^2}} - \frac{(bx) \text{Subst} \left( \int \frac{1}{1-\frac{ex^2}{c^2}} dx, x, x^2 \right)}{c\sqrt{c^2x^2}} \\
&= \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} + \frac{bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1+c^2x^2}} \right)}{e\sqrt{c^2x^2}} - \frac{bx \tanh^{-1} \left( \frac{\sqrt{e} \sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}} \right)}{\sqrt{e} \sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 211, normalized size = 1.60

$$\frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e} - \frac{bx \sqrt{1 - \frac{1}{c^2x^2}} \left( \sqrt{c^2} \sqrt{e} \sqrt{c^2d + e} \sqrt{\frac{c^2(d+ex^2)}{c^2d+e}} \sinh^{-1} \left( \frac{c\sqrt{e} \sqrt{c^2x^2-1}}{\sqrt{c^2} \sqrt{c^2d+e}} \right) + c^3 \sqrt{d} \sqrt{d + ex^2} \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1+c^2x^2}} \right) \right)}{c^2e\sqrt{c^2x^2-1} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]))/e - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])] + c^3\*Sqrt[d]\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]]))/(c^2\*e\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [A] time = 0.88, size = 869, normalized size = 6.58

$$\left[ \frac{bc\sqrt{-d} \log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2-4\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{-d+8d^2}}{x^4}\right) + b\sqrt{e} \log\left(8c^4e^2x^4 + c^4d^2 - 6c^2de + e^2\right)}{4c^2e^2x^4 + c^2d^2 - d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/4\*(b\*c\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + b\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*sqrt(e\*x^2 + d)\*(b\*c\*arcsec(c\*x) + a\*c))/(c\*e), 1/4\*(2\*b\*c\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + b\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*sqrt(e\*x^2 + d)\*(b\*c\*arcsec(c\*x) + a\*c))/(c\*e), 1/4\*(b\*c\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 2\*b\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 4\*sqrt(e\*x^2 + d)\*(b\*c\*arcsec(c\*x) + a\*c))/(c\*e), 1/2\*(b\*c\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + b\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*sqrt(e\*x^2 + d)\*(b\*c\*arcsec(c\*x) + a\*c))/(c\*e)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x/sqrt(e\*x^2 + d), x)

**maple** [F] time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arcsec}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)`

[Out] `int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \frac{c^4 \left( \frac{2\sqrt{ex^2+de}}{c^4} + \frac{e^2 \log\left(\frac{\sqrt{ex^2+d}c^2 - \sqrt{c^2d+ec}}{\sqrt{ex^2+d}c^2 + \sqrt{c^2d+ec}}\right)}{\sqrt{c^2d+ec^5}} \right) \log(c)}{e^2} - \frac{c^2 d \left( \frac{\log\left(\frac{\sqrt{ex^2+d}c^2 - \sqrt{c^2d+ec}}{\sqrt{ex^2+d}c^2 + \sqrt{c^2d+ec}}\right)}{\sqrt{c^2d+ec}} - \frac{\log\left(\frac{\sqrt{ex^2+d} - \sqrt{d}}{c^2 \sqrt{d}}\right)}{c^2 \sqrt{d}} \right)}{e} + \frac{c^4 \left( \frac{2\sqrt{ex^2+de}}{c^4} + \frac{e^2 \log\left(\frac{\sqrt{ex^2+d}c^2 - \sqrt{c^2d+ec}}{\sqrt{ex^2+d}c^2 + \sqrt{c^2d+ec}}\right)}{\sqrt{c^2d+ec^5}} \right)}{e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `-(e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c) + c^2)*e*x^3 + (c^2*d - e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (c^2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x))/((c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e)*sqrt(e*x^2 + d)), x) - sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/e + sqrt(e*x^2 + d)*a/e`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)`

[Out] `int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{asec}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)`

[Out] `Integral(x*(a + b*asec(c*x))/sqrt(d + e*x**2), x)`

$$3.134 \quad \int \frac{a+b \sec^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b \sec^{-1}(cx)}{x\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b \sec^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

Rubi steps

$$\int \frac{a+b \sec^{-1}(cx)}{x\sqrt{d+ex^2}} dx = \int \frac{a+b \sec^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{a+b \sec^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

fricas [A] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arcsec}(cx) + a)}{ex^3 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e\*x^3 + d\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(sqrt(e\*x^2 + d)\*x), x)

**maple** [A] time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan(\sqrt{cx+1} \sqrt{cx-1})}{\sqrt{ex^2 + d} x} dx - \frac{a \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(sqrt(e\*x^2 + d)\*x), x) - a\*arsinh(d/(sqrt(d\*e)\*abs(x)))/sqrt(d)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x\*(d + e\*x^2)^(1/2)),x)



[Out] `int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/x/(e*x**2+d)**(1/2),x)`

[Out] `Integral((a + b*asec(c*x))/(x*sqrt(d + e*x**2)), x)`

$$3.135 \quad \int \frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

Rubi steps

$$\int \frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx = \int \frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

Mathematica [A] time = 11.13, size = 0, normalized size = 0.00

$$\int \frac{a+b \sec^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

fricas [A] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arcsec}(cx) + a)}{ex^5 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e\*x^5 + d\*x^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(sqrt(e\*x^2 + d)\*x^3), x)

**maple** [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^3 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{3}{2}}} - \frac{\sqrt{ex^2 + d}}{dx^2} \right) + b \int \frac{\arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{\sqrt{ex^2 + d} x^3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/2\*a\*(e\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(3/2) - sqrt(e\*x^2 + d)/(d\*x^2)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(sqrt(e\*x^2 + d)\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^3 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x^3\*(d + e\*x^2)^(1/2)),x)

[Out] `int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x^3 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(1/2),x)`

[Out] `Integral((a + b*asec(c*x))/(x**3*sqrt(d + e*x**2)), x)`

$$3.136 \quad \int \frac{x^2(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^2(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable( $x^2*(a+b*\text{arcsec}(c*x))/(e*x^2+d)^{(1/2)}$ , x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[( $x^2*(a + b*\text{ArcSec}[c*x])$ )/Sqrt[d +  $e*x^2$ ], x]

[Out] Defer[Int] [( $x^2*(a + b*\text{ArcSec}[c*x])$ )/Sqrt[d +  $e*x^2$ ], x]

Rubi steps

$$\int \frac{x^2(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx = \int \frac{x^2(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 9.69, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b \sec^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[( $x^2*(a + b*\text{ArcSec}[c*x])$ )/Sqrt[d +  $e*x^2$ ], x]

[Out] Integrate[( $x^2*(a + b*\text{ArcSec}[c*x])$ )/Sqrt[d +  $e*x^2$ ], x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \text{arcsec}(cx) + ax^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*x^2\*arcsec(c\*x) + a\*x^2)/sqrt(e\*x^2 + d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^2}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^2/sqrt(e\*x^2 + d), x)

maple [A] time = 3.17, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arcsec}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{\sqrt{ex^2 + d} x}{e} - \frac{d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}} \right) + b \int \frac{x^2 \arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{\sqrt{ex^2+d}}\right) dx}{\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/2\*a\*(sqrt(e\*x^2 + d)\*x/e - d\*arcsinh(e\*x/sqrt(d\*e))/e^(3/2)) + b\*integrate(x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/sqrt(e\*x^2 + d), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

[Out] `int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{asec}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(1/2), x)`

[Out] `Integral(x**2*(a + b*asec(c*x))/sqrt(d + e*x**2), x)`

$$3.137 \quad \int \frac{a+b \sec^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}}, x\right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2), x)

**Rubi** [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/Sqrt[d + e\*x^2], x]

[Out] Defer[Int][(a + b\*ArcSec[c\*x])/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}} dx = \int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

**Mathematica** [A] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/Sqrt[d + e\*x^2], x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/Sqrt[d + e\*x^2], x]

**fricas** [A] time = 1.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)/sqrt(e\*x^2 + d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/sqrt(e\*x^2 + d), x)

maple [A] time = 2.53, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan(\sqrt{cx+1} \sqrt{cx-1})}{\sqrt{ex^2+d}} dx + \frac{a \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/sqrt(e\*x^2 + d), x) + a\*arc sinh(e\*x/sqrt(d\*e))/sqrt(e)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^(1/2),x)

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

$$3.138 \quad \int \frac{a+b \sec^{-1}(cx)}{x^2 \sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=246

$$\frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{dx} + \frac{bc\sqrt{c^2x^2-1} \sqrt{d+ex^2}}{d\sqrt{c^2x^2}} + \frac{bx\sqrt{1-c^2x^2} (c^2d+e) \sqrt{\frac{ex^2}{d}+1} F(\sin^{-1}(cx) | -\frac{e}{c^2d})}{d\sqrt{c^2x^2} \sqrt{c^2x^2-1} \sqrt{d+ex^2}} - \frac{bc^2}{d}$$

[Out]  $-(a+b*\text{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/d/x+b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}-b*c^2*x*\text{EllipticE}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+b*(c^2*d+e)*x*\text{EllipticF}(c*x, (-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.26, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {264, 5238, 12, 475, 21, 423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{dx} + \frac{bc\sqrt{c^2x^2-1} \sqrt{d+ex^2}}{d\sqrt{c^2x^2}} + \frac{bx\sqrt{1-c^2x^2} (c^2d+e) \sqrt{\frac{ex^2}{d}+1} F(\sin^{-1}(cx) | -\frac{e}{c^2d})}{d\sqrt{c^2x^2} \sqrt{c^2x^2-1} \sqrt{d+ex^2}} - \frac{bc^2}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSec}[c*x])/(x^2*\text{Sqrt}[d + e*x^2]), x]$

[Out]  $(b*c*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(d*\text{Sqrt}[c^2*x^2]) - (\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSec}[c*x]))/(d*x) - (b*c^2*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(d*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]) + (b*(c^2*d + e)*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(d*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])$

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_) /; \text{FreeQ}[b, x]]$

### Rule 21

$\text{Int}[(u_)*((a_) + (b_)*(v_))^{(m_)}*((c_) + (d_)*(v_))^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (\text{!IntegerQ}[n] || \text{SimplerQ}[c + d*x,$

$a + b*x$ )

### Rule 264

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[(m+1)/n + p + 1, 0] \&\& \text{NeQ}[m, -1]$

### Rule 419

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]*\text{Sqrt}[(c_*) + (d_*)*(x_*)^2]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& !(\text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])]$

### Rule 421

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]*\text{Sqrt}[(c_*) + (d_*)*(x_*)^2]), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[1 + (d*x^2)/c]), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& !\text{GtQ}[c, 0]$

### Rule 423

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_*)^2], x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Int}[\text{Sqrt}[c + d*x^2]/\text{Sqrt}[a + b*x^2], x], x] - \text{Dist}[(b*c - a*d)/d, \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[c + d*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{NegQ}[b/a]$

### Rule 424

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_*)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

### Rule 426

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_*)^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (b*x^2)/a], \text{Int}[\text{Sqrt}[1 + (b*x^2)/a]/\text{Sqrt}[c + d*x^2], x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& !\text{GtQ}[a, 0]$

### Rule 427

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_*)^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (d*x^2)/c], x], x]$

], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]

### Rule 475

Int[((e\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*e\*(m + 1)), x] - Dist[1/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*b\*(m + 1) + n\*(b\*c\*(p + 1) + a\*d\*q) + d\*(b\*(m + 1) + b\*n\*(p + q + 1))\*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^2 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{dx^2 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{-1+c^2x^2}} dx}{d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} - \frac{(bcx) \int \frac{-e+c^2ex^2}{\sqrt{-1+c^2x^2} \sqrt{d+ex^2}} dx}{d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} - \frac{(bcex) \int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}} dx}{d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} - \frac{(bc^3x) \int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}} dx}{d\sqrt{c^2x^2}} + \frac{(bc(c^2d))}{d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} - \frac{(bc^3x\sqrt{1 - c^2x^2}) \int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}} dx}{d\sqrt{c^2x^2} \sqrt{-1 + c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} - \frac{(bc^3x\sqrt{1 - c^2x^2} \sqrt{d + ex^2}) \int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}} dx}{d\sqrt{c^2x^2} \sqrt{-1 + c^2x^2} \sqrt{1 + c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{dx} - \frac{bc^2x\sqrt{1 - c^2x^2} \sqrt{d + ex^2} E(\sin^{-1}(\sqrt{-\frac{e}{d}}x))}{d\sqrt{c^2x^2} \sqrt{-1 + c^2x^2} \sqrt{1 + c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 143, normalized size = 0.58

$$\frac{\sqrt{d + ex^2} \left( -a + bcx \sqrt{1 - \frac{1}{c^2x^2}} - b \sec^{-1}(cx) \right)}{dx} - \frac{bcex \sqrt{1 - \frac{1}{c^2x^2}} \sqrt{\frac{ex^2}{d} + 1} E \left( \sin^{-1} \left( \sqrt{-\frac{e}{d}} x \right) \middle| -\frac{c^2d}{e} \right)}{d\sqrt{1 - c^2x^2} \sqrt{-\frac{e}{d}} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^2\*Sqrt[d + e\*x^2]),x]

[Out] (Sqrt[d + e\*x^2]\*(-a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x - b\*ArcSec[c\*x]))/(d\*x) - (b\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*EllipticE[ArcSin[Sqrt[

$-(e/d)]*x], -((c^2*d)/e)))/(d*\text{Sqrt}[-(e/d)]*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])$

**fricas** [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^4 + d*x^2), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)`

**maple** [F] time = 2.23, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)`

[Out] `int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\left(\sqrt{ex^2 + d}(\log(c) + \log(x)) - \sqrt{ex^2 + d} \arctan\left(\sqrt{cx + 1} \sqrt{cx - 1}\right)\right)b}{dx} - \frac{\sqrt{ex^2 + d} a}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `-(d*x*integrate((c^2*d*x^2*log(c) - (c^2*e*x^4 - (c^2*log(c) - c^2)*d*x^2 + d*log(c))*e^(log(c*x + 1) + log(c*x - 1)) - d*log(c) + (c^2*d*x^2 + (c^2*d`

```
*x^2 - d)*e^(log(c*x + 1) + log(c*x - 1)) - d)*log(x))/((c^2*d*x^4 - d*x^2
+ (c^2*d*x^4 - d*x^2)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x)
+ sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(d*x) - sqrt(e*x^
2 + d)*a/(d*x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x^2 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(1/2), x)
```

```
[Out] Integral((a + b*asec(c*x))/(x**2*sqrt(d + e*x**2)), x)
```



$$3.139 \quad \int \frac{a+b \sec^{-1}(cx)}{x^4 \sqrt{d+ex^2}} dx$$

Optimal. Leaf size=362

$$\frac{2e\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{3dx^3} + \frac{bc\sqrt{c^2x^2-1} (2c^2d-5e)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{2bx\sqrt{1-c^2x^2}}{9d^2\sqrt{c^2x^2}}$$

[Out]  $-1/3*(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/d/x^3+2/3*e*(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/d^2/x+1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}+1/9*b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(c^2*x^2)^{(1/2)}-1/9*b*c^2*(2*c^2*d-5*e)*x*\operatorname{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.45, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {271, 264, 5238, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2} (a+b \sec^{-1}(cx))}{3dx^3} + \frac{bc\sqrt{c^2x^2-1} (2c^2d-5e)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{2bx\sqrt{1-c^2x^2}}{9d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSec}[c*x])/(x^4*\operatorname{Sqrt}[d + e*x^2]),x]$

[Out]  $(b*c*(2*c^2*d - 5*e)*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(9*d^2*\operatorname{Sqrt}[c^2*x^2]) + (b*c*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(9*d*x^2*\operatorname{Sqrt}[c^2*x^2]) - (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcSec}[c*x]))/(3*d*x^3) + (2*e*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcSec}[c*x]))/(3*d^2*x) - (b*c^2*(2*c^2*d - 5*e)*x*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[c*x], -(e/(c^2*d))])/(9*d^2*\operatorname{Sqrt}[c^2*x^2]*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[1 + (e*x^2)/d]) + (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[1 + (e*x^2)/d]*\operatorname{EllipticF}[\operatorname{ArcSin}[c*x], -(e/(c^2*d))])/(9*d^2*\operatorname{Sqrt}[c^2*x^2]*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*
a + b*x^n)^(p + 1)/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*
(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^4 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3d^2x} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{3d^2x^4\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3d^2x} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4\sqrt{-1+c^2x^2}} dx}{3d^2\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3d^2x} + \\
&= \frac{bc(2c^2d - 5e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{bc(2c^2d - 5e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{bc(2c^2d - 5e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{bc(2c^2d - 5e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{bc(2c^2d - 5e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3} \\
&= \frac{bc(2c^2d - 5e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{3dx^3}
\end{aligned}$$

**Mathematica [C]** time = 0.67, size = 249, normalized size = 0.69

$$\frac{\sqrt{d + ex^2} \left( -3a(d - 2ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}} (2c^2dx^2 + d - 5ex^2) - 3b \sec^{-1}(cx) (d - 2ex^2) \right)}{9d^2x^3} - \frac{ibcx\sqrt{1 - \frac{1}{c^2x^2}} \sqrt{\frac{ex^2}{d}}}{d} +$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^4\*Sqrt[d + e\*x^2]), x]

[Out] (Sqrt[d + e\*x^2]\*(b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + 2\*c^2\*d\*x^2 - 5\*e\*x^2) - 3\*a\*(d - 2\*e\*x^2) - 3\*b\*(d - 2\*e\*x^2)\*ArcSec[c\*x]))/(9\*d^2\*x^3) - ((I/9)\*b

```
*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*Ellip
ticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e + 3
*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)))]/(Sqrt[-c^2]*d^2*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**fricas** [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{ex^6 + dx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^6 + d*x^4), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

**maple** [F] time = 4.69, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^4 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a \left( \frac{2 \sqrt{ex^2 + d} e}{d^2 x} - \frac{\sqrt{ex^2 + d}}{dx^3} \right) - \frac{(3(2ex^2 \log(c) - d \log(c) + (2ex^2 - d) \log(x))(ex^2 + d) - (2e^2x^4 + dex^2 - d^2))}{3 \sqrt{ex^2 + d} d^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*sqrt(e*x^2 + d)*e/(d^2*x) - sqrt(e*x^2 + d)/(d*x^3)) - 1/3*(3*sqrt
(e*x^2 + d)*d^2*x^3*integrate((3*c^2*d^2*x^2*log(c) - 3*d^2*log(c) + (2*c^2
*e^2*x^6 + c^2*d*e*x^4 + (3*c^2*log(c) - c^2)*d^2*x^2 - 3*d^2*log(c))*e^(lo
g(c*x + 1) + log(c*x - 1)) + 3*(c^2*d^2*x^2 - d^2 + (c^2*d^2*x^2 - d^2)*e^(
log(c*x + 1) + log(c*x - 1)))*log(x))/((c^2*d^2*x^6 - d^2*x^4 + (c^2*d^2*x^
6 - d^2*x^4)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x) - (2*e^2
*x^4 + d*e*x^2 - d^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(sqrt(e*x^2 +
d)*d^2*x^3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{x^4 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x^4 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(1/2), x)
```

```
[Out] Integral((a + b*asec(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

$$3.140 \quad \int \frac{a+b \sec^{-1}(cx)}{x^6 \sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=1006

$$\frac{8be^2x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)c^2}{15d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} + \frac{4be(2dc^2+e)x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)c^2}{45d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$

[Out]  $-1/5*(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/d/x^5+4/15*e*(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/d^2/x^3-8/15*e^2*(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/d^3/x+8/15*b*c*e^2*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}-4/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}+1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}+1/25*b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^4/(c^2*x^2)^{(1/2)}-4/45*b*c*e*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/x^2/(c^2*x^2)^{(1/2)}+1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/x^2/(c^2*x^2)^{(1/2)}-8/15*b*c^2*e^2*x*\operatorname{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+4/45*b*c^2*e*(2*c^2*d+e)*x*\operatorname{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}-1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*\operatorname{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}-8/45*b*c^2*e*(c^2*d+e)*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}+8/15*b*e^2*(c^2*d+e)*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 1.78, antiderivative size = 1006, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$ , Rules used = {271, 264, 5238, 12, 6742, 475, 583, 524, 427, 426, 424, 421, 419, 21, 423}

$$\frac{8be^2x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)c^2}{15d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} + \frac{4be(2dc^2+e)x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)c^2}{45d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcSec}[c*x])/(x^6*\operatorname{Sqrt}[d + e*x^2]),x]$

[Out]  $(8*b*c*e^2*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(15*d^3*\operatorname{Sqrt}[c^2*x^2]) - (4*b*c*e*(2*c^2*d + e)*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(45*d^3*\operatorname{Sqrt}[c^2*x^2])$

```

2]) + (b*c*(8*c^4*d^2 + 3*c^2*d*e - 2*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^
2])/(75*d^3*Sqrt[c^2*x^2]) + (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(25*d
*x^4*Sqrt[c^2*x^2]) - (4*b*c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(45*d^2*
x^2*Sqrt[c^2*x^2]) + (b*c*(4*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(75*d^2*x^2*Sqrt[c^2*x^2]) - (Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(5*d*x^
5) + (4*e*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(15*d^2*x^3) - (8*e^2*Sqrt[d
+ e*x^2]*(a + b*ArcSec[c*x]))/(15*d^3*x) - (8*b*c^2*e^2*x*Sqrt[1 - c^2*x^2
]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(15*d^3*Sqrt[c^2*x^
2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (4*b*c^2*e*(2*c^2*d + e)*x*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(45*d^
3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*c^2*(8*c^4*d^2
+ 3*c^2*d*e - 2*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[
c*x], -(e/(c^2*d))])/(75*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x
^2)/d]) + (b*c^2*(8*c^2*d - e)*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*
x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(75*d^2*Sqrt[c^2*x^2]*Sqrt[-1
+ c^2*x^2]*Sqrt[d + e*x^2]) - (8*b*c^2*e*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*S
qrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(45*d^2*Sqrt[c^2*x
^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) + (8*b*e^2*(c^2*d + e)*x*Sqrt[1 - c
^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(15*d^3*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

### Rule 21

```

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])

```

### Rule 264

```

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

```

### Rule 271

```

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*
(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

```



Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q
)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
```

```
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

### Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)])], x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

### Rule 583

```
Int[((g_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_))], x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

### Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.)), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rubi steps



**Mathematica** [C] time = 0.74, size = 329, normalized size = 0.33

$$\frac{\sqrt{d+ex^2} \left( -15a(3d^2 - 4dex^2 + 8e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}} \left( -dex^2(31c^2x^2 + 17) + 3d^2(8c^4x^4 + 4c^2x^2 + 3) + 94e^2x^4 \right) \right)}{225d^3x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^6\*Sqrt[d + e\*x^2]), x]

[Out] (Sqrt[d + e\*x^2]\*(-15\*a\*(3\*d^2 - 4\*d\*e\*x^2 + 8\*e^2\*x^4) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(94\*e^2\*x^4 - d\*e\*x^2\*(17 + 31\*c^2\*x^2) + 3\*d^2\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(3\*d^2 - 4\*d\*e\*x^2 + 8\*e^2\*x^4)\*ArcSec[c\*x]))/(225\*d^3\*x^5 - ((I/225)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(24\*c^4\*d^2 - 31\*c^2\*d\*e + 94\*e^2)\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] - (24\*c^6\*d^3 - 19\*c^4\*d^2\*e + 77\*c^2\*d\*e^2 + 120\*e^3)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))]))/(Sqrt[-c^2]\*d^3\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2]))

**fricas** [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{ex^8 + dx^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^6/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e\*x^8 + d\*x^6), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^6/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(sqrt(e\*x^2 + d)\*x^6), x)

**maple** [F] time = 6.31, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^6 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^6/(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))/x^6/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15} a \left( \frac{8 \sqrt{ex^2 + d} e^2}{d^3 x} - \frac{4 \sqrt{ex^2 + d} e}{d^2 x^3} + \frac{3 \sqrt{ex^2 + d}}{dx^5} \right) - \frac{\left( (8e^2x^4 - 4dex^2 + 3d^2) \sqrt{ex^2 + d} \arctan(\sqrt{cx + 1} \sqrt{cx - 1}) \right)}{d^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^6/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] -1/15\*a\*(8\*sqrt(e\*x^2 + d)\*e^2/(d^3\*x) - 4\*sqrt(e\*x^2 + d)\*e/(d^2\*x^3) + 3\*sqrt(e\*x^2 + d)/(d\*x^5)) - 1/15\*(15\*d^3\*x^5\*integrate((15\*c^2\*d^3\*x^2\*log(c) - 15\*d^3\*log(c) - (8\*c^2\*e^3\*x^8 + 4\*c^2\*d\*e^2\*x^6 - c^2\*d^2\*e\*x^4 - 3\*(5\*c^2\*log(c) - c^2)\*d^3\*x^2 + 15\*d^3\*log(c))\*e^(log(c\*x + 1) + log(c\*x - 1)) + 15\*(c^2\*d^3\*x^2 - d^3 + (c^2\*d^3\*x^2 - d^3)\*e^(log(c\*x + 1) + log(c\*x - 1))) \* log(x))/((c^2\*d^3\*x^8 - d^3\*x^6 + (c^2\*d^3\*x^8 - d^3\*x^6)\*e^(log(c\*x + 1) + log(c\*x - 1))) \* sqrt(e\*x^2 + d)), x) + (8\*e^2\*x^4 - 4\*d\*e\*x^2 + 3\*d^2) \* sqrt(e\*x^2 + d) \* arctan(sqrt(c\*x + 1) \* sqrt(c\*x - 1)) \* b / (d^3 \* x^5)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^6 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x^6\*(d + e\*x^2)^(1/2)),x)

[Out] int((a + b\*acos(1/(c\*x)))/(x^6\*(d + e\*x^2)^(1/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*6/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

$$3.141 \quad \int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=252

$$\frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{3e^3 \sqrt{c^2x^2}}$$

[Out]  $1/3*(e*x^2+d)^{(3/2)}*(a+b*\text{arcsec}(c*x))/e^3-8/3*b*c*d^{(3/2)}*x*\text{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})/e^3/(c^2*x^2)^{(1/2)}+1/6*b*(9*c^2*d-e)*x*\text{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^2/e^{(5/2)}/(c^2*x^2)^{(1/2)}-d^2*(a+b*\text{arcsec}(c*x))/e^3/(e*x^2+d)^{(1/2)}-2*d*(a+b*\text{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/e^3-1/6*b*x*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c/e^2/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 1.01, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 5238, 12, 1615, 157, 63, 217, 206, 93, 204}

$$\frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{3e^3 \sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5*(a + b*\text{ArcSec}[c*x]))/(d + e*x^2)^{(3/2)}, x]$

[Out]  $-(b*x*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(6*c*e^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcSec}[c*x]))/(e^3*\text{Sqrt}[d + e*x^2]) - (2*d*\text{Sqrt}[d + e*x^2]*(a + b*\text{ArcSec}[c*x]))/e^3 + ((d + e*x^2)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(3*e^3) - (8*b*c*d^{(3/2)}*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])])/(3*e^3*\text{Sqrt}[c^2*x^2]) + (b*(9*c^2*d - e)*x*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(c*\text{Sqrt}[d + e*x^2])])/(6*c^2*e^{(5/2)}*\text{Sqrt}[c^2*x^2])$

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 43

$\text{Int}[((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}$

$x]$  && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 206

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_.) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 1615

```
Int[(Px_)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f
_)*(x_)^(p_)), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
+ p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)])*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \sec^{-1}(cx))}{3e^3} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bx\sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3}
\end{aligned}$$

**Mathematica [A]** time = 0.64, size = 303, normalized size = 1.20

$$\frac{-2ac(8d^2 + 4dex^2 - e^2x^4) - bex\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) - 2bc \sec^{-1}(cx)(8d^2 + 4dex^2 - e^2x^4)}{6ce^3\sqrt{d + ex^2}} + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}} \left( \sqrt{c^2x^2} \sqrt{d + ex^2} \right)}{e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out]  $(-(b*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - 2*a*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*\text{ArcSec}[c*x])/(6*c*e^3*\text{Sqrt}[d +$

$$e*x^2)) + (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(\text{Sqrt}[c^2]*(9*c^2*d - e)*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[(c^2*(d + e*x^2))/(c^2*d + e)]*\text{ArcSinh}[(c*\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/(\text{Sqrt}[c^2]*\text{Sqrt}[c^2*d + e])]) + 16*c^5*d^{(3/2)}*\text{Sqrt}[d + e*x^2]*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])/(\text{Sqrt}[d + e*x^2])])/(6*c^4*e^3*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])$$

**fricas** [A] time = 1.03, size = 1483, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [-1/24\*((9\*b\*c^2\*d^2 - b\*d\*e + (9\*b\*c^2\*d\*e - b\*e^2)\*x^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) - 16\*(b\*c^3\*d\*e\*x^2 + b\*c^3\*d^2)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) - 4\*(2\*a\*c^3\*e^2\*x^4 - 8\*a\*c^3\*d\*e\*x^2 - 16\*a\*c^3\*d^2 + 2\*(b\*c^3\*e^2\*x^4 - 4\*b\*c^3\*d\*e\*x^2 - 8\*b\*c^3\*d^2)\*arcsec(c\*x) - (b\*c\*e^2\*x^2 + b\*c\*d\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^3\*e^4\*x^2 + c^3\*d\*e^3), -1/24\*(32\*(b\*c^3\*d\*e\*x^2 + b\*c^3\*d^2)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (9\*b\*c^2\*d^2 - b\*d\*e + (9\*b\*c^2\*d\*e - b\*e^2)\*x^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) - 4\*(2\*a\*c^3\*e^2\*x^4 - 8\*a\*c^3\*d\*e\*x^2 - 16\*a\*c^3\*d^2 + 2\*(b\*c^3\*e^2\*x^4 - 4\*b\*c^3\*d\*e\*x^2 - 8\*b\*c^3\*d^2)\*arcsec(c\*x) - (b\*c\*e^2\*x^2 + b\*c\*d\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^3\*e^4\*x^2 + c^3\*d\*e^3), -1/12\*((9\*b\*c^2\*d^2 - b\*d\*e + (9\*b\*c^2\*d\*e - b\*e^2)\*x^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) - 8\*(b\*c^3\*d\*e\*x^2 + b\*c^3\*d^2)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) - 2\*(2\*a\*c^3\*e^2\*x^4 - 8\*a\*c^3\*d\*e\*x^2 - 16\*a\*c^3\*d^2 + 2\*(b\*c^3\*e^2\*x^4 - 4\*b\*c^3\*d\*e\*x^2 - 8\*b\*c^3\*d^2)\*arcsec(c\*x) - (b\*c\*e^2\*x^2 + b\*c\*d\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^3\*e^4\*x^2 + c^3\*d\*e^3), -1/12\*(16\*(b\*c^3\*d\*e\*x^2 + b\*c^3\*d^2)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (9\*b\*c^2\*d^2 - b\*d\*e + (9\*b\*c^2\*d\*e - b\*e^2)\*x^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) - 2\*(2\*a\*c^3\*e^2\*x^4 - 8\*a\*c^3\*d\*e\*x^2 - 16\*a\*c^3\*d^2 + 2\*(b\*c^3\*e^2\*x^4 - 4\*b\*c^3\*d\*e\*x^2 - 8\*b\*c^3\*d^2)\*arcsec(c\*x) - (b\*c\*e^2\*x^2 + b\*c\*d\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^3\*e^4\*x^2 + c^3\*d\*e^3)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^5}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^5/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 5.16, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/3\*(x^4/(sqrt(e\*x^2 + d)\*e) - 4\*d\*x^2/(sqrt(e\*x^2 + d)\*e^2) - 8\*d^2/(sqrt(e\*x^2 + d)\*e^3))\*a - 1/3\*(3\*sqrt(e\*x^2 + d)\*e^3\*integrate((3\*c^2\*e^3\*x^7\*log(c) - 3\*e^3\*x^5\*log(c) + ((3\*c^2\*log(c) + c^2)\*e^3\*x^7 - 12\*c^2\*d^2\*e\*x^3 - 8\*c^2\*d^3\*x - 3\*(c^2\*d\*e^2 + e^3\*log(c))\*x^5)\*e^(log(c\*x + 1) + log(c\*x - 1)) + 3\*(c^2\*e^3\*x^7 - e^3\*x^5 + (c^2\*e^3\*x^7 - e^3\*x^5)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*log(x))/((c^2\*e^4\*x^4 - d\*e^3 + (c^2\*d\*e^3 - e^4)\*x^2 + (c^2\*e^4\*x^4 - d\*e^3 + (c^2\*d\*e^3 - e^4)\*x^2)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)), x) - (e^2\*x^4 - 4\*d\*e\*x^2 - 8\*d^2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/(sqrt(e\*x^2 + d)\*e^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

```
[Out] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(3/2), x)
```

```
[Out] Timed out
```

$$3.142 \quad \int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=157

$$\frac{\sqrt{d+ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{d (a + b \sec^{-1}(cx))}{e^2 \sqrt{d+ex^2}} - \frac{bx \tanh^{-1}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{d+ex^2}}\right)}{e^{3/2} \sqrt{c^2 x^2}} + \frac{2bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2 x^2 - 1}}\right)}{e^2 \sqrt{c^2 x^2}}$$

[Out]  $-b*x*\operatorname{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/e^{(3/2)}/(c^2*x^2)^{(1/2)}+2*b*c*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})*d^{(1/2)}/e^{2/(c^2*x^2)^{(1/2)}+d*(a+b*\operatorname{arcsec}(c*x))/e^2/(e*x^2+d)^{(1/2)}+(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/e^2}$

**Rubi [A]** time = 0.26, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 5238, 12, 573, 157, 63, 217, 206, 93, 204}

$$\frac{\sqrt{d+ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{d (a + b \sec^{-1}(cx))}{e^2 \sqrt{d+ex^2}} + \frac{2bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2 x^2 - 1}}\right)}{e^2 \sqrt{c^2 x^2}} - \frac{bx \tanh^{-1}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{d+ex^2}}\right)}{e^{3/2} \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*(a + b*\operatorname{ArcSec}[c*x]))/(d + e*x^2)^{(3/2)}, x]$

[Out]  $(d*(a + b*\operatorname{ArcSec}[c*x]))/(e^2*\operatorname{Sqrt}[d + e*x^2]) + (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcSec}[c*x]))/e^2 + (2*b*c*\operatorname{Sqrt}[d]*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 + c^2*x^2])])/(e^2*\operatorname{Sqrt}[c^2*x^2]) - (b*x*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 + c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(e^{(3/2)}*\operatorname{Sqrt}[c^2*x^2])$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

### Rule 43

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (\operatorname{!IntegerQ}[n] \operatorname{||} (\operatorname{EqQ}[c, 0] \&\& \operatorname{Le} Q[7*m + 4*n + 4, 0]) \operatorname{||} \operatorname{LtQ}[9*m + 5*(n + 1), 0]) \operatorname{||} \operatorname{GtQ}[m + n + 2, 0])$

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
)*(e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} - \frac{(bcx) \int \frac{2d+ex^2}{e^2 x \sqrt{-1+c^2x^2} \sqrt{d+ex^2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} - \frac{(bcx) \int \frac{2d+ex^2}{x \sqrt{-1+c^2x^2} \sqrt{d+ex^2}} dx}{e^2 \sqrt{c^2x^2}} \\
&= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} - \frac{(bcx) \text{Subst} \left( \int \frac{2d+ex}{x \sqrt{-1+c^2x} \sqrt{d+ex}} dx \right)}{2e^2 \sqrt{c^2x^2}} \\
&= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} - \frac{(bcdx) \text{Subst} \left( \int \frac{1}{x \sqrt{-1+c^2x} \sqrt{d+ex}} dx \right)}{e^2 \sqrt{c^2x^2}} \\
&= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} - \frac{(2bcdx) \text{Subst} \left( \int \frac{1}{-d-x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{d+ex}} \right)}{e^2 \sqrt{c^2x^2}} \\
&= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1+c^2x^2}} \right)}{e^2 \sqrt{c^2x^2}} \\
&= \frac{d(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1+c^2x^2}} \right)}{e^2 \sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.36, size = 221, normalized size = 1.41

$$\frac{(2d + ex^2)(a + b \sec^{-1}(cx))}{e^2 \sqrt{d + ex^2}} - \frac{bx \sqrt{1 - \frac{1}{c^2x^2}} \left( \sqrt{c^2} \sqrt{e} \sqrt{c^2d + e} \sqrt{\frac{c^2(d+ex^2)}{c^2d+e}} \sinh^{-1} \left( \frac{c\sqrt{e} \sqrt{c^2x^2-1}}{\sqrt{c^2} \sqrt{c^2d+e}} \right) + 2c^3 \sqrt{d} \sqrt{d + ex^2} \right)}{c^2 e^2 \sqrt{c^2x^2 - 1} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] ((2\*d + e\*x^2)\*(a + b\*ArcSec[c\*x]))/(e^2\*Sqrt[d + e\*x^2]) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/((Sqrt[c^2]\*Sqrt[c^2\*d + e])]) + 2\*c^3\*Sqrt[d]\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]]))/(c^2\*e^2\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])



**fricas** [A] time = 1.00, size = 1070, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [1/4\*((b\*e\*x^2 + b\*d)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 2\*(b\*c\*e\*x^2 + b\*c\*d)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 4\*(a\*c\*e\*x^2 + 2\*a\*c\*d + (b\*c\*e\*x^2 + 2\*b\*c\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d))/(c\*e^3\*x^2 + c\*d\*e^2), 1/4\*(4\*(b\*c\*e\*x^2 + b\*c\*d)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (b\*e\*x^2 + b\*d)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 - 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(a\*c\*e\*x^2 + 2\*a\*c\*d + (b\*c\*e\*x^2 + 2\*b\*c\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d))/(c\*e^3\*x^2 + c\*d\*e^2), 1/2\*((b\*e\*x^2 + b\*d)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + (b\*c\*e\*x^2 + b\*c\*d)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 2\*(a\*c\*e\*x^2 + 2\*a\*c\*d + (b\*c\*e\*x^2 + 2\*b\*c\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d))/(c\*e^3\*x^2 + c\*d\*e^2), 1/2\*(2\*(b\*c\*e\*x^2 + b\*c\*d)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (b\*e\*x^2 + b\*d)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(a\*c\*e\*x^2 + 2\*a\*c\*d + (b\*c\*e\*x^2 + 2\*b\*c\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d))/(c\*e^3\*x^2 + c\*d\*e^2)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^3/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 4.66, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)`

[Out] `int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left( \frac{x^2}{\sqrt{ex^2 + de}} + \frac{2d}{\sqrt{ex^2 + de^2}} \right) + \frac{\frac{1}{2} \left( 2(ex^2 + 2d)\sqrt{ex^2 + d} \arctan(\sqrt{cx + 1}\sqrt{cx - 1}) - (e^3x^2 + de^2) \right)}{2c^4d^2 \frac{2}{(c^4d + c^2e)\sqrt{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] `a*(x^2/(sqrt(e*x^2 + d)*e) + 2*d/(sqrt(e*x^2 + d)*e^2)) + ((e*x^2 + 2*d)*sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (e^3*x^2 + d*e^2)*integrate((c^2*e^2*x^5*log(c) - e^2*x^3*log(c) + ((c^2*log(c) + c^2)*e^2*x^5 + 2*c^2*d^2*x + (3*c^2*d*e - e^2*log(c))*x^3)*e^(log(c*x + 1) + log(c*x - 1)) + (c^2*e^2*x^5 - e^2*x^3 + (c^2*e^2*x^5 - e^2*x^3)*e^(log(c*x + 1) + log(c*x - 1))))*log(x))/((c^2*e^3*x^4 - d*e^2 + (c^2*d*e^2 - e^3)*x^2 + (c^2*e^3*x^4 - d*e^2 + (c^2*d*e^2 - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x)*b/(e^3*x^2 + d*e^2)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)`

[Out] `int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{asec}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(3/2), x)

[Out] Integral(x\*\*3\*(a + b\*asec(c\*x))/(d + e\*x\*\*2)\*\*(3/2), x)

$$3.143 \quad \int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=80

$$-\frac{a+b \sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

[Out]  $-b*c*x*\arctan((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})/e/d^{(1/2)}/(c^2*x^2)^{(1/2)}+(-a-b*\operatorname{arcsec}(c*x))/e/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {5236, 446, 93, 204}

$$-\frac{a+b \sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(a + b*\text{ArcSec}[c*x]))/(d + e*x^2)^{(3/2)}, x]$

[Out]  $-((a + b*\text{ArcSec}[c*x])/(e*\text{Sqrt}[d + e*x^2])) - (b*c*x*\text{ArcTan}[\text{Sqrt}[d + e*x^2]/(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])]/(\text{Sqrt}[d]*e*\text{Sqrt}[c^2*x^2]))$

#### Rule 93

$\text{Int}[(((a_.) + (b_.)*(x_))^{(m_.)*((c_.) + (d_.)*(x_))^{(n_.))}/((e_.) + (f_.)*(x_))), x\_Symbol] :> \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[m + n + 1, 0] \ \&\& \ \text{RationalQ}[n] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x]$

#### Rule 204

$\text{Int}[((a_.) + (b_.)*(x_)^2)^{(-1)}, x\_Symbol] :> -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 446

$\text{Int}[(x_)^{(m_.)*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)*((c_.) + (d_.)*(x_)^{(n_.)})^{(q_.)}}, x\_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p}$

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 5236

$\text{Int}[(a + \text{ArcSec}[c*x])*(b*x)^p*(d + e*x^2)^q, x\_Symbol] :> \text{Simp}[(d + e*x^2)^{p+1}*(a + b*\text{ArcSec}[c*x])/(2*e*(p + 1)), x] - \text{Dist}[(b*c*x)/(2*e*(p + 1)*\text{Sqrt}[c^2*x^2]), \text{Int}[(d + e*x^2)^{p+1}/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{x(a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{a + b \sec^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{e\sqrt{c^2x^2}} \\ &= -\frac{a + b \sec^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}} dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\ &= -\frac{a + b \sec^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{-d-x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} \\ &= -\frac{a + b \sec^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 95, normalized size = 1.19

$$\frac{bcx\sqrt{1 - \frac{1}{c^2x^2}} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2-1}} - \frac{a + b \sec^{-1}(cx)}{e\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] -((a + b\*ArcSec[c\*x])/(e\*Sqrt[d + e\*x^2])) + (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(Sqrt[d]\*e\*Sqrt[-1 + c^2\*x^2])

**fricas** [A] time = 0.64, size = 283, normalized size = 3.54

$$\left[ \frac{(bex^2 + bd)\sqrt{-d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 - 4\sqrt{c^2x^2 - 1}((c^2d - e)x^2 - 2d)\sqrt{ex^2 + d}\sqrt{-d} + 8d^2}{x^4}\right) + 4\sqrt{ex^2 + d}(bd \operatorname{arcsec}(cx))}{4(de^2x^2 + d^2e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [-1/4\*((b\*e\*x^2 + b\*d)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 4\*sqrt(e\*x^2 + d)\*(b\*d\*arcsec(c\*x) + a\*d))/(d\*e^2\*x^2 + d^2\*e), -1/2\*((b\*e\*x^2 + b\*d)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + 2\*sqrt(e\*x^2 + d)\*(b\*d\*arcsec(c\*x) + a\*d))/(d\*e^2\*x^2 + d^2\*e)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( \frac{c^4 d \left( \frac{2e}{(c^4 d + c^2 e) \sqrt{ex^2 + d}} + \frac{e \log\left(\frac{\sqrt{ex^2 + d} c^2 - \sqrt{c^2 d + ec}}{\sqrt{ex^2 + d} c^2 + \sqrt{c^2 d + ec}}\right)}{(c^2 d + e)^{\frac{3}{2}} c} \right)}{e^2} + \frac{c^4 \left( \frac{2de}{(c^4 d + c^2 e) \sqrt{ex^2 + d}} - \frac{e^2 \log\left(\frac{\sqrt{ex^2 + d} c^2 - \sqrt{c^2 d + ec}}{\sqrt{ex^2 + d} c^2 + \sqrt{c^2 d + ec}}\right)}{(c^4 d + c^2 e) \sqrt{c^2 d + ec}} \right) \log(c)}{e^2} - \frac{c^4 \left( \frac{2de}{(c^4 d + c^2 e) \sqrt{ex^2 + d}} - \frac{e^2 \log\left(\frac{\sqrt{ex^2 + d} c^2 - \sqrt{c^2 d + ec}}{\sqrt{ex^2 + d} c^2 + \sqrt{c^2 d + ec}}\right)}{(c^4 d + c^2 e) \sqrt{c^2 d + ec}} \right)}{e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -(sqrt(e\*x^2 + d)\*e\*integrate((c^2\*e\*x^3\*log(c) - e\*x\*log(c) + ((c^2\*log(c) - c^2)\*e\*x^3 - (c^2\*d + e\*log(c))\*x)\*e^(log(c\*x + 1) + log(c\*x - 1)) + (c^2\*e\*x^3 - e\*x + (c^2\*e\*x^3 - e\*x)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*log(x))/((c^2\*e^2\*x^4 + (c^2\*d\*e - e^2)\*x^2 - d\*e + (c^2\*e^2\*x^4 + (c^2\*d\*e - e^2)\*x^2 - d\*e)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)), x) + arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/(sqrt(e\*x^2 + d)\*e) - a/(sqrt(e\*x^2 + d)\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(3/2),x)

[Out] int((x\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + b \operatorname{asec}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Integral(x\*(a + b\*asec(c\*x))/(d + e\*x\*\*2)\*\*(3/2), x)

$$3.144 \quad \int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(3/2), x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{3/2}} dx = \int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 15.67, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{e^2 x^5 + 2 dex^3 + d^2 x}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x), x)

**maple** [A] time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(3/2),x)

[Out] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{\operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{3}{2}}} - \frac{1}{\sqrt{ex^2 + dd}} \right) + b \int \frac{\arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{(ex^3 + dx)\sqrt{ex^2 + d}}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a\*(arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(3/2) - 1/(sqrt(e\*x^2 + d)\*d)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e\*x^3 + d\*x)\*sqrt(e\*x^2 + d)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**(3/2), x)
```

```
[Out] Timed out
```

$$3.145 \quad \int \frac{a+b \sec^{-1}(cx)}{x^3(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(3/2), x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx = \int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 22.17, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

fricas [A] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{e^2 x^7 + 2 dex^5 + d^2 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^2\*x^7 + 2\*d\*e\*x^5 + d^2\*x^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x^3), x)

**maple** [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^3 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(3/2),x)

[Out] int((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{3e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{5}{2}}} - \frac{3e}{\sqrt{ex^2 + d} d^2} - \frac{1}{\sqrt{ex^2 + d} dx^2} \right) + b \int \frac{\arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{(ex^5 + dx^3)\sqrt{ex^2 + d}}\right) dx}{(ex^5 + dx^3)\sqrt{ex^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/2\*a\*(3\*e\*arsinh(d/(sqrt(d\*e)\*abs(x)))/d^(5/2) - 3\*e/(sqrt(e\*x^2 + d)\*d^2) - 1/(sqrt(e\*x^2 + d)\*d\*x^2)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e\*x^5 + d\*x^3)\*sqrt(e\*x^2 + d)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^3 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

$$3.146 \quad \int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int] [(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 8.19, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^4 \operatorname{arcsec}(cx) + ax^4) \sqrt{ex^2 + d}}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral((b\*x^4\*arcsec(c\*x) + a\*x^4)\*sqrt(e\*x^2 + d)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^4}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^4/(e\*x^2 + d)^(3/2), x)

**maple** [A] time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \frac{x^3}{\sqrt{ex^2 + de}} + \frac{3 dx}{\sqrt{ex^2 + de^2}} - \frac{3 d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}} \right) a + b \int \frac{x^4 \arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{(ex^2 + d)^{\frac{3}{2}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/2\*(x^3/(sqrt(e\*x^2 + d)\*e) + 3\*d\*x/(sqrt(e\*x^2 + d)\*e^2) - 3\*d\*arcsinh(e\*x/sqrt(d\*e))/e^(5/2))\*a + b\*integrate(x^4\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x^2 + d)^(3/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

```
[Out] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(3/2), x)
```

```
[Out] Timed out
```



$$3.147 \quad \int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int] [(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 3.87, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

fricas [A] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^2 \operatorname{arcsec}(cx) + ax^2) \sqrt{ex^2 + d}}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral((b\*x^2\*arcsec(c\*x) + a\*x^2)\*sqrt(e\*x^2 + d)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^2/(e\*x^2 + d)^(3/2), x)

**maple** [A] time = 3.15, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{x}{\sqrt{ex^2 + d}e} - \frac{\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}} \right) + b \int \frac{x^2 \arctan\left(\sqrt{cx + 1}\sqrt{cx - 1}\right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a\*(x/(sqrt(e\*x^2 + d)\*e) - arcsinh(e\*x/sqrt(d\*e))/e^(3/2)) + b\*integrate(x^2\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x^2 + d)^(3/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)`

[Out] `int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{asec}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(3/2), x)`

[Out] `Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)`

$$3.148 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=109

$$\frac{x(a+b \sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2} \sqrt{\frac{ex^2}{d}} + 1 F\left(\sin^{-1}(cx) \mid -\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2} \sqrt{c^2x^2-1} \sqrt{d+ex^2}}$$

[Out] x\*(a+b\*arcsec(c\*x))/d/(e\*x^2+d)^(1/2)-b\*x\*EllipticF(c\*x, (-e/c^2/d)^(1/2))\*(-c^2\*x^2+1)^(1/2)\*(1+e\*x^2/d)^(1/2)/d/(c^2\*x^2)^(1/2)/(c^2\*x^2-1)^(1/2)/(e\*x^2+d)^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {191, 5228, 12, 421, 419}

$$\frac{x(a+b \sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2} \sqrt{\frac{ex^2}{d}} + 1 F\left(\sin^{-1}(cx) \mid -\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2} \sqrt{c^2x^2-1} \sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^(3/2), x]

[Out] (x\*(a + b\*ArcSec[c\*x]))/(d\*Sqrt[d + e\*x^2]) - (b\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(d\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 5228

```
Int[((a_) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{3/2}} dx &= \frac{x(a + b \sec^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{1}{d\sqrt{-1+c^2x^2} \sqrt{d+ex^2}} dx}{\sqrt{c^2x^2}} \\
 &= \frac{x(a + b \sec^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{1}{\sqrt{-1+c^2x^2} \sqrt{d+ex^2}} dx}{d\sqrt{c^2x^2}} \\
 &= \frac{x(a + b \sec^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{\left(bcx\sqrt{1 + \frac{ex^2}{d}}\right) \int \frac{1}{\sqrt{-1+c^2x^2} \sqrt{1 + \frac{ex^2}{d}}} dx}{d\sqrt{c^2x^2} \sqrt{d + ex^2}} \\
 &= \frac{x(a + b \sec^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{\left(bcx\sqrt{1 - c^2x^2} \sqrt{1 + \frac{ex^2}{d}}\right) \int \frac{1}{\sqrt{1-c^2x^2} \sqrt{1 + \frac{ex^2}{d}}} dx}{d\sqrt{c^2x^2} \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}} \\
 &= \frac{x(a + b \sec^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{bx\sqrt{1 - c^2x^2} \sqrt{1 + \frac{ex^2}{d}} F\left(\sin^{-1}(cx) \middle| -\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2} \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 113, normalized size = 1.04

$$\frac{x(a + b \sec^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{bcx\sqrt{1 - \frac{1}{c^2x^2}} \sqrt{1 - c^2x^2} \sqrt{\frac{ex^2}{d}} + 1 F\left(\sin^{-1}(cx) \middle| -\frac{e}{c^2d}\right)}{d(c^3x^2 - c)\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^(3/2), x]

[Out] (x\*(a + b\*ArcSec[c\*x]))/(d\*Sqrt[d + e\*x^2]) - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(d\*(-c + c^3\*x^2)\*Sqrt[d + e\*x^2])

**fricas** [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 2.31, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x)

[Out] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan(\sqrt{cx+1} \sqrt{cx-1})}{(ex^2 + d)^{\frac{3}{2}}} dx + \frac{ax}{\sqrt{ex^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x^2 + d)^(3/2), x) + a\*x / (sqrt(e\*x^2 + d)\*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^(3/2),x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{(d + ex^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Integral((a + b\*asec(c\*x))/(d + e\*x\*\*2)\*\*(3/2), x)

$$3.149 \quad \int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=274

$$\frac{2ex(a+b \sec^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b \sec^{-1}(cx)}{dx\sqrt{d+ex^2}} + \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} + \frac{bx\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}} + 1F(\sin^{-1}(cx))}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

[Out]  $(-a-b*\text{arcsec}(c*x))/d/x/(e*x^2+d)^{(1/2)}-2*e*x*(a+b*\text{arcsec}(c*x))/d^2/(e*x^2+d)^{(1/2)}+b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}-b*c^2*x*E\text{llipticE}(c*x,(-e/c^2/d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)})/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+b*(c^2*d+2*e)*x*\text{EllipticF}(c*x,(-e/c^2/d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)})/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {271, 191, 5238, 12, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2ex(a+b \sec^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b \sec^{-1}(cx)}{dx\sqrt{d+ex^2}} + \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} + \frac{bx\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}} + 1F(\sin^{-1}(cx))}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSec}[c*x])/(x^2*(d + e*x^2)^{(3/2)}), x]$

[Out]  $(b*c*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(d^2*\text{Sqrt}[c^2*x^2]) - (a + b*\text{ArcSec}[c*x])/(d*x*\text{Sqrt}[d + e*x^2]) - (2*e*x*(a + b*\text{ArcSec}[c*x]))/(d^2*\text{Sqrt}[d + e*x^2]) - (b*c^2*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -e/(c^2*d)])/(d^2*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]) + (b*(c^2*d + 2*e)*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -e/(c^2*d)])/(d^2*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])$

### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

### Rule 191

$\text{Int}[((a_*) + (b_.)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := \text{Simp}[(x*(a + b*x^n)^{(p+1)})/a, x] /; \text{FreeQ}[\{a, b, n, p\}, x] \&\& \text{EqQ}[1/n + p + 1, 0]$



Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simplifier SqrtQ[-(b/a), -(d/c)]))))))

### Rule 583

Int[((g\_.)\*(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(e\*(g\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*c\*g\*(m + 1)), x] + Dist[1/(a\*c\*g^n\*(m + 1)), Int[(g\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^q\*Simp[a\*f\*c\*(m + 1) - e\*(b\*c + a\*d)\*(m + n + 1) - e\*n\*(b\*c\*p + a\*d\*q) - b\*e\*d\*(m + n\*(p + q + 2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrate[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx &= -\frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-d-2ex^2}{d^2x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-d-2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{d^2\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-2de+c^2d}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{d^3\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{(bc^3x) \int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}} dx}{d^2\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{(bc^3x\sqrt{1 - c^2x^2})}{d^2\sqrt{c^2x^2}\sqrt{-1}} \\
&= \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{(bc^3x\sqrt{1 - c^2x^2}\sqrt{-1})}{d^2\sqrt{c^2x^2}\sqrt{-1}} \\
&= \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \sec^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \sec^{-1}(cx))}{d^2\sqrt{d + ex^2}} - \frac{bc^2x\sqrt{1 - c^2x^2}\sqrt{d}}{d^2\sqrt{c^2x^2}\sqrt{-1}}
\end{aligned}$$

**Mathematica [C]** time = 0.48, size = 212, normalized size = 0.77

$$\frac{-a(d + 2ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) - b \sec^{-1}(cx)(d + 2ex^2)}{d^2x\sqrt{d + ex^2}} - \frac{ibcx\sqrt{1 - \frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d} + 1} \left( c^2dE \left( i \sinh^{-1} \left( \sqrt{-c^2x^2} \right) \right) \right)}{\sqrt{-c^2} d^2\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^2\*(d + e\*x^2)^(3/2)),x]

[Out] (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + e\*x^2) - a\*(d + 2\*e\*x^2) - b\*(d + 2\*e\*x^2)\*ArcSec[c\*x])/(d^2\*x\*Sqrt[d + e\*x^2]) - (I\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] - (c^2\*d + 2\*e)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))]))/(Sqrt[-c^2]\*d^2\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{e^2x^6 + 2dex^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^2\*x^6 + 2\*d\*e\*x^4 + d^2\*x^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x^2), x)

**maple** [F] time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(3/2),x)

[Out] int((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{2ex}{\sqrt{ex^2 + d}d^2} + \frac{1}{\sqrt{ex^2 + d}dx}\right) - \frac{\left((2ex^2 + d)\sqrt{ex^2 + d} \arctan(\sqrt{cx + 1}\sqrt{cx - 1}) - (2ex^2 \log(c) + d \log(c) + (2ex^2 + d)\sqrt{ex^2 + d})\right)}{d^2ex^3 + d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a\*(2\*e\*x/(sqrt(e\*x^2 + d)\*d^2) + 1/(sqrt(e\*x^2 + d)\*d\*x)) - ((2\*e\*x^2 + d)\*sqrt(e\*x^2 + d)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + (d^2\*e\*x^3 + d^3\*x)\*

```
integrate((c^2*d^2*x^2*log(c) - d^2*log(c) - (2*c^2*e^2*x^6 + 3*c^2*d*e*x^4
- (c^2*log(c) - c^2)*d^2*x^2 + d^2*log(c))*e^(log(c*x + 1) + log(c*x - 1))
+ (c^2*d^2*x^2 - d^2 + (c^2*d^2*x^2 - d^2)*e^(log(c*x + 1) + log(c*x - 1))
)*log(x))/((c^2*d^2*e*x^6 - d^3*x^2 + (c^2*d^3 - d^2*e)*x^4 + (c^2*d^2*e*x^
6 - d^3*x^2 + (c^2*d^3 - d^2*e)*x^4)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(
e*x^2 + d)), x))*b/(d^2*e*x^3 + d^3*x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 (ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

$$3.150 \quad \int \frac{a+b \sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=701

$$\frac{8e^2x(a+b \sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{4e(a+b \sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} - \frac{a+b \sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}} + 1F(\sin^{-1}(cx)|-\frac{e}{c^2d})}{3d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{2bc}{3d^3\sqrt{d+ex^2}}$$

[Out]  $\frac{1}{3}(-a-b\operatorname{arcsec}(c*x))/d/x^3/(e*x^2+d)^{(1/2)}+4/3*e*(a+b\operatorname{arcsec}(c*x))/d^2/x/(e*x^2+d)^{(1/2)}+8/3*e^2*x*(a+b\operatorname{arcsec}(c*x))/d^3/(e*x^2+d)^{(1/2)}+2/9*b*c*(c^2*d-e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}-4/3*b*c*e*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}+1/9*b*c*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/x^2/(c^2*x^2)^{(1/2)}-2/9*b*c^2*(c^2*d-e)*x*\operatorname{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+4/3*b*c^2*e*x*\operatorname{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+1/9*b*c^2*(2*c^2*d-e)*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}-4/3*b*c^2*e*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}-8/3*b*e^2*x*\operatorname{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 1.39, antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$ , Rules used = {271, 191, 5238, 12, 6742, 421, 419, 480, 583, 524, 427, 426, 424, 493}

$$\frac{8e^2x(a+b \sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{4e(a+b \sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} - \frac{a+b \sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}} + 1F(\sin^{-1}(cx)|-\frac{e}{c^2d})}{3d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{2bc}{3d^3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(x^4\*(d + e\*x^2)^(3/2)), x]

[Out]  $\frac{(2*b*c*(c^2*d - e)*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])}{(9*d^3*\operatorname{Sqrt}[c^2*x^2])} - \frac{(4*b*c*e*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])}{(3*d^3*\operatorname{Sqrt}[c^2*x^2])} + \frac{(b*c*\operatorname{Sqrt}[-1 + c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])}{(9*d^2*x^2*\operatorname{Sqrt}[c^2*x^2])} - \frac{(a + b*\operatorname{ArcSec}[c*x])}{(3*d*x^3*\operatorname{Sqrt}[d + e*x^2])} + \frac{(4*e*(a + b*\operatorname{ArcSec}[c*x]))}{(3*d^2*x*\operatorname{Sqrt}[d + e*x^2])} + \frac{(8*e^2*x*(a + b*\operatorname{ArcSec}[c*x]))}{(3*d^3*\operatorname{Sqrt}[d + e*x^2])} - \frac{(2*b*c^2*(c^2*d - e)*x*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[c*x], (-e/c^2/d)^{(1/2)}])}{(3*d^3*\operatorname{Sqrt}[c^2*x^2]*\operatorname{Sqrt}[c^2*x^2 - 1]*\operatorname{Sqrt}[d + e*x^2])} + \frac{2*b*c}{3*d^3*\operatorname{Sqrt}[d + e*x^2]}$

```

*x], -(e/(c^2*d)))]/(9*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2
)/d]) + (4*b*c^2*e*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d)))]/(3*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/
d]) + (b*c^2*(2*c^2*d - e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Elliptic
F[ArcSin[c*x], -(e/(c^2*d)))]/(9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[
d + e*x^2]) - (4*b*c^2*e*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[
ArcSin[c*x], -(e/(c^2*d)))]/(3*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d
+ e*x^2]) - (8*b*e^2*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcS
in[c*x], -(e/(c^2*d)))]/(3*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

```

### Rule 191

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

```

### Rule 271

```

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

```

### Rule 419

```

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

```

### Rule 421

```

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

```

### Rule 424

```

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c

```

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 426

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[a + b\*x^2]/Sqrt[1 + (b\*x^2)/a], Int[Sqrt[1 + (b\*x^2)/a]/Sqrt[c + d\*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

#### Rule 427

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[Sqrt[a + b\*x^2]/Sqrt[1 + (d\*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]

#### Rule 480

Int[((e\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*c\*e\*(m + 1)), x] - Dist[1/(a\*c\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^q\*Simp[(b\*c + a\*d)\*(m + n + 1) + n\*(b\*c\*p + a\*d\*q) + b\*d\*(m + n\*(p + q + 2) + 1)\*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

#### Rule 493

Int[(x\_)^(n\_)/(Sqrt[(a\_) + (b\_.)\*(x\_)^(n\_)]\*Sqrt[(c\_) + (d\_.)\*(x\_)^(n\_)]), x\_Symbol] := Dist[1/b, Int[Sqrt[a + b\*x^n]/Sqrt[c + d\*x^n], x], x] - Dist[a/b, Int[1/(Sqrt[a + b\*x^n]\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && SimplerSqrtQ[-(b/a), -(d/c)])

#### Rule 524

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/(Sqrt[(a\_) + (b\_.)\*(x\_)^(n\_)]\*Sqrt[(c\_) + (d\_.)\*(x\_)^(n\_)]), x\_Symbol] := Dist[f/b, Int[Sqrt[a + b\*x^n]/Sqrt[c + d\*x^n], x], x] + Dist[(b\*e - a\*f)/b, Int[1/(Sqrt[a + b\*x^n]\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

#### Rule 583



```

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

```

### Rule 5238

```

Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rule 6742

```

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^4 (d + ex^2)^{3/2}} dx &= -\frac{a + b \sec^{-1}(cx)}{3dx^3 \sqrt{d + ex^2}} + \frac{4e(a + b \sec^{-1}(cx))}{3d^2x \sqrt{d + ex^2}} + \frac{8e^2x(a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-d^2 + 4dex^2 + 8e^2x^4}{3d^3x^4 \sqrt{-1 + c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{3dx^3 \sqrt{d + ex^2}} + \frac{4e(a + b \sec^{-1}(cx))}{3d^2x \sqrt{d + ex^2}} + \frac{8e^2x(a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-d^2 + 4dex^2 + 8e^2x^4}{x^4 \sqrt{-1 + c^2x^2}} dx}{3d^3 \sqrt{c^2x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{3dx^3 \sqrt{d + ex^2}} + \frac{4e(a + b \sec^{-1}(cx))}{3d^2x \sqrt{d + ex^2}} + \frac{8e^2x(a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} - \frac{(bcx) \int \left( \frac{8e^2}{\sqrt{-1 + c^2x^2}} \sqrt{d + ex^2} \right) dx}{3d \sqrt{c^2x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{3dx^3 \sqrt{d + ex^2}} + \frac{4e(a + b \sec^{-1}(cx))}{3d^2x \sqrt{d + ex^2}} + \frac{8e^2x(a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} + \frac{(bcx) \int \frac{1}{x^4 \sqrt{-1 + c^2x^2}} dx}{3d \sqrt{c^2x^2}} \\
&= -\frac{4bce \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{3d^3 \sqrt{c^2x^2}} + \frac{bc \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2x^2 \sqrt{c^2x^2}} - \frac{a + b \sec^{-1}(cx)}{3dx^3 \sqrt{d + ex^2}} + \frac{4e(a + b \sec^{-1}(cx))}{3d^2x \sqrt{d + ex^2}} \\
&= \frac{2bc(c^2d - e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^3 \sqrt{c^2x^2}} - \frac{4bce \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{3d^3 \sqrt{c^2x^2}} + \frac{bc \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2x^2 \sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d - e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^3 \sqrt{c^2x^2}} - \frac{4bce \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{3d^3 \sqrt{c^2x^2}} + \frac{bc \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2x^2 \sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d - e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^3 \sqrt{c^2x^2}} - \frac{4bce \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{3d^3 \sqrt{c^2x^2}} + \frac{bc \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2x^2 \sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d - e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^3 \sqrt{c^2x^2}} - \frac{4bce \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{3d^3 \sqrt{c^2x^2}} + \frac{bc \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2x^2 \sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d - e) \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^3 \sqrt{c^2x^2}} - \frac{4bce \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{3d^3 \sqrt{c^2x^2}} + \frac{bc \sqrt{-1 + c^2x^2} \sqrt{d + ex^2}}{9d^2x^2 \sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica** [C] time = 0.74, size = 292, normalized size = 0.42

$$\frac{-3a(d^2 - 4dex^2 - 8e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(d^2(2c^2x^2 + 1) + dex^2(2c^2x^2 - 13) - 14e^2x^4) - 3b\sec^{-1}(cx)(d^2 - 4dex^2 - 8e^2x^4)}{9d^3x^3\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^4\*(d + e\*x^2)^(3/2)), x]

[Out]  $(-3*a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4) + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(-14*e^2*x^4 + d*e*x^2*(-13 + 2*c^2*x^2) + d^2*(1 + 2*c^2*x^2)) - 3*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*\text{ArcSec}[c*x])/(9*d^3*x^3*\text{Sqrt}[d + e*x^2]) - ((I/9)*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 7*e)*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d))]) + (-2*c^4*d^2 + 13*c^2*d*e + 24*e^2)*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d))]))/(\text{Sqrt}[-c^2]*d^3*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])$

**fricas** [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{e^2x^8 + 2dex^6 + d^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^4/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^2\*x^8 + 2\*d\*e\*x^6 + d^2\*x^4), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^4/(e\*x^2+d)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x^4), x)

**maple** [F] time = 4.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^4 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)`

[Out] `int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a \left( \frac{8e^2x}{\sqrt{ex^2+d}d^3} + \frac{4e}{\sqrt{ex^2+d}d^2x} - \frac{1}{\sqrt{ex^2+d}dx^3} \right) - \frac{\left( \frac{3(8e^2x^4 \log(c) + 4dex^2 \log(c) - d^2 \log(c) + (8e^2x^4 + 4dex^2 - d^2) \log(x))(ex^2+d)d^3x}{d^3ex^5+d^4x^3} \right)}{3\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] `1/3*a*(8*e^2*x/(sqrt(e*x^2 + d)*d^3) + 4*e/(sqrt(e*x^2 + d)*d^2*x) - 1/(sqrt(e*x^2 + d)*d*x^3)) - 1/3*(3*sqrt(e*x^2 + d)*d^3*x^3*integrate((3*c^2*d^3*x^2*log(c) - 3*d^3*log(c) + (8*c^2*e^3*x^8 + 12*c^2*d*e^2*x^6 + 3*c^2*d^2*e*x^4 + (3*c^2*log(c) - c^2)*d^3*x^2 - 3*d^3*log(c))*e^(log(c*x + 1) + log(c*x - 1)) + 3*(c^2*d^3*x^2 - d^3 + (c^2*d^3*x^2 - d^3)*e^(log(c*x + 1) + log(c*x - 1)))*log(x))/((c^2*d^3*e*x^8 - d^4*x^4 + (c^2*d^4 - d^3*e)*x^6 + (c^2*d^3*e*x^8 - d^4*x^4 + (c^2*d^4 - d^3*e)*x^6)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x) - (8*e^2*x^4 + 4*d*e*x^2 - d^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(sqrt(e*x^2 + d)*d^3*x^3)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{x^4 (ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)),x)`

[Out] `int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)),x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(3/2),x)`

[Out] Timed out

$$3.151 \quad \int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=244

$$\frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{bx \tanh^{-1}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{d + ex^2}}\right)}{e^{5/2} \sqrt{c^2 x^2}} + \frac{8bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{d + ex^2}}\right)}{3e^3}$$

[Out]  $-1/3*d^2*(a+b*\operatorname{arcsec}(c*x))/e^3/(e*x^2+d)^{(3/2)}-b*x*\operatorname{arctanh}(e^{(1/2)}*(c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/e^{(5/2)}/(c^2*x^2)^{(1/2)}+8/3*b*c*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(c^2*x^2-1)^{(1/2)})*d^{(1/2)}/e^3/(c^2*x^2)^{(1/2)}+2*d*(a+b*\operatorname{arcsec}(c*x))/e^3/(e*x^2+d)^{(1/2)}-1/3*b*c*d*x*(c^2*x^2-1)^{(1/2)}/e^2/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(e*x^2+d)^{(1/2)}+(a+b*\operatorname{arcsec}(c*x))*(e*x^2+d)^{(1/2)}/e^3$

Rubi [A] time = 1.06, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 5238, 12, 1614, 157, 63, 217, 206, 93, 204}

$$\frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{bcdx \sqrt{c^2 x^2 - 1}}{3e^2 \sqrt{c^2 x^2} (c^2 d + e) \sqrt{d + ex^2}} + \frac{8bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{c \sqrt{d + ex^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^5*(a + b*\operatorname{ArcSec}[c*x]))/(d + e*x^2)^{(5/2)}, x]$

[Out]  $-(b*c*d*x*\operatorname{Sqrt}[-1 + c^2*x^2])/(3*e^2*(c^2*d + e)*\operatorname{Sqrt}[c^2*x^2]*\operatorname{Sqrt}[d + e*x^2]) - (d^2*(a + b*\operatorname{ArcSec}[c*x]))/(3*e^3*(d + e*x^2)^{(3/2)}) + (2*d*(a + b*\operatorname{ArcSec}[c*x]))/(e^3*\operatorname{Sqrt}[d + e*x^2]) + (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcSec}[c*x]))/e^3 + (8*b*c*\operatorname{Sqrt}[d]*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 + c^2*x^2])])/(3*e^3*\operatorname{Sqrt}[c^2*x^2]) - (b*x*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 + c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(e^{(5/2)}*\operatorname{Sqrt}[c^2*x^2])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 43

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\},$

$x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& ( !\text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

### Rule 63

$\text{Int}[\frac{(a + b*x)^m * (c + d*x)^n}{(a + b*x)^{1/p}}, x] \text{ :> With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m + 1) - 1} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{1/p}], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 93

$\text{Int}[\frac{(a + b*x)^m * (c + d*x)^n}{(e + f*x)^q}, x] \text{ :> With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{q*(m + 1) - 1} / (b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{1/q} / (c + d*x)^{1/q}], x] \text{ ; FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n] \&\& \text{LtQ}[-1, m, 0] \&\& \text{SimplerQ}[a + b*x, c + d*x]$

### Rule 157

$\text{Int}[\frac{(c + d*x)^n * (e + f*x)^p * (g + h*x)}{(a + b*x)}, x] \text{ :> Dist}[h/b, \text{Int}[(c + d*x)^n * (e + f*x)^p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n * (e + f*x)^p / (a + b*x), x], x] \text{ ; FreeQ}[\{a, b, c, d, e, f, g, h, n, p\}, x]$

### Rule 204

$\text{Int}[\frac{(a + b*x^2)^{-1}}{x}, x] \text{ :> -Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] * \text{Rt}[-b, 2]), x] \text{ ; FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 206

$\text{Int}[\frac{(a + b*x^2)^{-1}}{x}, x] \text{ :> Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2]*x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] \text{ ; FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 217

$\text{Int}[1 / \sqrt{a + b*x^2}, x] \text{ :> Subst}[\text{Int}[1 / (1 - b*x^2), x], x, x / \sqrt{a + b*x^2}] \text{ ; FreeQ}[\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 1614

```
Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_
.)*(x_))^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[Px, a + b*x, x],
R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
- b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 5238

```
Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{(bcx)}{\dots} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{(bcx)}{\dots} \\
&= -\frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} - \frac{(bcx)}{\dots} \\
&= -\frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3} \\
&= -\frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \sec^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \sec^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \sec^{-1}(cx))}{e^3}
\end{aligned}$$

**Mathematica [A]** time = 0.54, size = 312, normalized size = 1.28

$$\frac{a (c^2d + e) (8d^2 + 12dex^2 + 3e^2x^4) + b (c^2d + e) \sec^{-1}(cx) (8d^2 + 12dex^2 + 3e^2x^4) - bcdex \sqrt{1 - \frac{1}{c^2x^2}} (d + ex^2)}{3e^3 (c^2d + e) (d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.





) $x^2 - 2d$ ) $\sqrt{ex^2 + d}\sqrt{-d} + 8d^2/x^4 + 2(8ac^3d^3 + 8ac^3d^2e + 3(a^3c^3d^2e^2 + a^3c^3e^3)x^4 + 12(a^3c^3d^2e + a^3c^3d^2e^2)x^2 + (8b^3c^3d^3 + 8b^3c^3d^2e + 3(b^3c^3d^2e^2 + b^3c^3e^3)x^4 + 12(b^3c^3d^2e + b^3c^3d^2e^2)x^2)\arccsc(cx) - (b^3c^3d^2e^2x^2 + b^3c^3d^2e)\sqrt{c^2x^2 - 1})\sqrt{ex^2 + d})/(c^3d^3e^3 + c^3d^2e^4 + (c^3d^3e^5 + c^3e^6)x^4 + 2(c^3d^2e^4 + c^3d^2e^5)x^2), 1/6(8(b^3c^3d^3 + b^3c^3d^2e + (b^3c^3d^2e^2 + b^3c^3e^3)x^4 + 2(b^3c^3d^2e + b^3c^3d^2e^2)x^2)\sqrt{d}\arctan(-1/2\sqrt{c^2x^2 - 1})((c^2d - e)x^2 - 2d)\sqrt{ex^2 + d}\sqrt{d}/(c^2d^2e^3x^4 + (c^2d^2 - d^2e)x^2 - d^2)) + 3(b^3c^2d^3 + (b^3c^2d^2e^2 + b^3e^3)x^4 + b^3d^2e + 2(b^3c^2d^2e + b^3d^2e^2)x^2)\sqrt{-e}\arctan(1/2(2c^2e^2x^2 + c^2d - e)\sqrt{c^2x^2 - 1})\sqrt{ex^2 + d}\sqrt{-e}/(c^3e^2x^4 - c^3d^2e + (c^3d^2e - c^3e^2)x^2)) + 2(8ac^3d^3 + 8ac^3d^2e + 3(a^3c^3d^2e^2 + a^3c^3e^3)x^4 + 12(a^3c^3d^2e + a^3c^3d^2e^2)x^2 + (8b^3c^3d^3 + 8b^3c^3d^2e + 3(b^3c^3d^2e^2 + b^3c^3e^3)x^4 + 12(b^3c^3d^2e + b^3c^3d^2e^2)x^2)\arccsc(cx) - (b^3c^3d^2e^2x^2 + b^3c^3d^2e)\sqrt{c^2x^2 - 1})\sqrt{ex^2 + d})/(c^3d^3e^3 + c^3d^2e^4 + (c^3d^3e^5 + c^3e^6)x^4 + 2(c^3d^2e^4 + c^3d^2e^5)x^2)]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^5(a+b\operatorname{arcsec}(cx))/(ex^2+d)^{(5/2)}$ ), x, algorithm="giac")

[Out] integrate( $(b\operatorname{arcsec}(cx) + a)x^5/(ex^2 + d)^{(5/2)}$ ), x)

**maple** [F] time = 5.33, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int( $x^5(a+b\operatorname{arcsec}(cx))/(ex^2+d)^{(5/2)}$ ), x)

[Out] int( $x^5(a+b\operatorname{arcsec}(cx))/(ex^2+d)^{(5/2)}$ ), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/3\*(3\*x^4/((e\*x^2 + d)^(3/2)\*e) + 12\*d\*x^2/((e\*x^2 + d)^(3/2)\*e^2) + 8\*d^2/((e\*x^2 + d)^(3/2)\*e^3))\*a - 1/3\*(3\*(e^4\*x^2 + d\*e^3)\*sqrt(e\*x^2 + d)\*integrate((3\*c^2\*e^3\*x^7\*log(c) - 3\*e^3\*x^5\*log(c) + (3\*(c^2\*log(c) + c^2)\*e^3\*x^7 + 20\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x + 3\*(5\*c^2\*d\*e^2 - e^3\*log(c))\*x^5)\*e^(log(c\*x + 1) + log(c\*x - 1)) + 3\*(c^2\*e^3\*x^7 - e^3\*x^5 + (c^2\*e^3\*x^7 - e^3\*x^5)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*log(x))/((c^2\*e^5\*x^6 - d^2\*e^3 + (2\*c^2\*d\*e^4 - e^5)\*x^4 + (c^2\*d^2\*e^3 - 2\*d\*e^4)\*x^2 + (c^2\*e^5\*x^6 - d^2\*e^3 + (2\*c^2\*d\*e^4 - e^5)\*x^4 + (c^2\*d^2\*e^3 - 2\*d\*e^4)\*x^2)\*e^(log(c\*x + 1) + log(c\*x - 1)))\*sqrt(e\*x^2 + d)), x) - (3\*e^2\*x^4 + 12\*d\*e\*x^2 + 8\*d^2)\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*b/((e^4\*x^2 + d\*e^3)\*sqrt(e\*x^2 + d))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x^5\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.152 \quad \int \frac{x^3(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=163

$$-\frac{a+b \sec^{-1}(cx)}{e^2 \sqrt{d+ex^2}} + \frac{d(a+b \sec^{-1}(cx))}{3e^2 (d+ex^2)^{3/2}} - \frac{2bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{3\sqrt{d} e^2 \sqrt{c^2x^2}} + \frac{bcx \sqrt{c^2x^2-1}}{3e \sqrt{c^2x^2} (c^2d+e) \sqrt{d+ex^2}}$$

[Out] 1/3\*d\*(a+b\*arcsec(c\*x))/e^2/(e\*x^2+d)^(3/2)-2/3\*b\*c\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(c^2\*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2\*x^2)^(1/2)+(-a-b\*arcsec(c\*x))/e^2/(e\*x^2+d)^(1/2)+1/3\*b\*c\*x\*(c^2\*x^2-1)^(1/2)/e/(c^2\*d+e)/(c^2\*x^2)^(1/2)/(e\*x^2+d)^(1/2)

**Rubi [A]** time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {266, 43, 5238, 12, 573, 152, 93, 204}

$$-\frac{a+b \sec^{-1}(cx)}{e^2 \sqrt{d+ex^2}} + \frac{d(a+b \sec^{-1}(cx))}{3e^2 (d+ex^2)^{3/2}} - \frac{2bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{3\sqrt{d} e^2 \sqrt{c^2x^2}} + \frac{bcx \sqrt{c^2x^2-1}}{3e \sqrt{c^2x^2} (c^2d+e) \sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*x\*Sqrt[-1 + c^2\*x^2])/(3\*e\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*Sqrt[d + e\*x^2]) + (d\*(a + b\*ArcSec[c\*x]))/(3\*e^2\*(d + e\*x^2)^(3/2)) - (a + b\*ArcSec[c\*x])/(e^2\*Sqrt[d + e\*x^2]) - (2\*b\*c\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*Sqrt[d]\*e^2\*Sqrt[c^2\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 93

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/Rt[-a, 2]*Rt[-b, 2], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
```

x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-2d - 3ex^2}{3e^2 x \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}} dx}{\sqrt{c^2 x^2}} \\
 &= \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-2d - 3ex^2}{x \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}} dx}{3e^2 \sqrt{c^2 x^2}} \\
 &= \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \text{Subst} \left( \int \frac{-2d - 3ex}{x \sqrt{-1 + c^2 x} (d + ex)^{3/2}} dx, x, x^2 \right)}{6e^2 \sqrt{c^2 x^2}} \\
 &= \frac{bcx \sqrt{-1 + c^2 x^2}}{3e (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \text{Subst}}{3de} \\
 &= \frac{bcx \sqrt{-1 + c^2 x^2}}{3e (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \text{Subst}}{e^2 \sqrt{d + ex^2}} \\
 &= \frac{bcx \sqrt{-1 + c^2 x^2}}{3e (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(2bcx) \text{Subst}}{e^2 \sqrt{d + ex^2}} \\
 &= \frac{bcx \sqrt{-1 + c^2 x^2}}{3e (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \sec^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \sec^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{2bcx \tan^{-1} \left( \frac{\sqrt{-1 + c^2 x^2}}{\sqrt{d + ex^2}} \right)}{3\sqrt{d} e^2 \sqrt{c^2 x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.27, size = 172, normalized size = 1.06

$$\frac{-a(c^2 d + e)(2d + 3ex^2) + bcex \sqrt{1 - \frac{1}{c^2 x^2}} (d + ex^2) - b(c^2 d + e) \sec^{-1}(cx)(2d + 3ex^2)}{3e^2 (c^2 d + e) (d + ex^2)^{3/2}} + \frac{2bcx \sqrt{1 - \frac{1}{c^2 x^2}} \tan^{-1} \left( \frac{\sqrt{-1 + c^2 x^2}}{\sqrt{d + ex^2}} \right)}{3\sqrt{d} e^2 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + e\*x^2) - a\*(c^2\*d + e)\*(2\*d + 3\*e\*x^2) - b\*(c^2\*d + e)\*(2\*d + 3\*e\*x^2)\*ArcSec[c\*x])/(3\*e^2\*(c^2\*d + e)\*(d + e\*x^2)^(3/2)) + (2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(3\*Sqrt[d]\*e^2\*Sqrt[-1 + c^2\*x^2])

**fricas** [B] time = 0.98, size = 664, normalized size = 4.07

$$\left[ \frac{(bc^2d^3 + (bc^2de^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bde^2)x^2)\sqrt{-d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 - 4\sqrt{c^2x^2 - 1}((c^2d - e)x^2 - 2d)\sqrt{e*x^2 + d}\sqrt{-d} + 8*d^2)/x^4}{x^4}\right)}{6(c^2d^2e^2 + d^3e^3 + (c^2d^2e^4 + d^2e^5)x^4 + 2(c^2d^3e^3 + d^2e^4)x^2)}, - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] [-1/6\*((b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + b\*e^3)\*x^4 + b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 2\*(2\*a\*c^2\*d^3 + 2\*a\*d^2\*e + 3\*(a\*c^2\*d^2\*e + a\*d\*e^2)\*x^2 + (2\*b\*c^2\*d^3 + 2\*b\*d^2\*e + 3\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*arcsec(c\*x) - (b\*d\*e^2\*x^2 + b\*d^2\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^2\*d^4\*e^2 + d^3\*e^3 + (c^2\*d^2\*e^4 + d^2\*e^5)\*x^4 + 2\*(c^2\*d^3\*e^3 + d^2\*e^4)\*x^2), - 1/3\*((b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + b\*e^3)\*x^4 + b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (2\*a\*c^2\*d^3 + 2\*a\*d^2\*e + 3\*(a\*c^2\*d^2\*e + a\*d\*e^2)\*x^2 + (2\*b\*c^2\*d^3 + 2\*b\*d^2\*e + 3\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*arcsec(c\*x) - (b\*d\*e^2\*x^2 + b\*d^2\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^2\*d^4\*e^2 + d^3\*e^3 + (c^2\*d^2\*e^4 + d^2\*e^5)\*x^4 + 2\*(c^2\*d^3\*e^3 + d^2\*e^4)\*x^2)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^3/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 4.63, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)`

[Out] `int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a \left( \frac{3x^2}{(ex^2 + d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2 + d)^{\frac{3}{2}}e^2} \right) + b \int \frac{x^3 \arctan(\sqrt{cx+1}\sqrt{cx-1})}{(e^2x^4 + 2dex^2 + d^2)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] `-1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*integrate(x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)`

[Out] `int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{asec}(cx))}{(d + ex^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)`

[Out] `Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)`



$$3.153 \quad \int \frac{x(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=138

$$\frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} - \frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

[Out] 1/3\*(-a-b\*arcsec(c\*x))/e/(e\*x^2+d)^(3/2)-1/3\*b\*c\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(c^2\*x^2-1)^(1/2))/d^(3/2)/e/(c^2\*x^2)^(1/2)-1/3\*b\*c\*x\*(c^2\*x^2-1)^(1/2)/d/(c^2\*d+e)/(c^2\*x^2)^(1/2)/(e\*x^2+d)^(1/2)

**Rubi [A]** time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {5236, 446, 96, 93, 204}

$$\frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} - \frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] -(b\*c\*x\*sqrt[-1 + c^2\*x^2])/(3\*d\*(c^2\*d + e)\*sqrt[c^2\*x^2]\*sqrt[d + e\*x^2]) - (a + b\*ArcSec[c\*x])/(3\*e\*(d + e\*x^2)^(3/2)) - (b\*c\*x\*ArcTan[sqrt[d + e\*x^2]/(sqrt[d]\*sqrt[-1 + c^2\*x^2])])/(3\*d^(3/2)\*e\*sqrt[c^2\*x^2])

### Rule 93

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 96

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)], x]
```

```
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 5236

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSec[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{3e\sqrt{c^2x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex)^{3/2}} dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}} dx, x, x^2\right)}{6de\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{-d-x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3de\sqrt{c^2x^2}} \\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a + b \sec^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 158, normalized size = 1.14

$$\frac{-ad(c^2d + e) - bcex\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) - bd(c^2d + e)\sec^{-1}(cx)}{3de(c^2d + e)(d + ex^2)^{3/2}} + \frac{bcx\sqrt{1 - \frac{1}{c^2x^2}}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out]  $(-(a*d*(c^2*d + e)) - b*c*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - b*d*(c^2*d + e)*\text{ArcSec}[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])/(\text{Sqrt}[d + e*x^2])])/(3*d^(3/2)*e*\text{Sqrt}[-1 + c^2*x^2])$

**fricas [B]** time = 1.54, size = 571, normalized size = 4.14

$$\left[ \frac{(bc^2d^3 + (bc^2de^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bde^2)x^2)\sqrt{-d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 - 4\sqrt{c^2x^2-1}((c^2d - e))}{x^4}\right)}{12(c^2d^5e + d^4e^2 + (c^2d^3e^3 + a\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + \\ & b*d*e^2)*x^2)*\sqrt{-d}*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - \\ & d*e)*x^2 - 4*\sqrt{c^2*x^2 - 1}*((c^2*d - e)*x^2 - 2*d)*\sqrt{e*x^2 + d})*\sqrt{t(-d) + 8*d^2}/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*\text{arcsec} \\ & (c*x) + (b*d*e^2*x^2 + b*d^2*e)*\sqrt{c^2*x^2 - 1})*\sqrt{e*x^2 + d})/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2 \\ & ), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + \\ & b*d*e^2)*x^2)*\sqrt{d}*\arctan(-1/2*\sqrt{c^2*x^2 - 1}*((c^2*d - e)*x^2 - 2*d)*\sqrt{e*x^2 + d})*\sqrt{d}/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 2*( \\ & a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*\text{arcsec}(c*x) + (b*d*e^2*x^2 + b*d^2*e)*\sqrt{c^2*x^2 - 1})*\sqrt{e*x^2 + d})/(c^2*d^5*e + d^4*e^2 + (c^2*d^3* \\ & e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)] \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 3.47, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{x \arctan(\sqrt{cx+1} \sqrt{cx-1})}{(e^2x^4 + 2dex^2 + d^2)\sqrt{ex^2 + d}} dx - \frac{a}{3(ex^2 + d)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] b\*integrate(x\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)), x) - 1/3\*a/((e\*x^2 + d)^(3/2)\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + b \operatorname{asec}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Integral(x\*(a + b\*asec(c\*x))/(d + e\*x\*\*2)\*\*(5/2), x)

$$3.154 \quad \int \frac{a+b \sec^{-1}(cx)}{x(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(5/2), x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{5/2}} dx = \int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 26.80, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{e^3 x^7 + 3 d e^2 x^5 + 3 d^2 e x^3 + d^3 x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(5/2)\*x), x)

**maple** [A] time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x (ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(5/2),x)

[Out] int((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a \left( \frac{3 \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{5}{2}}} - \frac{3}{\sqrt{ex^2 + d} d^2} - \frac{1}{(ex^2 + d)^{\frac{3}{2}} d} \right) + b \int \frac{\arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{(e^2x^5 + 2dex^3 + d^2x)\sqrt{ex^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*a\*(3\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(5/2) - 3/(sqrt(e\*x^2 + d)\*d^2) - 1/((e\*x^2 + d)^(3/2)\*d)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x)\*sqrt(e\*x^2 + d), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x (ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**(5/2), x)
```

```
[Out] Timed out
```



$$3.155 \quad \int \frac{a+b \sec^{-1}(cx)}{x^3(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(5/2), x)

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx = \int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 33.12, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a)}{e^3 x^9 + 3 d e^2 x^7 + 3 d^2 e x^5 + d^3 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^3\*x^9 + 3\*d\*e^2\*x^7 + 3\*d^2\*e\*x^5 + d^3\*x^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(5/2)\*x^3), x)

**maple** [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^3 (ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(5/2),x)

[Out] int((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} a \left( \frac{15 e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{7}{2}}} - \frac{15 e}{\sqrt{ex^2 + d} d^3} - \frac{5 e}{(ex^2 + d)^{\frac{3}{2}} d^2} - \frac{3}{(ex^2 + d)^{\frac{3}{2}} dx^2} \right) + b \int \frac{\arctan(\sqrt{cx + 1} \sqrt{cx - 1})}{(e^2 x^7 + 2 dex^5 + d^2 x^3) \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^3/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/6\*a\*(15\*e\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(7/2) - 15\*e/(sqrt(e\*x^2 + d)\*d^3) - 5\*e/((e\*x^2 + d)^(3/2)\*d^2) - 3/((e\*x^2 + d)^(3/2)\*d\*x^2)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e^2\*x^7 + 2\*d\*e\*x^5 + d^2\*x^3)\*sqrt(e\*x^2 + d)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^3 (ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x^3\*(d + e\*x^2)^(5/2)),x)

[Out] int((a + b\*acos(1/(c\*x)))/(x^3\*(d + e\*x^2)^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/x\*\*3/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.156 \quad \int \frac{x^6 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{x^6 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable(x^6\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2), x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^6 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^6\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Defer[Int] [(x^6\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^6 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^6 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 10.32, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^6\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Integrate[(x^6\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

fricas [A] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^6 \operatorname{arcsec}(cx) + ax^6) \sqrt{ex^2 + d}}{e^3 x^6 + 3 de^2 x^4 + 3 d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral((b\*x^6\*arcsec(c\*x) + a\*x^6)\*sqrt(e\*x^2 + d)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^6}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^6/(e\*x^2 + d)^(5/2), x)

**maple** [A] time = 3.69, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^6\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} \left( \frac{3x^5}{(ex^2 + d)^{\frac{3}{2}}e} + \frac{5dx \left( \frac{3x^2}{(ex^2+d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2+d)^{\frac{3}{2}}e^2} \right)}{e} + \frac{5dx}{\sqrt{ex^2 + d}e^3} - \frac{15d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{7}{2}}} \right) a + b \int \frac{x^6 \arctan(\sqrt{cx + 1})}{(e^2x^4 + 2dex^2 + d^2)\sqrt{e^2x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/6\*(3\*x^5/((e\*x^2 + d)^(3/2)\*e) + 5\*d\*x\*(3\*x^2/((e\*x^2 + d)^(3/2)\*e) + 2\*d/((e\*x^2 + d)^(3/2)\*e^2))/e + 5\*d\*x/(sqrt(e\*x^2 + d)\*e^3) - 15\*d\*arcsinh(e\*x/sqrt(d\*e))/e^(7/2))\*a + b\*integrate(x^6\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

[Out] int((x^6\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(5/2), x)

[Out] Timed out

$$3.157 \quad \int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2), x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Defer[Int] [(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 9.67, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Integrate[(x^4\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

fricas [A] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^4 \text{arcsec}(cx) + ax^4) \sqrt{ex^2 + d}}{e^3 x^6 + 3 de^2 x^4 + 3 d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral((b\*x^4\*arcsec(c\*x) + a\*x^4)\*sqrt(e\*x^2 + d)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*x^4/(e\*x^2 + d)^(5/2), x)

**maple** [A] time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left( x \left( \frac{3x^2}{(ex^2 + d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2 + d)^{\frac{3}{2}}e^2} \right) + \frac{x}{\sqrt{ex^2 + d}e^2} - \frac{3 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}} \right) a + b \int \frac{x^4 \arctan\left(\frac{\sqrt{cx+1}\sqrt{cx-1}}{(e^2x^4 + 2dex^2 + d^2)\sqrt{ex^2 + d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*(x\*(3\*x^2/((e\*x^2 + d)^(3/2)\*e) + 2\*d/((e\*x^2 + d)^(3/2)\*e^2)) + x/(sqrt(e\*x^2 + d)\*e^2) - 3\*arcsinh(e\*x/sqrt(d\*e))/e^(5/2))\*a + b\*integrate(x^4\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)), x)



mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4 \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

[Out] int((x^4\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(5/2), x)

[Out] Timed out

$$3.158 \quad \int \frac{x^2(a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=276

$$\frac{x^3(a+b \sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F(\sin^{-1}(cx)|-\frac{e}{c^2d})}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

[Out]  $1/3*x^3*(a+b*\text{arcsec}(c*x))/d/(e*x^2+d)^{(3/2)}-1/3*b*c*x^2*(c^2*x^2-1)^{(1/2)}/d/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(e*x^2+d)^{(1/2)}+1/3*b*c^2*x*\text{EllipticE}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/e/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}-1/3*b*x*\text{EllipticF}(c*x,(-e/c^2/d)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d/e/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.28, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {264, 5238, 12, 471, 423, 427, 426, 424, 421, 419}

$$\frac{x^3(a+b \sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F(\sin^{-1}(cx)|-\frac{e}{c^2d})}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(a + b*\text{ArcSec}[c*x]))/(d + e*x^2)^{(5/2)}, x]$

[Out]  $-(b*c*x^2*\text{Sqrt}[-1 + c^2*x^2])/(3*d*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[d + e*x^2]) + (x^3*(a + b*\text{ArcSec}[c*x]))/(3*d*(d + e*x^2)^{(3/2)}) + (b*c^2*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(3*d*e*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]) - (b*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + (e*x^2)/d]*\text{EllipticF}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(3*d*e*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])$

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 264

$\text{Int}[((c_*)(x_))^{(m_)}*((a_*) + (b_*)(x_)^{(n_))^{(p_)}], x\_Symbol] \rightarrow \text{Simp}[((c*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n,$

p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

#### Rule 421

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/(Sqrt[a + b\*x^2]\*Sqrt[1 + (d\*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

#### Rule 423

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Dist[b/d, Int[Sqrt[c + d\*x^2]/Sqrt[a + b\*x^2], x], x] - Dist[(b\*c - a\*d)/d, Int[1/(Sqrt[a + b\*x^2]\*Sqrt[c + d\*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]

#### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 426

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[a + b\*x^2]/Sqrt[1 + (b\*x^2)/a], Int[Sqrt[1 + (b\*x^2)/a]/Sqrt[c + d\*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

#### Rule 427

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[Sqrt[a + b\*x^2]/Sqrt[1 + (d\*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]

#### Rule 471

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Simp[(e^(n - 1)\*(e\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1)

```

*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

```

### Rule 5238

```

Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \sec^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{x^2}{3d \sqrt{-1+c^2x^2} (d+ex^2)^{3/2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{x^2}{\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}} dx}{3d \sqrt{c^2x^2}} \\
&= -\frac{bcx^2 \sqrt{-1+c^2x^2}}{3d (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{(bcx) \int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}} dx}{3d (c^2d + e) \sqrt{c^2x^2}} \\
&= -\frac{bcx^2 \sqrt{-1+c^2x^2}}{3d (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{1}{\sqrt{-1+c^2x^2} \sqrt{d+ex^2}} dx}{3de \sqrt{c^2x^2}} \\
&= -\frac{bcx^2 \sqrt{-1+c^2x^2}}{3d (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{(bc^3x \sqrt{1 - c^2x^2}) \int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}} dx}{3de (c^2d + e) \sqrt{c^2x^2} \sqrt{-1+c^2x^2}} \\
&= -\frac{bcx^2 \sqrt{-1+c^2x^2}}{3d (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{(bc^3x \sqrt{1 - c^2x^2} \sqrt{d + ex^2})}{3de (c^2d + e) \sqrt{c^2x^2} \sqrt{-1+c^2x^2}} \\
&= -\frac{bcx^2 \sqrt{-1+c^2x^2}}{3d (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \sec^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{bc^2x \sqrt{1 - c^2x^2} \sqrt{d + ex^2} E}{3de (c^2d + e) \sqrt{c^2x^2} \sqrt{-1+c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.29, size = 186, normalized size = 0.67

$$\frac{x^2 \left( ax (c^2d + e) - bc \sqrt{1 - \frac{1}{c^2x^2}} (d + ex^2) + bx (c^2d + e) \sec^{-1}(cx) \right)}{3d (c^2d + e) (d + ex^2)^{3/2}} + \frac{bcx \sqrt{1 - \frac{1}{c^2x^2}} \sqrt{\frac{ex^2}{d} + 1} E \left( \sin^{-1} \left( \sqrt{-\frac{e}{d}} x \right) \right)}{3d \sqrt{1 - c^2x^2} \sqrt{-\frac{e}{d}} (c^2d + e) \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (x^2\*(a\*(c^2\*d + e)\*x - b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(d + e\*x^2) + b\*(c^2\*d + e)\*x\*ArcSec[c\*x]))/(3\*d\*(c^2\*d + e)\*(d + e\*x^2)^(3/2)) + (b\*c\*Sqrt[1 - 1/(c

$\wedge 2 * x^2) * x * \text{Sqrt}[1 + (e * x^2) / d] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[-(e / d)] * x], -((c^2 * d) / e)] / (3 * d * \text{Sqrt}[-(e / d)] * (c^2 * d + e) * \text{Sqrt}[1 - c^2 * x^2] * \text{Sqrt}[d + e * x^2])$

**fricas** [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^2 \operatorname{arcsec}(cx) + ax^2) \sqrt{ex^2 + d}}{e^3 x^6 + 3de^2 x^4 + 3d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)`

**maple** [F] time = 3.15, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)`

[Out] `int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a \left( \frac{x}{(ex^2 + d)^{\frac{3}{2}} e} - \frac{x}{\sqrt{ex^2 + d} de} \right) + b \int \frac{x^2 \arctan(\sqrt{cx + 1} \sqrt{cx - 1})}{(e^2 x^4 + 2dex^2 + d^2) \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out]  $-1/3*a*(x/((e*x^2 + d)^{(3/2)*e}) - x/(\sqrt{e*x^2 + d}*d*e)) + b*\text{integrate}(x^2*\arctan(\sqrt{c*x + 1}*\sqrt{c*x - 1})/((e^2*x^4 + 2*d*e*x^2 + d^2)*\sqrt{e*x^2 + d}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{(e x^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x^2\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.159 \quad \int \frac{a+b \sec^{-1}(cx)}{(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=296

$$\frac{2x(a+b \sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b \sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} - \frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F(\sin^{-1}(cx)|-)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

[Out] 1/3\*x\*(a+b\*arcsec(c\*x))/d/(e\*x^2+d)^(3/2)+2/3\*x\*(a+b\*arcsec(c\*x))/d^2/(e\*x^2+d)^(1/2)+1/3\*b\*c\*e\*x^2\*(c^2\*x^2-1)^(1/2)/d^2/(c^2\*d+e)/(c^2\*x^2)^(1/2)/(e\*x^2+d)^(1/2)-1/3\*b\*c^2\*x\*EllipticE(c\*x,(-e/c^2/d)^(1/2))\*(-c^2\*x^2+1)^(1/2)\*(e\*x^2+d)^(1/2)/d^2/(c^2\*d+e)/(c^2\*x^2)^(1/2)/(c^2\*x^2-1)^(1/2)/(1+e\*x^2/d)^(1/2)-2/3\*b\*x\*EllipticF(c\*x,(-e/c^2/d)^(1/2))\*(-c^2\*x^2+1)^(1/2)\*(1+e\*x^2/d)^(1/2)/d^2/(c^2\*x^2)^(1/2)/(c^2\*x^2-1)^(1/2)/(e\*x^2+d)^(1/2)

**Rubi [A]** time = 0.24, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$ , Rules used = {192, 191, 5228, 12, 527, 524, 427, 426, 424, 421, 419}

$$\frac{2x(a+b \sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b \sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} - \frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F(\sin^{-1}(cx)|-)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*e\*x^2\*sqrt[-1 + c^2\*x^2])/(3\*d^2\*(c^2\*d + e)\*sqrt[c^2\*x^2]\*sqrt[d + e\*x^2]) + (x\*(a + b\*ArcSec[c\*x]))/(3\*d\*(d + e\*x^2)^(3/2)) + (2\*x\*(a + b\*ArcSec[c\*x]))/(3\*d^2\*sqrt[d + e\*x^2]) - (b\*c^2\*x\*sqrt[1 - c^2\*x^2]\*sqrt[d + e\*x^2]\*EllipticE[ArcSin[c\*x], -(e/(c^2\*d))])/(3\*d^2\*(c^2\*d + e)\*sqrt[c^2\*x^2]\*sqrt[-1 + c^2\*x^2]\*sqrt[1 + (e\*x^2)/d]) - (2\*b\*x\*sqrt[1 - c^2\*x^2]\*sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(3\*d^2\*sqrt[c^2\*x^2]\*sqrt[-1 + c^2\*x^2]\*sqrt[d + e\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]



Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)
)/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
&& NeQ[p, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler  
SqrtQ[-(b/a), -(d/c)]))))))

### Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

### Rule 5228

```
Int[((a_.) + ArcSec[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{5/2}} dx &= \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{3d+2ex^2}{3d^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{3d+2ex^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{3d^2\sqrt{c^2x^2}} \\
&= \frac{bcex^2\sqrt{-1 + c^2x^2}}{3d^2(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} + \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{d}{\sqrt{-1+c^2x^2}} dx}{3d^3} \\
&= \frac{bcex^2\sqrt{-1 + c^2x^2}}{3d^2(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} + \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(2bcx) \int \frac{1}{\sqrt{-1+c^2x^2}} dx}{3d^2} \\
&= \frac{bcex^2\sqrt{-1 + c^2x^2}}{3d^2(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} + \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bc^3x\sqrt{1 - 1/c^2x^2})}{3d^2(c^2d + e)} \\
&= \frac{bcex^2\sqrt{-1 + c^2x^2}}{3d^2(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} + \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bc^3x\sqrt{1 - 1/c^2x^2})}{3d^2(c^2d + e)} \\
&= \frac{bcex^2\sqrt{-1 + c^2x^2}}{3d^2(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} + \frac{x(a + b \sec^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \sec^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{bc^2x\sqrt{1 - 1/c^2x^2}}{3d^2(c^2d + e)}
\end{aligned}$$

**Mathematica [C]** time = 0.55, size = 248, normalized size = 0.84

$$\frac{x \left( a(c^2d + e)(3d + 2ex^2) + bcex\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) + b(c^2d + e)\sec^{-1}(cx)(3d + 2ex^2) \right) - ibcx\sqrt{1 - \frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}}}{3d^2(c^2d + e)(d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(d + e\*x^2)^(5/2), x]

[Out] (x\*(b\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + e\*x^2) + a\*(c^2\*d + e)\*(3\*d + 2\*e\*x^2) + b\*(c^2\*d + e)\*(3\*d + 2\*e\*x^2)\*ArcSec[c\*x])/(3\*d^2\*(c^2\*d + e)\*(d + e\*x^2)^(5/2)) - (b\*c\*x\*Sqrt[1 - 1/(c^2\*x^2)]\*Sqrt[e\*x^2/d])/d

$x^2)^{(3/2)} - ((I/3)*b*c*sqrt[1 - 1/(c^2*x^2)]*x*sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(sqrt[-c^2]*d^2*(c^2*d + e)*sqrt[1 - c^2*x^2]*sqrt[d + e*x^2])$

**fricas** [F] time = 2.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 2.31, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int((a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a \left( \frac{2x}{\sqrt{ex^2 + d}d^2} + \frac{x}{(ex^2 + d)^{\frac{3}{2}}d} \right) + b \int \frac{\arctan(\sqrt{cx + 1}\sqrt{cx - 1})}{(e^2x^4 + 2dex^2 + d^2)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/3\*a\*(2\*x/(sqrt(e\*x^2 + d)\*d^2) + x/((e\*x^2 + d)^(3/2)\*d)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^(5/2),x)

[Out] int((a + b\*acos(1/(c\*x)))/(d + e\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*asec(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.160 \quad \int \frac{a+b \sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=631

$$\frac{8ex(a+b \sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{4ex(a+b \sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} - \frac{a+b \sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} - \frac{4bce^2x^2\sqrt{c^2x^2-1}}{3d^3\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} + \frac{bc\sqrt{c^2x^2-1}(c^2d+e)}{d^3\sqrt{c^2x^2}}$$

[Out]  $(-a-b*\text{arcsec}(c*x))/d/x/(e*x^2+d)^{(3/2)}-4/3*e*x*(a+b*\text{arcsec}(c*x))/d^2/(e*x^2+d)^{(3/2)}-8/3*e*x*(a+b*\text{arcsec}(c*x))/d^3/(e*x^2+d)^{(1/2)}-b*c*e*(c^2*x^2-1)^{(1/2)}/d^2/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(e*x^2+d)^{(1/2)}-4/3*b*c*e^2*x^2*(c^2*x^2-1)^{(1/2)}/d^3/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(e*x^2+d)^{(1/2)}+b*c*(c^2*d+2*e)*(c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*d+e)/(c^2*x^2)^{(1/2)}+4/3*b*c^2*e*x*\text{EllipticE}(c*x,(-e/c^2/d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}-b*c^2*(c^2*d+2*e)*x*\text{EllipticE}(c*x,(-e/c^2/d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(c^2*d+e)/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(1+e*x^2/d)^{(1/2)}+b*c^2*x*\text{EllipticF}(c*x,(-e/c^2/d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^2/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)}+8/3*b*e*x*\text{EllipticF}(c*x,(-e/c^2/d)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(1+e*x^2/d)^{(1/2)}/d^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/(e*x^2+d)^{(1/2)})$

**Rubi [A]** time = 1.40, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 18, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$ , Rules used = {271, 192, 191, 5238, 12, 6742, 414, 21, 427, 426, 424, 472, 583, 524, 421, 419, 471, 423}

$$\frac{8ex(a+b \sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{4ex(a+b \sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} - \frac{a+b \sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} - \frac{4bce^2x^2\sqrt{c^2x^2-1}}{3d^3\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} + \frac{bc\sqrt{c^2x^2-1}(c^2d+e)}{d^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcSec}[c*x])/(x^2*(d + e*x^2)^{(5/2))}, x]$

[Out]  $-((b*c*e*\text{Sqrt}[-1 + c^2*x^2])/(d^2*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[d + e*x^2])) - (4*b*c*e^2*x^2*\text{Sqrt}[-1 + c^2*x^2])/(3*d^3*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[d + e*x^2]) + (b*c*(c^2*d + 2*e)*\text{Sqrt}[-1 + c^2*x^2]*\text{Sqrt}[d + e*x^2])/(d^3*(c^2*d + e)*\text{Sqrt}[c^2*x^2]) - (a + b*\text{ArcSec}[c*x])/(d*x*(d + e*x^2)^{(3/2)}) - (4*e*x*(a + b*\text{ArcSec}[c*x]))/(3*d^2*(d + e*x^2)^{(3/2)}) - (8*e*x*(a + b*\text{ArcSec}[c*x]))/(3*d^3*\text{Sqrt}[d + e*x^2]) + (4*b*c^2*e*x*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcSin}[c*x], -(e/(c^2*d))])/(3*d^3*(c^2*d + e)*\text{Sqrt}[c^2*x^2])$

$$x^2 \sqrt{-1 + c^2 x^2} \sqrt{1 + (e x^2)/d} - (b c^2 (c^2 d + 2e) x \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \text{EllipticE}[\text{ArcSin}[c x], -(e/(c^2 d))]) / (d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{1 + (e x^2)/d}) + (b c^2 x \sqrt{1 - c^2 x^2} \sqrt{1 + (e x^2)/d} \text{EllipticF}[\text{ArcSin}[c x], -(e/(c^2 d))]) / (d^2 \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}) + (8 b e x \sqrt{1 - c^2 x^2} \sqrt{1 + (e x^2)/d} \text{EllipticF}[\text{ArcSin}[c x], -(e/(c^2 d))]) / (3 d^3 \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2})$$
Rule 12

$$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$$
Rule 21

$$\text{Int}[(u_*)((a_*) + (b_*)(v_))^{(m_)}*((c_*) + (d_*)(v_))^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (\text{!IntegerQ}[n] \parallel \text{SimplerQ}[c + d*x, a + b*x])$$
Rule 191

$$\text{Int}[(a_*) + (b_*)(x_*)^{(n_)}]^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^{(p+1)})/a, x] /; \text{FreeQ}[\{a, b, n, p\}, x] \&\& \text{EqQ}[1/n + p + 1, 0]$$
Rule 192

$$\text{Int}[(a_*) + (b_*)(x_*)^{(n_)}]^{(p_)}, x\_Symbol] \rightarrow -\text{Simp}[(x*(a + b*x^n)^{(p+1)})/(a*n*(p+1)), x] + \text{Dist}[(n*(p+1) + 1)/(a*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, n, p\}, x] \&\& \text{ILtQ}[\text{Simplify}[1/n + p + 1], 0] \&\& \text{NeQ}[p, -1]$$
Rule 271

$$\text{Int}[(x_*)^{(m_)}*((a_*) + (b_*)(x_*)^{(n_)}]^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(x^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1) + 1))/(a*(m+1)), \text{Int}[x^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{ILtQ}[\text{Simplify}[(m+1)/n + p + 1], 0] \&\& \text{NeQ}[m, -1]$$
Rule 414

$$\text{Int}[(a_*) + (b_*)(x_*)^{(n_)}]^{(p_)}*((c_*) + (d_*)(x_*)^{(n_)}]^{(q_)}, x\_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q * \text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1]$$

```
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

#### Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

#### Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

#### Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

#### Rule 471



```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

```

### Rule 472

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

```

### Rule 524

```

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler SqrtQ[-(b/a), -(d/c)]))))))

```

### Rule 583

```

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

```

### Rule 5238

```

Int[((a_) + ArcSec[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2

```

\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^{-1}(cx)}{x^2 (d + ex^2)^{5/2}} dx &= -\frac{a + b \sec^{-1}(cx)}{dx (d + ex^2)^{3/2}} - \frac{4ex (a + b \sec^{-1}(cx))}{3d^2 (d + ex^2)^{3/2}} - \frac{8ex (a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-3d^2 - 12dex}{3d^3 x^2 \sqrt{-1 + c^2 x^2}}}{\sqrt{c^2 x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{dx (d + ex^2)^{3/2}} - \frac{4ex (a + b \sec^{-1}(cx))}{3d^2 (d + ex^2)^{3/2}} - \frac{8ex (a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-3d^2 - 12dex}{x^2 \sqrt{-1 + c^2 x^2}}}{3d^3 \sqrt{c^2 x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{dx (d + ex^2)^{3/2}} - \frac{4ex (a + b \sec^{-1}(cx))}{3d^2 (d + ex^2)^{3/2}} - \frac{8ex (a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} - \frac{(bcx) \int \left( -\frac{12a}{\sqrt{-1 + c^2 x^2}} \right)}{\sqrt{c^2 x^2}} \\
&= -\frac{a + b \sec^{-1}(cx)}{dx (d + ex^2)^{3/2}} - \frac{4ex (a + b \sec^{-1}(cx))}{3d^2 (d + ex^2)^{3/2}} - \frac{8ex (a + b \sec^{-1}(cx))}{3d^3 \sqrt{d + ex^2}} + \frac{(bcx) \int \frac{1}{x^2 \sqrt{-1 + c^2 x^2}}}{d \sqrt{c^2 x^2}} \\
&= -\frac{bce \sqrt{-1 + c^2 x^2}}{d^2 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{4bce^2 x^2 \sqrt{-1 + c^2 x^2}}{3d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{a + b \sec^{-1}(cx)}{dx (d + ex^2)^{3/2}} - \frac{4ex}{3d^3 \sqrt{d + ex^2}} \\
&= -\frac{bce \sqrt{-1 + c^2 x^2}}{d^2 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{4bce^2 x^2 \sqrt{-1 + c^2 x^2}}{3d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{bc (c^2 d + 2e) \sqrt{-1 + c^2 x^2}}{d^3 (c^2 d + e) \sqrt{c^2 x^2}} \\
&= -\frac{bce \sqrt{-1 + c^2 x^2}}{d^2 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{4bce^2 x^2 \sqrt{-1 + c^2 x^2}}{3d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{bc (c^2 d + 2e) \sqrt{-1 + c^2 x^2}}{d^3 (c^2 d + e) \sqrt{c^2 x^2}} \\
&= -\frac{bce \sqrt{-1 + c^2 x^2}}{d^2 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{4bce^2 x^2 \sqrt{-1 + c^2 x^2}}{3d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{bc (c^2 d + 2e) \sqrt{-1 + c^2 x^2}}{d^3 (c^2 d + e) \sqrt{c^2 x^2}} \\
&= -\frac{bce \sqrt{-1 + c^2 x^2}}{d^2 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{4bce^2 x^2 \sqrt{-1 + c^2 x^2}}{3d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{bc (c^2 d + 2e) \sqrt{-1 + c^2 x^2}}{d^3 (c^2 d + e) \sqrt{c^2 x^2}} \\
&= -\frac{bce \sqrt{-1 + c^2 x^2}}{d^2 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} - \frac{4bce^2 x^2 \sqrt{-1 + c^2 x^2}}{3d^3 (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + ex^2}} + \frac{bc (c^2 d + 2e) \sqrt{-1 + c^2 x^2}}{d^3 (c^2 d + e) \sqrt{c^2 x^2}}
\end{aligned}$$

**Mathematica** [C] time = 0.78, size = 323, normalized size = 0.51

$$\frac{-a(c^2d + e)(3d^2 + 12dex^2 + 8e^2x^4) - b(c^2d + e)\sec^{-1}(cx)(3d^2 + 12dex^2 + 8e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2)(3c^2d^2 + 12c^2dex^2 + 8c^2e^2x^4)}{3d^3x(c^2d + e)(d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^2\*(d + e\*x^2)^(5/2)), x]

[Out] 
$$\begin{aligned} & -(a*(c^2*d + e)*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)) + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(3*c^2*d*(d + e*x^2) + e*(3*d + 2*e*x^2)) - b*(c^2*d + e) \\ & *(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)*\text{ArcSec}[c*x]/(3*d^3*(c^2*d + e)*x*(d + e*x^2)^{(3/2)}) - ((I/3)*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[1 + (e*x^2)/d]*(c^2*d \\ & *(3*c^2*d + 2*e)*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d))] - (3*c^4*d^2 + 11*c^2*d*e + 8*e^2)*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d))]) \\ & )/(\text{Sqrt}[-c^2]*d^3*(c^2*d + e)*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2]) \end{aligned}$$

**fricas** [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsec}(cx) + a)}{e^3x^8 + 3de^2x^6 + 3d^2ex^4 + d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)/(e^3\*x^8 + 3\*d\*e^2\*x^6 + 3\*d^2\*e\*x^4 + d^3\*x^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^2/(e\*x^2+d)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/((e\*x^2 + d)^(5/2)\*x^2), x)

**maple** [F] time = 2.08, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2(e x^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)`

[Out] `int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a \left( \frac{8ex}{\sqrt{ex^2+d}d^3} + \frac{4ex}{(ex^2+d)^{\frac{3}{2}}d^2} + \frac{3}{(ex^2+d)^{\frac{3}{2}}dx} \right) - \frac{\left( 3(d^3ex^3+d^4x)(8e^2x^4\log(c)+12dex^2\log(c)+3d^2\log(c)+(8e^2x^4+12dex^2+d^3e^2x^5+2d^4ex^3+d^5x) \right)}{3(d^3e^2x^5+2d^4ex^3+d^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] `-1/3*a*(8*e*x/(sqrt(e*x^2+d)*d^3) + 4*e*x/((e*x^2+d)^(3/2)*d^2) + 3/((e*x^2+d)^(3/2)*d*x)) - 1/3*(3*(d^3*e*x^3+d^4*x)*sqrt(e*x^2+d)*integrate((3*c^2*d^3*x^2*log(c) - 3*d^3*log(c) - (8*c^2*e^3*x^8 + 20*c^2*d*e^2*x^6 + 15*c^2*d^2*e*x^4 - 3*(c^2*log(c) - c^2)*d^3*x^2 + 3*d^3*log(c))*e^(log(c*x+1) + log(c*x-1)) + 3*(c^2*d^3*x^2 - d^3 + (c^2*d^3*x^2 - d^3)*e^(log(c*x+1) + log(c*x-1)))*log(x))/((c^2*d^3*e^2*x^8 - d^5*x^2 + (2*c^2*d^4*e - d^3*e^2)*x^6 + (c^2*d^5 - 2*d^4*e)*x^4 + (c^2*d^3*e^2*x^8 - d^5*x^2 + (2*c^2*d^4*e - d^3*e^2)*x^6 + (c^2*d^5 - 2*d^4*e)*x^4)*e^(log(c*x+1) + log(c*x-1)))*sqrt(e*x^2+d)), x) + (8*e^2*x^4 + 12*d*e*x^2 + 3*d^2)*arctan(sqrt(c*x+1)*sqrt(c*x-1))*b/((d^3*e*x^3 + d^4*x)*sqrt(e*x^2+d))`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 (ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)),x)`

[Out] `int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)),x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(5/2),x)`

[Out] Timed out

$$3.161 \quad \int (fx)^m (d + ex^2)^3 (a + b \sec^{-1}(cx)) dx$$

**Optimal.** Leaf size=589

$$\frac{d^3(fx)^{m+1} (a + b \sec^{-1}(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3} (a + b \sec^{-1}(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5} (a + b \sec^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7} (a + b \sec^{-1}(cx))}{f^7(m+7)}$$

[Out]  $d^3(fx)^{(1+m)}(a+b\operatorname{arcsec}(cx))/f/(1+m)+3d^2e(fx)^{(3+m)}(a+b\operatorname{arcsec}(cx))/f^3/(3+m)+3d^2e^2(fx)^{(5+m)}(a+b\operatorname{arcsec}(cx))/f^5/(5+m)+e^3(fx)^{(7+m)}(a+b\operatorname{arcsec}(cx))/f^7/(7+m)-b(c^6d^3(2+m)(4+m)(6+m)/(1+m)+e(1+m)(e^2(m^2+8m+15)^2+3c^2d^2e(3+m)^2(m^2+13m+42)+3c^4d^2(m^4+22m^3+179m^2+638m+840))/(m^3+15m^2+71m+105))x(fx)^{(1+m)}\operatorname{hypergeom}([1/2, 1/2+1/2m], [3/2+1/2m], c^2x^2)(-c^2x^2+1)^{(1/2)}/c^5/f/(1+m)/(2+m)/(4+m)/(6+m)/(c^2x^2)^{(1/2)}/(c^2x^2-1)^{(1/2)}-b^2e(e^2(m^2+8m+15)^2+3c^2d^2e(3+m)^2(m^2+13m+42)+3c^4d^2(m^4+22m^3+179m^2+638m+840))x(fx)^{(1+m)}(c^2x^2-1)^{(1/2)}/c^5/f/(6+m)/(m^2+6m+8)/(m^3+15m^2+71m+105)/(c^2x^2)^{(1/2)}-b^2e^2(e(5+m)^2+3c^2d^2e(m^2+13m+42))x(fx)^{(3+m)}(c^2x^2-1)^{(1/2)}/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2x^2)^{(1/2)}-b^2e^3x(fx)^{(5+m)}(c^2x^2-1)^{(1/2)}/c/f^5/(6+m)/(7+m)/(c^2x^2)^{(1/2)}$

**Rubi [A]** time = 2.42, antiderivative size = 570, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {270, 5238, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3} (a + b \sec^{-1}(cx))}{f^3(m+3)} + \frac{d^3(fx)^{m+1} (a + b \sec^{-1}(cx))}{f(m+1)} + \frac{3de^2(fx)^{m+5} (a + b \sec^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7} (a + b \sec^{-1}(cx))}{f^7(m+7)}$$

Antiderivative was successfully verified.

[In] Int[(fx)^m\*(d + e\*x^2)^3\*(a + b\*ArcSec[c\*x]), x]

[Out]  $-((b^2e(e^2(15+8m+m^2)^2+3c^2d^2e(3+m)^2(42+13m+m^2))+3c^4d^2(840+638m+179m^2+22m^3+m^4))x(fx)^{(1+m)}\sqrt{-1+c^2x^2})/(c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}) - (b^2e^2(e(5+m)^2+3c^2d^2e(42+13m+m^2))x(fx)^{(3+m)}\sqrt{-1+c^2x^2})/(c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}) - (b^2e^3x(fx)^{(5+m)}\sqrt{-1+c^2x^2})/(c^3f^5(6+m)(7+m)\sqrt{c^2x^2}) + (d^3(fx)^{(1+m)}(a+b\operatorname{ArcSec}[c*x]))/(f(1+m)) + (3d^2e(fx)^{(3+m)}(a+b\operatorname{ArcSec}[c*x]))/(f^3(3+m)) + (3d^2e^2(fx)^{(5+m)}(a+b\operatorname{ArcSec}[c*x]))/(f^5(5+m)) + (e^3(fx)^{(7+m)}(a+b\operatorname{ArcSec}[c*x]))/(f^7(7+m))$

$$7*(7 + m) - (b*c*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m))) * x*(f*x)^(1 + m) * \text{Sqrt}[1 - c^2*x^2] * \text{Hypergeometric2F1}[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) / (f * \text{Sqrt}[c^2*x^2] * \text{Sqrt}[-1 + c^2*x^2])$$

### Rule 270

$$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] \text{ ; FreeQ}\{a, b, c, m, n\}, x\} \&\& \text{IGtQ}[p, 0]$$

### Rule 364

$$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{m+1} * \text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a]) / (c*(m+1)), x] \text{ ; FreeQ}\{a, b, c, m, n, p\}, x\} \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILtQ}[p, 0] \text{ || GtQ}[a, 0])$$

### Rule 365

$$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]} * (a + b*x^n)^{\text{FracPart}[p]}) / (1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(c*x)^{m*(1 + (b*x^n)/a)^p}, x], x] \text{ ; FreeQ}\{a, b, c, m, n, p\}, x\} \&\& \text{!IGtQ}[p, 0] \&\& \text{!(ILtQ}[p, 0] \text{ || GtQ}[a, 0])$$

### Rule 459

$$\text{Int}[(e_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(d*(e*x)^{m+1}*(a + b*x^n)^{p+1}) / (b*e*(m + n*(p + 1) + 1)), x] - \text{Dist}[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1)) / (b*(m + n*(p + 1) + 1)), \text{Int}[(e*x)^m*(a + b*x^n)^p, x], x] \text{ ; FreeQ}\{a, b, c, d, e, m, n, p\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m + n*(p + 1) + 1, 0]$$

### Rule 1267

$$\text{Int}[(f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c^p*(f*x)^{m+4*p-1}*(d + e*x^2)^{(q+1}) / (e*f^{4*p-1}*(m + 4*p + 2*q + 1)), x] + \text{Dist}[1/(e*(m + 4*p + 2*q + 1)), \text{Int}[(f*x)^m*(d + e*x^2)^q * \text{ExpandToSum}[e*(m + 4*p + 2*q + 1)*((a + b*x^2 + c*x^4)^p - c^p*x^{4*p}) - d*c^p*(m + 4*p - 1)*x^{4*p-2}], x], x] \text{ ; FreeQ}\{a, b, c, d, e, f, m, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{!IntegerQ}[q] \&\& \text{NeQ}[m + 4*p + 2*q + 1, 0]$$

### Rule 1809

```

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])

```

### Rule 5238

```

Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rubi steps



$$\begin{aligned}
\int (fx)^m (d + ex^2)^3 (a + b \sec^{-1}(cx)) dx &= \frac{d^3 (fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m} (a + b \sec^{-1}(cx))}{f^3(3+m)} + \frac{3de^2 (fx)^{5+m} (a + b \sec^{-1}(cx))}{f^5(5+m)} \\
&= -\frac{be^3 x (fx)^{5+m} \sqrt{-1 + c^2 x^2}}{c f^5 (6+m)(7+m) \sqrt{c^2 x^2}} + \frac{d^3 (fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m} (a + b \sec^{-1}(cx))}{f^3(3+m)} \\
&= -\frac{be^2 (e(5+m)^2 + 3c^2 d (42 + 13m + m^2)) x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{c^3 f^3 (4+m)(5+m)(6+m)(7+m) \sqrt{c^2 x^2}} - \frac{bd^3 (fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} \\
&= -\frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (5+m)) (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{c^5 f(2+m)(3+m)(4+m)(5+m)} \\
&= -\frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (5+m)) (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{c^5 f(2+m)(3+m)(4+m)(5+m)} \\
&= -\frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (5+m)) (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{c^5 f(2+m)(3+m)(4+m)(5+m)}
\end{aligned}$$

**Mathematica** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2)^3 (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^3\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^3\*(a + b\*ArcSec[c\*x]), x]

**fricas** [F] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^3x^6 + 3ade^2x^4 + 3ad^2ex^2 + ad^3 + (be^3x^6 + 3bde^2x^4 + 3bd^2ex^2 + bd^3) \text{arcsec}(cx)\right) (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



$(m^2 x^2) \sqrt{cx + 1} \sqrt{cx - 1} x^m / (m^4 + 16m^3 - (c^2 m^4 + 16c^2 m^3 + 86c^2 m^2 + 176c^2 m + 105c^2) x^2 + 86m^2 + 176m + 105), x) / (m^4 + 16m^3 + 86m^2 + 176m + 105)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f x)^m (e x^2 + d)^3 \left( a + b \operatorname{acos} \left( \frac{1}{c x} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))), x)`

[Out] `int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(e*x**2+d)**3*(a+b*asec(c*x)), x)`

[Out] Timed out

$$3.162 \quad \int (fx)^m (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$$

**Optimal.** Leaf size=374

$$\frac{d^2(fx)^{m+1} (a + b \sec^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3} (a + b \sec^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5} (a + b \sec^{-1}(cx))}{f^5(m+5)} - \frac{be^2x\sqrt{c^2x^2-1} (fx)^{m+1}}{cf^3(m+4)(m+5)\sqrt{c^2}}$$

[Out]  $d^2*(f*x)^{(1+m)*(a+b*\text{arcsec}(c*x))/f/(1+m)+2*d*e*(f*x)^{(3+m)*(a+b*\text{arcsec}(c*x))}/f^3/(3+m)+e^2*(f*x)^{(5+m)*(a+b*\text{arcsec}(c*x))/f^5/(5+m)-b*(c^4*d^2*(2+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^{(1+m)*\text{hypergeom}([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2*x^2)*(-c^2*x^2+1)^{(1/2)}/c^3/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)-b*e*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^{(1+m)*(c^2*x^2-1)^{(1/2)}/c^3/f/(4+m)/(5+m)/(m^2+5*m+6)/(c^2*x^2)^{(1/2)-b*e^2*x*(f*x)^{(3+m)*(c^2*x^2-1)^{(1/2)}/c/f^3/(4+m)/(5+m)/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.43, antiderivative size = 355, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {270, 5238, 12, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1} (a + b \sec^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3} (a + b \sec^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5} (a + b \sec^{-1}(cx))}{f^5(m+5)} - \frac{bcx\sqrt{1-c^2x^2} (fx)^{m+1}}{f^3(m+4)(m+5)\sqrt{c^2}}$$

Antiderivative was successfully verified.

[In] Int[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out]  $-((b*e*(e*(3+m)^2 + 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^{(1+m)*\text{Sqrt}[-1 + c^2*x^2]})/(c^3*f*(2+m)*(3+m)*(4+m)*(5+m)*\text{Sqrt}[c^2*x^2])) - (b*e^2*x*(f*x)^{(3+m)*\text{Sqrt}[-1 + c^2*x^2]})/(c*f^3*(4+m)*(5+m)*\text{Sqrt}[c^2*x^2]) + (d^2*(f*x)^{(1+m)*(a+b*\text{ArcSec}[c*x])})/(f*(1+m)) + (2*d*e*(f*x)^{(3+m)*(a+b*\text{ArcSec}[c*x])})/(f^3*(3+m)) + (e^2*(f*x)^{(5+m)*(a+b*\text{ArcSec}[c*x])})/(f^5*(5+m)) - (b*c*(d^2/(1+m)^2 + (e*(e*(3+m)^2 + 2*c^2*d*(20 + 9*m + m^2))))/(c^4*(2+m)*(3+m)*(4+m)*(5+m))*x*(f*x)^{(1+m)*\text{Sqrt}[1 - c^2*x^2]*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2]})/(f*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2])$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 270**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rule 365

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 1267

Int[((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Simp[(c^p\*(f\*x)^(m + 4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*f^(4\*p - 1)\*(m + 4\*p + 2\*q + 1)), x] + Dist[1/(e\*(m + 4\*p + 2\*q + 1)), Int[(f\*x)^m\*(d + e\*x^2)^q\*ExpandToSum[e\*(m + 4\*p + 2\*q + 1)\*((a + b\*x^2 + c\*x^4)^p - c^p\*x^(4\*p)) - d\*c^p\*(m + 4\*p - 1)\*x^(4\*p - 2), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4\*p + 2\*q + 1, 0]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2

\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
 \int (fx)^m (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx &= \frac{d^2 (fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{2de(fx)^{3+m} (a + b \sec^{-1}(cx))}{f^3(3+m)} + \frac{e^2 (fx)^5}{f^5} \\
 &= \frac{d^2 (fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{2de(fx)^{3+m} (a + b \sec^{-1}(cx))}{f^3(3+m)} + \frac{e^2 (fx)^5}{f^5} \\
 &= -\frac{be^2 x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{cf^3(4+m)(5+m)\sqrt{c^2 x^2}} + \frac{d^2 (fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{2de(fx)^3}{f^3} \\
 &= -\frac{be(e(3+m)^2 + 2c^2 d(20 + 9m + m^2)) x (fx)^{1+m} \sqrt{-1 + c^2 x^2}}{c^3 f(2+m)(4+m)(15 + 8m + m^2) \sqrt{c^2 x^2}} - \frac{be^2 x}{cf^3(4+m)} \\
 &= -\frac{be(e(3+m)^2 + 2c^2 d(20 + 9m + m^2)) x (fx)^{1+m} \sqrt{-1 + c^2 x^2}}{c^3 f(2+m)(4+m)(15 + 8m + m^2) \sqrt{c^2 x^2}} - \frac{be^2 x}{cf^3(4+m)} \\
 &= -\frac{be(e(3+m)^2 + 2c^2 d(20 + 9m + m^2)) x (fx)^{1+m} \sqrt{-1 + c^2 x^2}}{c^3 f(2+m)(4+m)(15 + 8m + m^2) \sqrt{c^2 x^2}} - \frac{be^2 x}{cf^3(4+m)}
 \end{aligned}$$

**Mathematica** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcSec[c\*x]), x]

**fricas** [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \text{arcsec}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arcsec(c\*x))\*(f\*x)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^2 (b \operatorname{arcsec}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

**maple** [F] time = 7.52, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d)^2 (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^2 f^m x^5 x^m}{m+5} + \frac{2ade f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} ad^2}{f(m+1)} + \frac{((be^2 f^m m^2 + 4be^2 f^m m + 3be^2 f^m)x^5 + 2(bdef^m m^2 + 6bdef^m m + 5))}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] a\*e^2\*f^m\*x^5\*x^m/(m + 5) + 2\*a\*d\*e\*f^m\*x^3\*x^m/(m + 3) + (f\*x)^(m + 1)\*a\*d^2/(f\*(m + 1)) + (((b\*e^2\*f^m\*m^2 + 4\*b\*e^2\*f^m\*m + 3\*b\*e^2\*f^m)\*x^5 + 2\*(b\*d\*e\*f^m\*m^2 + 6\*b\*d\*e\*f^m\*m + 5\*b\*d\*e\*f^m)\*x^3 + (b\*d^2\*f^m\*m^2 + 8\*b\*d^2\*f^m\*m + 15\*b\*d^2\*f^m)\*x)\*x^m\*arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - (m^3 + 9\*m^2 + 23\*m + 15)\*integrate(-(b\*d^2\*f^m\*m^2 + 8\*b\*d^2\*f^m\*m + (b\*e^2\*f^m\*m^2 + 4\*b\*e^2\*f^m\*m + 3\*b\*e^2\*f^m)\*x^4 + 15\*b\*d^2\*f^m + 2\*(b\*d\*e\*f^m\*m^2 + 6\*b\*d\*e\*f^m\*m + 5\*b\*d\*e\*f^m)\*x^2)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*x^m/(m^3 - (c^2\*m^3 + 9\*c^2\*m^2 + 23\*c^2\*m + 15\*c^2)\*x^2 + 9\*m^2 + 23\*m + 15), x))/(m^3 + 9\*m^2 + 23\*m + 15)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (fx)^m (ex^2 + d)^2 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)`

[Out] `int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{asec}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(e*x**2+d)**2*(a+b*asec(c*x)), x)`

[Out] `Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2)**2, x)`



### 3.163 $\int (fx)^m (d + ex^2) (a + b \sec^{-1}(cx)) dx$

**Optimal.** Leaf size=178

$$\frac{dx^{m+1} (a + b \sec^{-1}(cx))}{m+1} + \frac{ex^{m+3} (a + b \sec^{-1}(cx))}{m+3} + \frac{b\sqrt{c^2x^2-1}x^{m+2} (c^2d(m+2)(m+3) + e(m+1)^2)}{c(m+1)^2(m+2)(m+3)\sqrt{c^2x^2}} {}_2F_1\left(1, \frac{m+1}{2}\right)$$

[Out]  $d*x^{(1+m)}*(a+b*\text{arcsec}(c*x))/(1+m)+e*x^{(3+m)}*(a+b*\text{arcsec}(c*x))/(3+m)-b*e*x^{(2+m)}*(c^2*x^2-1)^{(1/2)}/c/(m^2+5*m+6)/(c^2*x^2)^{(1/2)}+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*x^{(2+m)}*\text{hypergeom}([1, 1+1/2*m], [3/2+1/2*m], c^2*x^2)*(c^2*x^2-1)^{(1/2)}/c/(1+m)^2/(2+m)/(3+m)/(c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {14, 5238, 12, 459, 365, 364}

$$\frac{d(fx)^{m+1} (a + b \sec^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3} (a + b \sec^{-1}(cx))}{f^3(m+3)} - \frac{bcx\sqrt{1-c^2x^2} (fx)^{m+1} \left(\frac{e}{c^2(m+2)(m+3)} + \frac{d}{(m+1)^2}\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}\right)$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[(f*x)^m*(d + e*x^2)*(a + b*\text{ArcSec}[c*x]), x]$

[Out]  $-((b*e*x*(f*x)^{(1+m)}*\text{Sqrt}[-1 + c^2*x^2])/(c*f*(6 + 5*m + m^2)*\text{Sqrt}[c^2*x^2-1]) + (d*(f*x)^{(1+m)}*(a + b*\text{ArcSec}[c*x]))/(f*(1+m)) + (e*(f*x)^{(3+m)}*(a + b*\text{ArcSec}[c*x]))/(f^3*(3+m)) - (b*c*(d/(1+m)^2 + e/(c^2*(2+m)*(3+m))))*x*(f*x)^{(1+m)}*\text{Sqrt}[1 - c^2*x^2]*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/(f*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2])$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x \&\& \text{SumQ}[u] \&\& \text{!LinearQ}[u, x] \&\& \text{!MatchQ}[u, (a_)+(b_)*(v_)] /; \text{FreeQ}\{a, b\}, x \&\& \text{InverseFunctionQ}[v]$

#### Rule 364

$\text{Int}[(c_)*(x_))^{(m_)*((a_)+(b_)*(x_)^{(n_))^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a$

)]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rule 365

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 5238

Int[((a\_.) + ArcSec[(c\_.)\*(x\_)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSec[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2) (a + b \sec^{-1}(cx)) dx &= \frac{d(fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \sec^{-1}(cx))}{f^3(3+m)} - \frac{(bcx) \int \frac{(fx)^m}{f} dx}{(3+4)} \\
&= \frac{d(fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \sec^{-1}(cx))}{f^3(3+m)} - \frac{(bcx) \int \frac{(fx)^m}{f} dx}{(3+4)} \\
&= -\frac{bex(fx)^{1+m} \sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m}}{f^3} \\
&= -\frac{bex(fx)^{1+m} \sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m}}{f^3} \\
&= -\frac{bex(fx)^{1+m} \sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m} (a + b \sec^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m}}{f^3}
\end{aligned}$$

**Mathematica** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)\*(a + b\*ArcSec[c\*x]), x]

**fricas** [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)\right) (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*(f\*x)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \text{arcsec}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

maple [F] time = 6.10, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d) (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (fx)^m (ex^2 + d) \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int((f\*x)^m\*(d + e\*x^2)\*(a + b\*acos(1/(c\*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{asec}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*(a+b\*asec(c\*x)),x)

[Out] Integral((f\*x)\*\*m\*(a + b\*asec(c\*x))\*(d + e\*x\*\*2), x)

$$3.164 \quad \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{d + ex^2} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(fx)^m (a + b \sec^{-1}(cx))}{d + ex^2}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d), x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

[Out] Defer[Int](((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x)

Rubi steps

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{d + ex^2} dx = \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{d + ex^2} dx$$

Mathematica [A] time = 2.04, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2), x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b \operatorname{arcsec}(cx) + a) (fx)^m}{ex^2 + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

**maple** [A] time = 3.14, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsec}(cx))}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x)

[Out] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f\*x)^m\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2),x)

[Out] int(((f\*x)^m\*(a + b\*acos(1/(c\*x))))/(d + e\*x^2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{asec}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*asec(c\*x))/(e\*x\*\*2+d), x)

[Out] Integral((f\*x)\*\*m\*(a + b\*asec(c\*x))/(d + e\*x\*\*2), x)

$$3.165 \quad \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^2}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] Defer[Int] [((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2, x]

Rubi steps

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Mathematica [A] time = 5.39, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2,x]

[Out] Integrate [((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^2, x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b \operatorname{arcsec}(cx) + a) (fx)^m}{e^2 x^4 + 2 d e x^2 + d^2}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^2, x)

**maple** [A] time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

[Out] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

$$3.166 \quad \int (fx)^m (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Optimal. Leaf size=28

$$\text{Int}\left((d + ex^2)^{3/2} (fx)^m (a + b \sec^{-1}(cx)), x\right)$$

[Out] Unintegrable((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int] [(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx = \int (fx)^m (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Mathematica [A] time = 0.90, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcSec[c\*x]), x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \text{arcsec}(cx)\right) \sqrt{ex^2 + d} (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arcsec(c\*x))\*sqrt(e\*x^2 + d)\*(f\*x)^m, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

**maple** [A] time = 2.84, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (fx)^m (ex^2 + d)^{3/2} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int((f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*acos(1/(c\*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*asec(c\*x)),x)

[Out] Timed out

$$3.167 \quad \int (fx)^m \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt{d + ex^2} (fx)^m (a + b \sec^{-1}(cx)), x\right)$$

[Out] Unintegrable((f\*x)^m\*(e\*x^2+d)^(1/2)\*(a+b\*arcsec(c\*x)), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (fx)^m \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

[Out] Defer[Int] [(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

Rubi steps

$$\int (fx)^m \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx = \int (fx)^m \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Mathematica [A] time = 0.11, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{d + ex^2} (a + b \sec^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

[Out] Integrate[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcSec[c\*x]), x]

fricas [A] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex^2 + d} (b \text{arcsec}(cx) + a) (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(1/2)\*(a+b\*arcsec(c\*x)), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(1/2)\*(a+b\*arcsec(c\*x)),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

**maple** [A] time = 2.80, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{ex^2 + d} (a + b \operatorname{arcsec}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^(1/2)\*(a+b\*arcsec(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)^(1/2)\*(a+b\*arcsec(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(1/2)\*(a+b\*arcsec(c\*x)),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (fx)^m \sqrt{ex^2 + d} \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))),x)

[Out] int((f\*x)^m\*(d + e\*x^2)^(1/2)\*(a + b\*acos(1/(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{asec}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```



$$3.168 \quad \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Optimal. Leaf size=28

$$\text{Int} \left( \frac{(fx)^m (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Defer[Int](((f\*x)^m\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x)

Rubi steps

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx = \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Mathematica [A] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate(((f\*x)^m\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x)

[Out] Integrate(((f\*x)^m\*(a + b\*ArcSec[c\*x]))/Sqrt[d + e\*x^2], x)

fricas [A] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b \operatorname{arcsec}(cx) + a) (fx)^m}{\sqrt{ex^2 + d}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*arcsec(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

**maple** [A] time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsec}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

[Out] `int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{asec}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(1/2), x)`

[Out] `Integral((f*x)**m*(a + b*asec(c*x))/sqrt(d + e*x**2), x)`

$$3.169 \quad \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=28

$$\text{Int} \left( \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int] [((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \sec^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate [((f\*x)^m\*(a + b\*ArcSec[c\*x]))/(d + e\*x^2)^(3/2), x]

fricas [A] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsec}(cx) + a) (fx)^m}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arcsec(c\*x) + a)\*(f\*x)^m/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^(3/2), x)

**maple** [A] time = 3.79, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsec}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arcsec(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] integrate((b\*arcsec(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^(3/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{acos}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

```
[Out] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(3/2), x)
```

```
[Out] Timed out
```

$$3.170 \quad \int \frac{x^{11}(a+b \sec^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$$

Optimal. Leaf size=401

$$\frac{(1-c^4x^4)^{5/2}(a+b \sec^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b \sec^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b \sec^{-1}(cx))}{2c^{12}} + \frac{b\sqrt{1-c^2x^2}(c^2x)}{90c^{13}x\sqrt{1-c^2x^2}}$$

[Out]  $1/3*(-c^4*x^4+1)^{(3/2)}*(a+b*\operatorname{arcsec}(c*x))/c^{12}-1/10*(-c^4*x^4+1)^{(5/2)}*(a+b*\operatorname{arcsec}(c*x))/c^{12}-7/90*b*(c^2*x^2+1)^{(3/2)}*(-c^2*x^2+1)^{(1/2)}/c^{13}/x/(1-1/c^2/x^2)^{(1/2)}+13/150*b*(c^2*x^2+1)^{(5/2)}*(-c^2*x^2+1)^{(1/2)}/c^{13}/x/(1-1/c^2/x^2)^{(1/2)}-3/70*b*(c^2*x^2+1)^{(7/2)}*(-c^2*x^2+1)^{(1/2)}/c^{13}/x/(1-1/c^2/x^2)^{(1/2)}+1/90*b*(c^2*x^2+1)^{(9/2)}*(-c^2*x^2+1)^{(1/2)}/c^{13}/x/(1-1/c^2/x^2)^{(1/2)}-4/15*b*\operatorname{arctanh}((c^2*x^2+1)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}/c^{13}/x/(1-1/c^2/x^2)^{(1/2)}+4/15*b*(-c^2*x^2+1)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/c^{13}/x/(1-1/c^2/x^2)^{(1/2)}-1/2*(a+b*\operatorname{arcsec}(c*x))*(-c^4*x^4+1)^{(1/2)}/c^{12}$

**Rubi [A]** time = 2.48, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {266, 43, 5246, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{(1-c^4x^4)^{5/2}(a+b \sec^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b \sec^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b \sec^{-1}(cx))}{2c^{12}} + \frac{b\sqrt{1-c^2x^2}(c^2x)}{90c^{13}x\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^11\*(a + b\*ArcSec[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out]  $(4*b*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{Sqrt}[1+c^2*x^2])/(15*c^{13}*\operatorname{Sqrt}[1-1/(c^2*x^2)])*x - (7*b*\operatorname{Sqrt}[1-c^2*x^2]*(1+c^2*x^2)^{(3/2)})/(90*c^{13}*\operatorname{Sqrt}[1-1/(c^2*x^2)])*x + (13*b*\operatorname{Sqrt}[1-c^2*x^2]*(1+c^2*x^2)^{(5/2)})/(150*c^{13}*\operatorname{Sqrt}[1-1/(c^2*x^2)])*x - (3*b*\operatorname{Sqrt}[1-c^2*x^2]*(1+c^2*x^2)^{(7/2)})/(70*c^{13}*\operatorname{Sqrt}[1-1/(c^2*x^2)])*x + (b*\operatorname{Sqrt}[1-c^2*x^2]*(1+c^2*x^2)^{(9/2)})/(90*c^{13}*\operatorname{Sqrt}[1-1/(c^2*x^2)])*x - (\operatorname{Sqrt}[1-c^4*x^4]*(a+b*\operatorname{ArcSec}[c*x]))/(2*c^{12}) + ((1-c^4*x^4)^{(3/2)}*(a+b*\operatorname{ArcSec}[c*x]))/(3*c^{12}) - ((1-c^4*x^4)^{(5/2)}*(a+b*\operatorname{ArcSec}[c*x]))/(10*c^{12}) - (4*b*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1+c^2*x^2]])/(15*c^{13}*\operatorname{Sqrt}[1-1/(c^2*x^2)])*x$

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 783

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_
.), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_)^2
```



```
)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

### Rule 5246

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)])], x], x], x] /; InverseFunctionFreeQ[v, x] /; F
reeQ[{a, b, c}, x]
```

### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && !LtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (a + b \sec^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \sec^{-1}(cx))}{10c^{12}} \\
&= \frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{3c^{12}} \\
&= \frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{7b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{90c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{13b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{150c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{7b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{90c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{13b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{150c^{13} \sqrt{1 - \frac{1}{c^2 x^2}} x}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 194, normalized size = 0.48

$$\frac{-105a\sqrt{1-c^4x^4}(3c^8x^8+4c^4x^4+8)-105b\sqrt{1-c^4x^4}(3c^8x^8+4c^4x^4+8)\sec^{-1}(cx)+840b\tan^{-1}\left(\frac{cx\sqrt{1-\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\right)}{3150c^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11\*(a + b\*ArcSec[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out]  $(-105*a*\text{Sqrt}[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 + 35*c^8*x^8)))/(-1 + c^2*x^2) - 105*b*\text{Sqrt}[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8)*\text{ArcSec}[c*x] + 840*b*\text{ArcTan}[(c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/\text{Sqrt}[1 - c^4*x^4]]/(3150*c^{12})$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(a+b\*arcsec(c\*x))/(-c^4\*x^4+1)^(1/2), x, algorithm="fricas")

[Out] Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(a+b\*arcsec(c\*x))/(-c^4\*x^4+1)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [F]** time = 10.87, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (a + b \operatorname{arcsec}(cx))}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)`

[Out] `int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)`

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11} \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{\sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)`

[Out] `int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)`

[Out] Timed out

$$3.171 \quad \int \frac{x^7 (a + b \sec^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

**Optimal.** Leaf size=268

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{5/2}}{30c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^3}{18c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}}$$

[Out]  $1/6*(-c^4*x^4+1)^{(3/2)}*(a+b*\text{arcsec}(c*x))/c^8-1/18*b*(c^2*x^2+1)^{(3/2)}*(-c^2*x^2+1)^{(1/2)}/c^9/x/(1-1/c^2/x^2)^{(1/2)}+1/30*b*(c^2*x^2+1)^{(5/2)}*(-c^2*x^2+1)^{(1/2)}/c^9/x/(1-1/c^2/x^2)^{(1/2)}-1/3*b*\text{arctanh}((c^2*x^2+1)^{(1/2)})*(-c^2*x^2+1)^{(1/2)}/c^9/x/(1-1/c^2/x^2)^{(1/2)}+1/3*b*(-c^2*x^2+1)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/c^9/x/(1-1/c^2/x^2)^{(1/2)}-1/2*(a+b*\text{arcsec}(c*x))*(-c^4*x^4+1)^{(1/2)}/c^8$

**Rubi [A]** time = 2.02, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {266, 43, 5246, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{5/2}}{30c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^3}{18c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^7*(a + b*\text{ArcSec}[c*x]))/\text{Sqrt}[1 - c^4*x^4], x]$

[Out]  $(b*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + c^2*x^2])/(3*c^9*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) - (b*\text{Sqrt}[1 - c^2*x^2]*(1 + c^2*x^2)^{(3/2)})/(18*c^9*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) + (b*\text{Sqrt}[1 - c^2*x^2]*(1 + c^2*x^2)^{(5/2)})/(30*c^9*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) - (\text{Sqrt}[1 - c^4*x^4]*(a + b*\text{ArcSec}[c*x]))/(2*c^8) + ((1 - c^4*x^4)^{(3/2)}*(a + b*\text{ArcSec}[c*x]))/(6*c^8) - (b*\text{Sqrt}[1 - c^2*x^2]*\text{ArcTanh}[\text{Sqrt}[1 + c^2*x^2]])/(3*c^9*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)$

**Rule 12**

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

**Rule 43**

$\text{Int}[(a_*)(u_*) + (b_*)(x_*)^m*((c_*) + (d_*)(x_*)^n), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{Le}$

$Q[7*m + 4*n + 4, 0] \parallel LtQ[9*m + 5*(n + 1), 0] \parallel GtQ[m + n + 2, 0]$

### Rule 50

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] \rightarrow Simp[$   
 $((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/$   
 $(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] \rightarrow With[$   
 $\{p = Denominator[m]\}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +$   
 $(d*x^p)/b]^n, x], x, (a + b*x)^(1/p)], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

$Int[((a_) + (b_.)*(x_)^2)^(-1), x\_Symbol] \rightarrow Simp[(Rt[-(a/b), 2]*ArcTanh[x/$   
 $Rt[-(a/b), 2]])/a, x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 266

$Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x\_Symbol] \rightarrow Dist[1/n, Subst[$   
 $Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /;$  FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 783

$Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_$   
 $.), x\_Symbol] \rightarrow Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /;$  FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

### Rule 848

$Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2$   
 $)^(p_.), x\_Symbol] \rightarrow Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,$   
 $x] /;$  FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e\*f - d\*g, 0] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 5246

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7 (a + b \sec^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} - \frac{b \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{6c^8 \sqrt{1 - \frac{1}{c^2 x^2}} x^2}}{c} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} - \frac{b \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2}}{6c^9} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2}}{6c^9 \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Sub}}{12} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Sub}}{6c^9} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Sub}}{6c^9} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Sub}}{6c^9} \\
&= \frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \sec^{-1}(cx))}{6c^8} \\
&= \frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{18c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{30c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8} \\
&= \frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{18c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{30c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^8}
\end{aligned}$$



**Mathematica [A]** time = 0.33, size = 159, normalized size = 0.59

$$\frac{-15a\sqrt{1-c^4x^4}(c^4x^4+2) - 15b\sqrt{1-c^4x^4}(c^4x^4+2)\sec^{-1}(cx) + \frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{1-c^4x^4}(3c^4x^4+c^2x^2+28)}{c^2x^2-1} + 30b\tan^{-1}\left(\frac{cx\sqrt{1-\frac{1}{c^2x^2}}}{c^2x^2-1}\right)}{90c^8}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(a + b\*ArcSec[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] (-15\*a\*Sqrt[1 - c^4\*x^4]\*(2 + c^4\*x^4) + (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^4\*x^4]\*(28 + c^2\*x^2 + 3\*c^4\*x^4))/(-1 + c^2\*x^2) - 15\*b\*Sqrt[1 - c^4\*x^4]\*(2 + c^4\*x^4)\*ArcSec[c\*x] + 30\*b\*ArcTan[(c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/Sqrt[1 - c^4\*x^4]])/(90\*c^8)

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arcsec(c\*x))/(-c^4\*x^4+1)^(1/2), x, algorithm="fricas")

[Out] Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arcsec(c\*x))/(-c^4\*x^4+1)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple [F]** time = 7.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (a + b \operatorname{arcsec}(cx))}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(a+b\*arcsec(c\*x))/(-c^4\*x^4+1)^(1/2), x)

[Out] `int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)`

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 \left( a + b \operatorname{acos} \left( \frac{1}{cx} \right) \right)}{\sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)`

[Out] `int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)`

[Out] Timed out

$$3.172 \quad \int \frac{x^3(a+b \sec^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$$

Optimal. Leaf size=126

$$-\frac{\sqrt{1-c^4x^4}(a+b \sec^{-1}(cx))}{2c^4} + \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}} - \frac{bx \tan^{-1}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{c^2x^2-1}}\right)}{2c^3\sqrt{c^2x^2}}$$

[Out]  $-1/2*b*x*\arctan((-c^4*x^4+1)^{(1/2)/(c^2*x^2-1)^{(1/2)})/c^3/(c^2*x^2)^{(1/2)}-1/2*(a+b*\operatorname{arcsec}(c*x))*(-c^4*x^4+1)^{(1/2)}/c^4+1/2*b*x*(-c^4*x^4+1)^{(1/2)}/c^3/(c^2*x^2)^{(1/2)}/(c^2*x^2-1)^{(1/2)}$

**Rubi [A]** time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {261, 5246, 12, 1572, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1-c^4x^4}(a+b \sec^{-1}(cx))}{2c^4} + \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{b\sqrt{1-c^2x^2} \tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcSec[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out]  $(b*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[1 + c^2*x^2])/(2*c^5*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x) - (\operatorname{Sqrt}[1 - c^4*x^4]*(a + b*\operatorname{ArcSec}[c*x]))/(2*c^4) - (b*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + c^2*x^2]])/(2*c^5*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x)$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

### Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

### Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

### Rule 1572

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] := Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*Frac
Part[q]))*(1 + d/(x^mn*e))^FracPart[q]), Int[x^(m + mn*q)*(1 + d/(x^mn*e))^q
*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && EqQ[n2, -
2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]
```

### Rule 5246

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \sec^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} - \frac{b \int -\frac{\sqrt{1 - c^4 x^4}}{2c^4 \sqrt{1 - \frac{1}{c^2 x^2} x^2}} dx}{c} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} + \frac{b \int \frac{\sqrt{1 - c^4 x^4}}{\sqrt{1 - \frac{1}{c^2 x^2} x^2}} dx}{2c^5} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{\sqrt{1 - c^4 x^4}}{x\sqrt{1 - c^2 x^2}} dx}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{\sqrt{1 - c^4 x^2}}{x\sqrt{1 - c^2 x}} dx, x, x^2 \right)}{4c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{\sqrt{1 + c^2 x}}{x} dx, x, x^2 \right)}{4c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= \frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{1}{x} dx, x, x^2 \right)}{4c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= \frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{1}{x} dx, x, x^2 \right)}{2c^7 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= \frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} - \frac{\sqrt{1 - c^4 x^4} (a + b \sec^{-1}(cx))}{2c^4} - \frac{b\sqrt{1 - c^2 x^2} \tanh^{-1} \left( \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 118, normalized size = 0.94

$$\frac{\sqrt{1 - c^4 x^4} \left( -ac^2 x^2 + a + bcx \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{c^2 x^2 - 1} - b\sqrt{1 - c^4 x^4} \sec^{-1}(cx) + b \tan^{-1} \left( \frac{cx \sqrt{1 - \frac{1}{c^2 x^2}}}{\sqrt{1 - c^4 x^4}} \right)$$


---

2c<sup>4</sup>

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4], x]
```

```
[Out] (((a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4])/(-1 + c^2*x^2) - b*Sqrt[1 - c^4*x^4]*ArcSec[c*x] + b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2), x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError >> the translation of the FriCAS object sage2 to sage is not yet implemented
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsec}(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2), x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

**maple** [F] time = 4.45, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsec}(cx))}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2), x)
```

```
[Out] int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2), x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2), x, algorithm="maxima")
```

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \arccos\left(\frac{1}{cx}\right) \right)}{\sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)`

[Out] `int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{asec}(cx))}{\sqrt{-(cx - 1)(cx + 1)(c^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2), x)`

[Out] `Integral(x**3*(a + b*asec(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)), x)`

$$3.173 \quad \int \frac{a+b \sec^{-1}(cx)}{x \sqrt{1-c^4x^4}} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{a + b \sec^{-1}(cx)}{x \sqrt{1 - c^4x^4}}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x/(-c^4\*x^4+1)^(1/2), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{x \sqrt{1 - c^4x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{x \sqrt{1 - c^4x^4}} dx = \int \frac{a + b \sec^{-1}(cx)}{x \sqrt{1 - c^4x^4}} dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x \sqrt{1 - c^4x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

fricas [A] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{\sqrt{-c^4x^4 + 1} (b \operatorname{arcsec}(cx) + a)}{c^4x^5 - x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*arcsec(c\*x))/x/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4\*x^4 + 1)\*(b\*arcsec(c\*x) + a)/(c^4\*x^5 - x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{-c^4 x^4 + 1} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(-c^4\*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(sqrt(-c^4\*x^4 + 1)\*x), x)

**maple** [A] time = 4.08, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x \sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x/(-c^4\*x^4+1)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))/x/(-c^4\*x^4+1)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4} a \left( \log \left( \sqrt{-c^4 x^4 + 1} + 1 \right) - \log \left( \sqrt{-c^4 x^4 + 1} - 1 \right) \right) + b \int \frac{\arctan \left( \frac{\sqrt{cx + 1} \sqrt{cx - 1}}{\sqrt{c^2 x^2 + 1}} \right)}{\sqrt{cx + 1} \sqrt{-cx + 1} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/4\*a\*(log(sqrt(-c^4\*x^4 + 1) + 1) - log(sqrt(-c^4\*x^4 + 1) - 1)) + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{acos} \left( \frac{1}{cx} \right)}{x \sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x\*(1 - c^4\*x^4)^(1/2)),x)

[Out] `int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x \sqrt{-(cx - 1)(cx + 1)(c^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/x/(-c**4*x**4+1)**(1/2), x)`

[Out] `Integral((a + b*asec(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))), x)`

$$3.174 \quad \int \frac{a+b \sec^{-1}(cx)}{x^5 \sqrt{1-c^4x^4}} dx$$

Optimal. Leaf size=29

$$\text{Int} \left( \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4x^4}}, x \right)$$

[Out] Unintegrable((a+b\*arcsec(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcSec[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

[Out] Defer[Int] [(a + b\*ArcSec[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

Rubi steps

$$\int \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4x^4}} dx = \int \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4x^4}} dx$$

Mathematica [A] time = 6.55, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcSec[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

[Out] Integrate[(a + b\*ArcSec[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

fricas [A] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{\sqrt{-c^4x^4 + 1} (b \operatorname{arcsec}(cx) + a)}{c^4x^9 - x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4\*x^4 + 1)\*(b\*arcsec(c\*x) + a)/(c^4\*x^9 - x^5), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{-c^4 x^4 + 1} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arcsec(c\*x) + a)/(sqrt(-c^4\*x^4 + 1)\*x^5), x)

**maple** [A] time = 4.77, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^5 \sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsec(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x)

[Out] int((a+b\*arcsec(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8} \left( c^4 \log(\sqrt{-c^4 x^4 + 1} + 1) - c^4 \log(\sqrt{-c^4 x^4 + 1} - 1) + \frac{2\sqrt{-c^4 x^4 + 1}}{x^4} \right) a + b \int \frac{\arctan(\sqrt{cx+1}\sqrt{cx-1})}{\sqrt{c^2 x^2 + 1} \sqrt{cx+1} \sqrt{-cx+1} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsec(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/8\*(c^4\*log(sqrt(-c^4\*x^4 + 1) + 1) - c^4\*log(sqrt(-c^4\*x^4 + 1) - 1) + 2\*sqrt(-c^4\*x^4 + 1)/x^4)\*a + b\*integrate(arctan(sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^5), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acos(1/(c\*x)))/(x^5\*(1 - c^4\*x^4)^(1/2)),x)

[Out] `int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asec}(cx)}{x^5 \sqrt{-(cx-1)(cx+1)(c^2x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asec(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)`

[Out] `Integral((a + b*asec(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))), x)`



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
  If[Head[expn]===Plus || Head[expn]===Times,
```

```
    Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
  If[ElementaryFunctionQ[Head[expn]],
```

```
    Max[3,ExpnType[expn[[1]]],
```

```
  If[SpecialFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```



```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```



## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```